12.804 — One Layer Inversion/ Rossby Waves — Num. Expts.

Model

$$\frac{\partial}{\partial t}q' + J(\psi, q') + \beta\frac{\partial}{\partial x}\psi = filtering \tag{1}$$

$$q' = \nabla^2\psi - F_1\psi \tag{2}$$

The filtering is required for numerical stability and is intended to remove small scale enstrophy without dissipating too much energy — a typical numerical trick. The parameter F_1 is zero for the barotropic vorticity equation and non–zero for the shallow water QG system. The model works on a doubly periodic domain with $-pi \le x < \pi$, $-\pi \le y < \pi$. It is a pseudospectral model, meaning the inversion of (2) is done by working in Fourier space, as is the evaluation of the frictional term, the β term, and the time–stepping. The advection term is rewritten as

$$\nabla \cdot (\vec{u}q')$$

and is solved by evaluating the transform of the velocities [multiplying the transform of ψ by $(-\imath\ell,\ \imath k)$], transforming them to real space and multiplying by the real space value of q'. The result is transformed back to wavenumber space and dotted with $(\imath k,\ \imath\ell)$.

PV inversion

The interface allows you to specify the q' values and determine the resulting ψ fields. For the inversion, you specify the parameters F_1 and β. You express the PV anomaly q' as functions of the x and y matrices and then the program will calculate ψ and contour both the PV anomaly q' and the full PV field $q' + \beta y$. For example, consider

```
q=3*cos(2*x-3*y)+0.01*sin(x)
```

The program can also be run in the opposite direction: specifying ψ and calculating q, e.g.

```
psi=1.5*exp(-2*x.^2-2*y.^2);
```

In addition you can give values to the parameters β and F_1 $(= r_d^{-2})$.

PV evolution

Once you have specified the PV and/or streamfunction field, use the link appropriate to your machine to see how the flow evolves. You can alter the time step $\Delta t = 1/$`nsteps`, the contour interval `ci` for the display, and the total time length for the run `timelength`. The contours will be shown every `nsnap` time steps.

When you're finished be sure to hit the Quit button before closing the browser.

Things you can try

1) Verify the dispersion relation with and without F_1. Look at how waves with their crests at an angle propagate. Remember — initial conditions should be periodic in x and y with period 2π.
2) Consider how superpositions of two waves propagate. When does nonlinearity enter?
3) Look at the dispersion of one– and two–dimensional isolated features. How can you understand the latter? How does nonlinearity affect it?
4) What happens to an elliptical eddy on an f–plane? A β–plane?

```
psi=exp(-2*x.^2-4*y.^2);
f1=0;
beta=0 to 0.5;
```

5) Look at the stability of a Rossby wave

```
psi=0.2*cos(2*x-3*y)+0.01*sin(x);
timelength=30;
f1=0.0;
beta=1;
```

6) Look at the stability of a vortex
7) Examine the propagation of dipoles with and without β.