

# Chapter 2

## Simple energy balance climate models

### Supplemental reading:<sup>1</sup>

Budyko (1969)

Held and Suarez (1974)

Lindzen and Farrell (1977)

North (1975)

Sellers (1969)

My initial purpose in beginning with an energy balance climate model was to provide a simple model wherein the interactions of radiation (including the effect of snow on albedo) and dynamic heat transport could be considered. The above references mostly date back to the period where there was concern over global cooling and the possibility of a new ice age. In the above references some attempt is made to justify the realism of the models. This is certainly worth thinking about; however, as we will see, most of the attempts amounted to little more than special pleading. Given the current popularity of global warming rather than global cooling, one might have thought that

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<sup>1</sup>A complete list of references is given at the end of this book. Those references that are particularly useful to a given chapter are listed at the beginning of that chapter. Sometimes specific pages and/or chapters will be noted.

this model would have been largely forgotten. However, it has returned to some popularity in connection with the debate over whether there were periods when the earth was totally ice covered (the so-called snowball earth). In this chapter, we will not discuss such matters in detail, since we are primarily concerned with the illustrative aspects of this model rather than detailed realism.

As far as this chapter is concerned, there are two major points to this exercise:

1. To ‘invent’ a model which illustrates the impact of dynamic transport on climate, and
2. To consider the impact of seemingly innocent modeling assumptions.

This inevitably leads to a consideration of broader issues concerned with modeling, and, in particular, with the profound difference between modeling and approximation. Over the years, I have noticed that students often respond to this exercise by attaching undue credibility to this model, and quickly fall into the trap of discussing the model output in terms of nature. If such a trivial model can provoke such a response, there can be little surprise at the temptation to treat more complex models in a similar manner.

The specific models that we will work with are typically characterized as follows:

1. Only latitude dependences are considered; that is, the models are spatially one-dimensional (though time dependence is also sometimes considered).
2. Global energy budgets are assumed to be expressible in terms of surface temperatures.
3. Planetary albedo is taken to depend primarily on ice and/or snow cover or the lack thereof.
4. The convergence of dynamic heat fluxes is generally represented by either a simple diffusion law or by a linear heating law wherein local heating is proportional to deviations of the global mean temperature from the local surface temperature. *The primary feature of the heat transport is that it carries heat from warmer to colder regions.* Both of the above representations do this.

5. Generally, only annual mean conditions are considered, although sometimes annual mean conditions are replaced by equinoctial conditions.

(It is worth pointing out, in passing, that all of the above modeling assumptions are actually incorrect.) The starting point for such models is an equation of the form

$$C \frac{\partial T(x, t)}{\partial t} = \begin{aligned} &\text{incoming solar radiation} \\ &\text{–infrared cooling} \\ &\text{–divergence of heat flux,} \end{aligned} \quad (2.1)$$

where  $C$  is some heat capacity of the atmosphere-ocean system,  $T$  the surface temperature ( $^{\circ}\text{C}$ ),  $t$  is time, and  $x = \sin \theta$ , where  $\theta$  is the latitude. It is somewhat more convenient to deal with  $x$  rather than  $\theta$ .

Under the assumption that the *total* global energy budget can be expressed in terms of the surface temperature, the first term on the right-hand side of Equation 2.1 is generally taken to be the total insolation as might be determined by a satellite above the atmosphere. It is typically written as

$$\text{incoming solar radiation} = Qs(x)\mathcal{A}(T), \quad (2.2)$$

where  $Q$  is one quarter of the solar constant (Why?) and  $s(x)$  is a function whose integral from the equator to the pole is unity and which represents the annually averaged latitude distribution of incoming radiation. This function is discussed in Held and Suarez (1974). Finally,  $\mathcal{A}(T)$  is 1 minus the planetary albedo;  $\mathcal{A}$  is allowed to depend on temperature. In most simple climate models, a temperature  $T_s$  is identified with the onset of ice (snow) cover such that for  $T > T_s$  there is no ice (snow), and for  $T < T_s$  there is. The most important change in  $\mathcal{A}$ , *in these models*, is due to  $T$  passing through  $T_s$ . We will specify  $\mathcal{A}$  more explicitly later.

