12.800: Fall 2004 Problem set 3

1) A linear uncertainty propagator is given by:

$$\underline{\underline{M}} = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

Find the perpendicular directions at initial time (the initial singular vectors), that evolve into the perpendicular directions at final time that have experienced the most growth (the final time singular vectors), and give the factors by which the vectors have grown or shrunk (the singular values). Sketch the orientation of the initial and final axes. Scale the final axes by the singular values. Perform your eigen analysis by hand (you can check your answers using Matlab (or equivalent) if you like).

2) Show that the vorticity field for <u>any</u> flow satisfies

$$\nabla \cdot \underline{\omega} = 0$$

3) Explain (in words) how $\iiint_V \underline{u} \cdot \nabla c + c \nabla \cdot \underline{u} \, dV$ is equivalent to $\iint_A c\underline{u} \cdot d\underline{A}$.

4) Let a 1d velocity field be $u = u(x,t)$, with $v = 0$ and $w = 0$. The density varies as $\rho = \rho_0 (2 - \cos \omega t)$. Find and expression for $u(x,t)$ if $u(0,t) = U$.
[KC04 Ex. 4.1]

5) Derive the full continuity equation by starting from the integral form of conservation of mass for a fixed Lagrangian volume. Carefully explain each step.
[KC04 Ex 4.2]

6) Derive a general form of conservation of momentum (KC04 Eq 4.15) by starting from a Lagrangian expression of $F = ma$ (KC04 Eq 4.22) Carefully explain each step.
[Variant of KC04 Ex 4.4]