Again under the assumption that global energy budgets can be expressed in terms of surface temperature, one writes

$$\text{infrared cooling} = I(T). \quad (2.3)$$

The justification for Equation 2.3 is that temperature profiles have more

or less the same shape at all latitudes. Hence cooling, which depends on the temperature at all levels, ought to be expressible in terms of surface temperature, since the temperature at all levels is related to the surface temperature. In fact, temperature profiles at different latitudes are somewhat different (*viz.* Figure 2.1). In principle, there would also need to be similarity with latitude for water vapor, and that is certainly far from being satisfied.

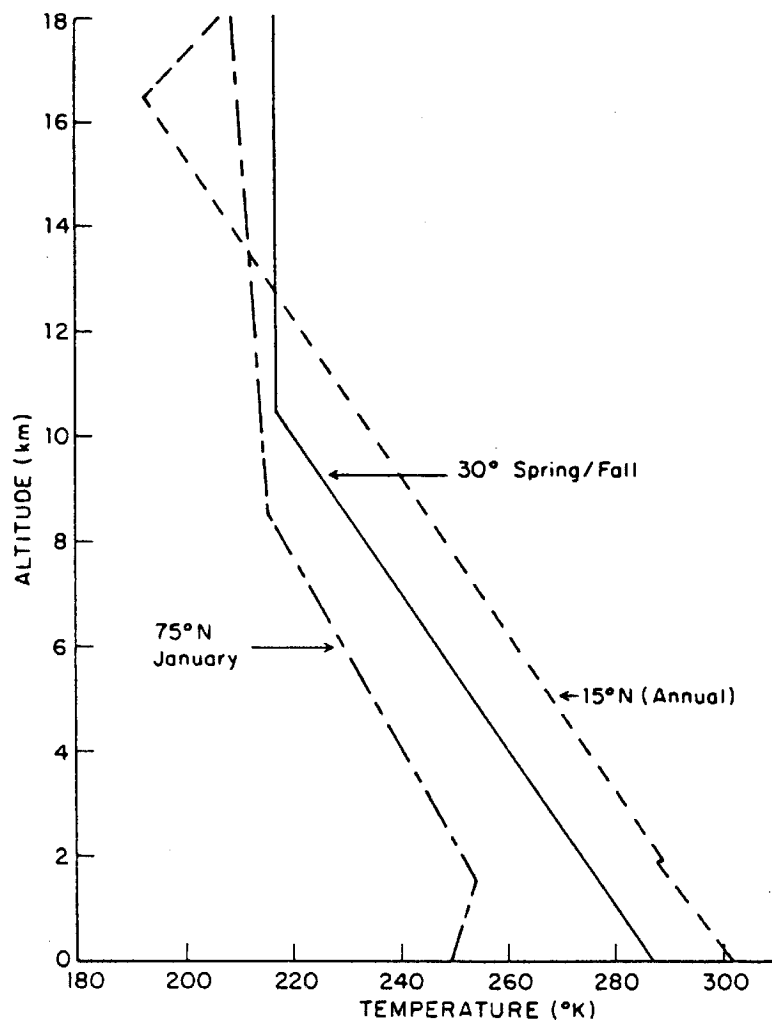


Figure 2.1: Vertical temperature profiles for various latitudes. (From U.S. Standard Atmosphere Supplements, 1966.)

Moreover, Held and Suarez (1974) have shown that 500mb temperatures correlate better with infrared emission than do surface temperatures. Nevertheless, it is the surface temperature which relates to the formation of ice, and which therefore must be used in simple climate models. (Note that such compromises with the underlying physics in order that the resulting model does what we want it to do are not uncommon. Almost all parameterizations fall into this situation to some extent.) The fact that total infrared emission is not perfectly related to surface temperature is merely an indication that a significant portion of the emitted radiation originates in the atmosphere (What is the reason for this?). Similarly, not all of the incoming radiation is absorbed at the surface; in practice, some of the incoming radiation is *not* directly involved in the surface energy budget.

As a rule, models based on Equation 2.1 take little account of clouds and cloud feedbacks. In truth we hardly know how to include such feedbacks. It is probably impossible in such a simple model. However, to the extent that clouds can be specified in terms of latitude and surface temperature, their effects on incoming radiation and on infrared emission can be included in Equations 2.2 and 2.3. In reality, this is almost certainly a bad approximation. Consider, for example, the difference in cloud cover between summer and winter; they are quite similar.

The divergence of atmospheric and oceanic heat flux must also be expressed in terms of an operator on surface temperature; that is,

$$\text{div flux} = -F[T], \quad (2.4)$$

where  $F$  is some operator. Usually  $F$  is a linear operator, although Held and Suarez (1974) and North (1975) have also considered nonlinear operators as suggested by Green (1970) and Stone (1973). The common choices for  $F[T]$  are a linear relation first suggested by Budyko (1969):

$$F[T] = C (T - \bar{T}), \quad (2.5)$$

where  $\bar{T}$  is the average of  $T$  over all latitudes<sup>2</sup>, and a diffusion law (first used in this context by Sellers (1969)):

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<sup>2</sup>A general constraint on  $F[T]$  is that its integral over the globe be zero (Why?).

$$F[T] = \frac{\partial}{\partial x} \left[ (1 - x^2) D \frac{\partial T}{\partial x} \right], \quad (2.6)$$

where  $C$  and  $D$  in (2.5) and (2.6) are constants; they are generally chosen to simulate some features of the existing climate.

The most common application of (2.1) involves assuming a steady state and seeking a relation between the equilibrium position of the ice line and the solar constant. Usually, one linearizes (2.3) to obtain

$$I = A + BT, \quad (2.7)$$

and replaces  $T$  by  $I$  as defined in (2.7). Equation 2.1 becomes<sup>3</sup>

$$Qs(x)\mathcal{A}(I) - I + F^*[I] = 0. \quad (2.8)$$

One identifies the ice line with a temperature  $T_s$ , or equivalently  $I_s = A + BT_s$ . We shall use  $x_s$  to identify the value of  $x$  at  $I = I_s$ . Moreover, variations in  $\mathcal{A}$  are taken to be due solely to whether or not there is an ice surface, that is,

$$\mathcal{A} = \mathcal{A}(x, x_s). \quad (2.9)$$

‘ $C$ ’ or ‘ $D$ ’ in (2.4) will be chosen so that for the present climate  $T = T_s$  at the present annually averaged value of  $x_s$  (i.e.,  $x_s \simeq 0.95$ ). Obtaining the dependence of  $x_s$  on  $Q$  (or more conveniently, the dependence of  $Q$  on  $x_s$ ) is straightforward. If we write

$$I = Q\tilde{I}(x), \quad (2.10)$$

and assume  $F$  to be a linear operator, then we may divide (2.8) by  $Q$ , yielding

$$-F^*[\tilde{I}] + \tilde{I} = s(x)\mathcal{A}(x, x_s). \quad (2.11)$$

For any choice of  $x_s$  we may solve (2.11) for  $\tilde{I} = \tilde{I}(x, x_s)$ . It is now a trivial

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<sup>3</sup> $F^* \equiv \frac{1}{B}F$

matter to obtain the solar constant (or, equivalently,  $Q$ ) as a function of  $x_s$ . We already have

$$I(x_s) = I_s. \quad (2.12)$$

But since

$$I(x_s) = \tilde{I}(x_s, x_s)Q, \quad (2.13)$$

by combining (2.12) and (2.13) we have

$$\frac{Q}{I_s} = \frac{1}{\tilde{I}(x_s, x_s)}, \quad (2.14)$$

which is the desired relation.

Normally, we expect advancing ice (decreasing  $x_s$ ) to be associated with decreasing  $Q$ . Such a situation is generally stable in the sense that the time-dependent version (2.1) indicates that perturbations away from the equilibria defined by (2.14) decay in time. This stability is easy to understand intuitively. If, for example, one decreased  $x_s$  while holding  $Q$  constant, then  $Q$  would be larger than needed for that value of  $x_s$  and the resulting warming would cause  $x_s$  to increase. This is, in fact, the situation when we do *not* have transport. Surprisingly, the introduction of transport always leads to some values of  $x_s$  where decreasing  $x_s$  is associated with increasing  $Q$  – an unstable situation leading to an ice covered earth (at least in the context of the simple model).

We shall examine how this occurs under particularly simple conditions. First we shall use (2.5) for  $F[T]$ . Next we shall take the following expression for  $\mathcal{A}$ :

$$\begin{aligned} \mathcal{A} &= \alpha \text{ for } T < T_s \\ \mathcal{A} &= \beta \text{ for } T > T_s. \end{aligned} \quad (2.15)$$

Common choices for  $\alpha$  and  $\beta$  are:

$$\begin{aligned}\alpha &= 0.4 \\ \beta &= 0.7.\end{aligned}\tag{2.16}$$

(Note, that by letting there be a discrete jump in  $\mathcal{A}$  at the latitude where  $T = T_s$ , we have implicitly replaced the annual mean with something like a permanent equinox. Why?) For  $T_s$  we, for the moment, take  $T_s = -10^\circ\text{C}$ , and for the constants  $A$  and  $B$  in (2.7) we use  $A = 211.1 \text{ Wm}^{-2}$ , and  $B = 1.55 \text{ Wm}^{-2}(\text{ }^\circ\text{C})^{-1}$ . Hence,  $I_s = 195.7 \text{ Wm}^{-2}$ . For  $s(x)$  we use the annual average function as approximated by North (1975):

$$s(x) \approx 1 - 0.241(3x^2 - 1).\tag{2.17}$$

For the present solar constant,

$$Q = 344 \text{ Wm}^{-2}.$$

For the above choices, straightforward analytic solutions exist. Let us begin by neglecting all transport. Equation 2.8 becomes

$$Qs(x)\mathcal{A}(x, x_s) - A - BT = 0,$$

or

$$T = \frac{Qs(x)\mathcal{A}(x, x_s) - A}{B}.\tag{2.18}$$

This radiative equilibrium value of  $T$  depends only on the local radiative budget. Its distribution is shown in Figure 2.2 as is the observed distribution.

Note that the observed gradients are much smaller than in the equilibrium distribution. The value of  $Q$  associated with  $x_s$  is not unique. Any value of  $Q$  less than the value needed for  $T(x_s) = T_s$  with  $\mathcal{A} = \alpha$ , and greater than the value needed for  $T(x_s) = T_s$  with  $\mathcal{A} = \beta$ , is consistent with  $x_s$ . This leads to the two curves for  $Q(x_s)$  shown in Figure 2.3:



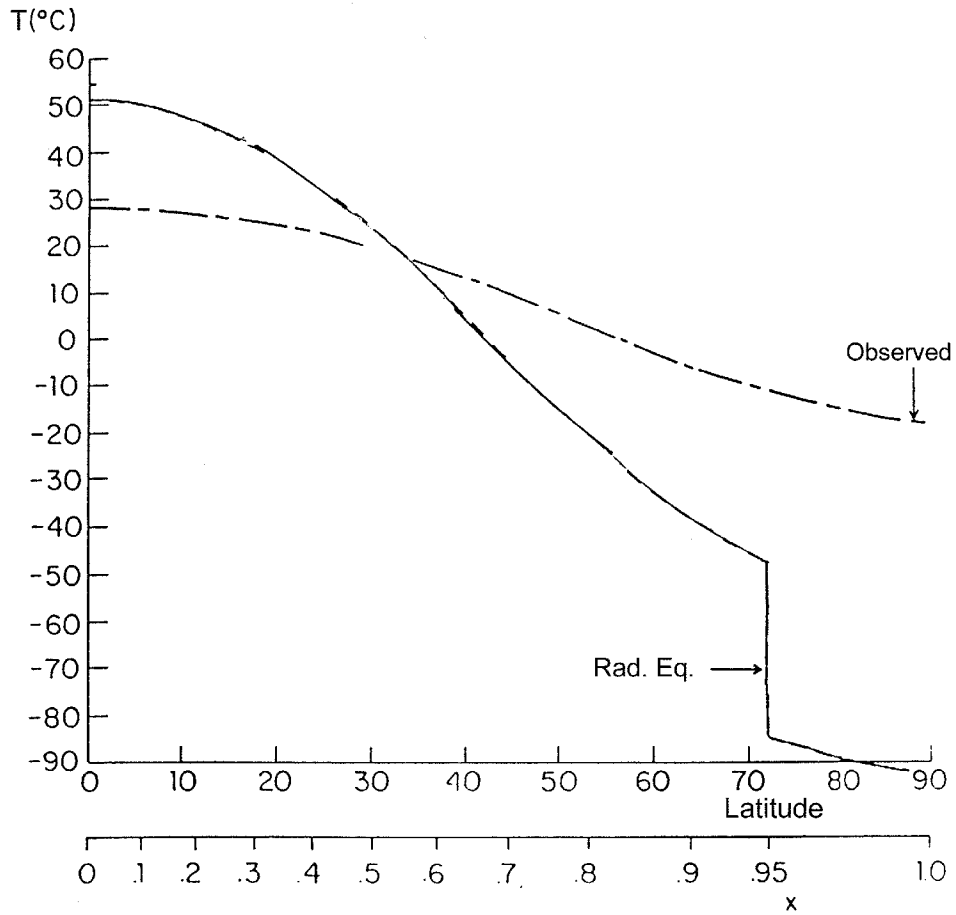


Figure 2.2:  $T(\phi)$  for radiative equilibrium. Also shown is the observed  $T(\phi)$ . For reference purposes,  $x$  as well as  $\phi$  is shown. Taken from Lindzen and Farrell (1977).

$$Q_+(x_s) = \frac{A + BT_s}{s(x_s)\alpha}, \quad (2.19)$$

$$Q_-(x_s) = \frac{A + BT_s}{s(x_s)\beta}. \quad (2.20)$$

It will be left to the student to figure out how  $x_s$  will vary as  $Q$  is changed, but it is evident that for both (2.19) and (2.20) that decreasing  $Q$  leads to decreasing  $x_s$  and *vice versa*. (Note that  $s(x_s)$  decreases monotonically with  $x_s$ .)

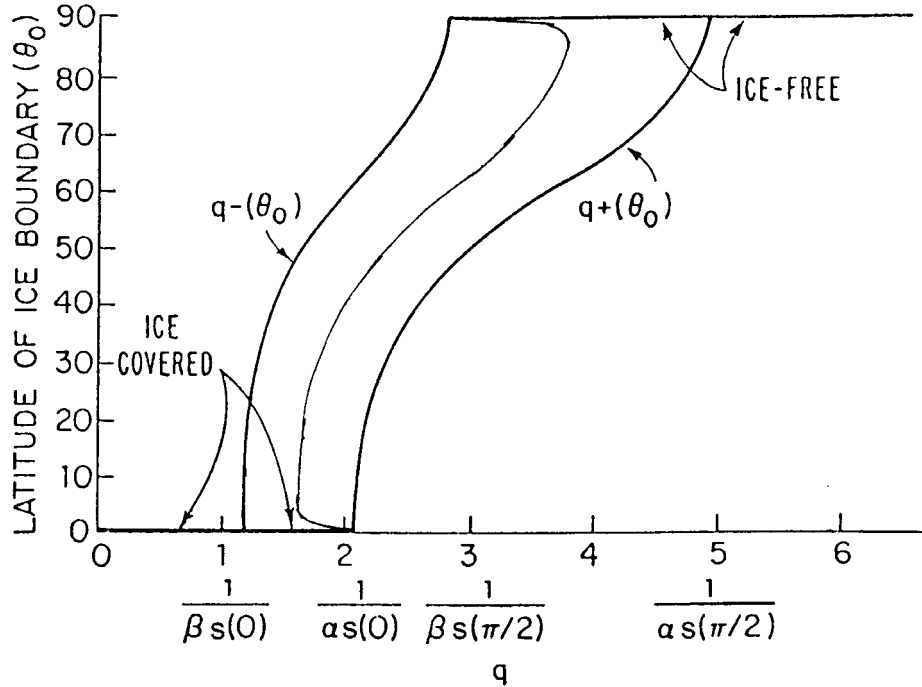


Figure 2.3: Variation of  $q$  ( $q \equiv Q/I_s$ ) vs.  $\theta_s$  ( $\sin^{-1} x_s$ ) for radiative equilibrium. The curve  $q_-$  represents  $T = T_s$  for the ice-free side of  $x_s$ , while  $q_+$  represents  $T = T_s$  for the ice-covered side of  $x_s$ . Also shown is the curve obtained with a very small amount of diffusive heat transport. From Held and Suarez (1974).

Introducing transport via Equation 2.5 does not eliminate the ambiguity in  $Q$ . A device for eliminating the ambiguity is to choose

$$\mathcal{A}(x_s) = \frac{\alpha + \beta}{2}. \quad (2.21)$$

This device turns out to be almost equivalent to introducing a very small conductivity which, in turn, renders  $T$  continuous at  $x_s$  (*viz* Figure 2.3).

Now

$$F(\tilde{I}) = (C/B)(\bar{\tilde{I}} - \tilde{I}) \quad (2.22)$$

and

$$\bar{\tilde{I}} = \int_0^1 s(x) \mathcal{A}(x, x_s) dx \quad (2.23)$$

(Why?).

Evaluating (2.23) (using (2.15) and (2.17)) we get

$$\bar{\tilde{I}} = (\beta - \alpha)(1.241 x_s - .241 x_s^3) + \alpha. \quad (2.24)$$

Substituting (2.24) into (2.22), and (2.22) in (2.11), we get

$$\tilde{I} = \frac{\frac{C}{B}\bar{\tilde{I}}(x_s) + s(x) \mathcal{A}(x, x_s)}{1 + \frac{C}{B}}. \quad (2.25)$$

Equation 2.25 allows us to determine  $C$  such that  $\tilde{I}(.95) = \tilde{I}_s$  for present conditions ( $C/B = 2.45$  is obtained). Assuming this value of  $C$  remains constant as  $Q$  varies, we then get from (2.14)

$$Q = \frac{(1 + \frac{C}{B})(A + BT_s)}{\frac{C}{B}\bar{\tilde{I}}(x_s) + s(x_s)(\frac{\alpha+\beta}{2})}. \quad (2.26)$$

Examining the denominator of (2.26) in detail we get

$$\begin{aligned} \text{den} &= \frac{\beta + \alpha}{2} \times 1.241 + \alpha \frac{C}{B} \\ &+ \frac{C}{B}(\beta - \alpha) \times 1.241 x_s \\ &- \frac{(\beta + \alpha)}{2} \times .723 x_s^2 - \frac{C}{B}(\beta - \alpha) \times .241 x_s^3. \end{aligned} \quad (2.27)$$

When  $C = 0$ , the denominator decreases as  $x_s$  increases, as already noted. But, when  $C \neq 0$ , there always exists some neighbourhood of  $x_s = 0$  where the linear term in (2.27) dominates, and the denominator *increases* as  $x_s$  increases. This leads to the distribution of  $Q$  versus  $x_s$  shown in Figure 2.4.

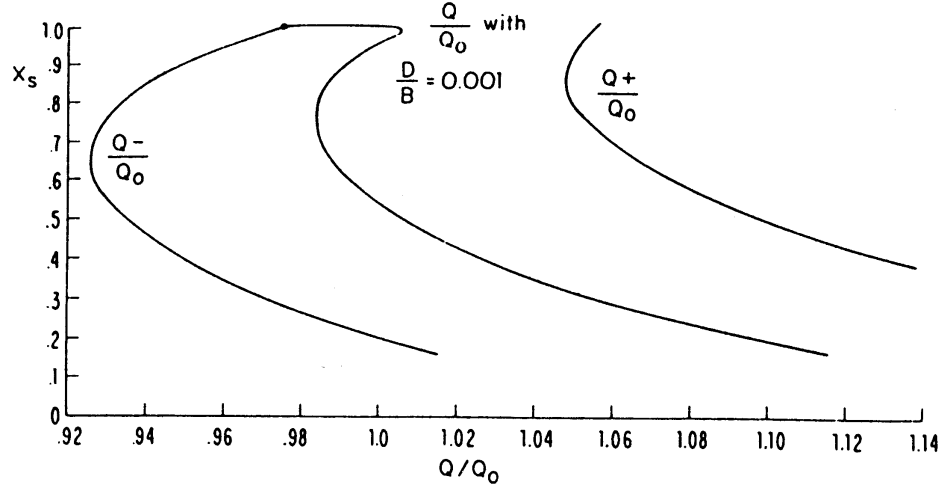


Figure 2.4: Equilibrium ice line position  $x_s$  as a function of normalized solar constant  $Q/Q_0$  (where  $Q_0$  is the current value of the solar constant) when Budyko-type heat transport is used. The curve  $Q_-/Q_0$  corresponds to  $T = T_s$  on the equatorward side of the ice line, while  $Q_+/Q_0$  corresponds to  $T = T_s$  on the poleward side. The single remaining curve results from adding a small amount of diffusion to the Budyko-type transport.

Two features should be noted in Figure 2.4, both being due to the existence of horizontal heat transport:

1. A much smaller value of  $Q$  is needed for ice/snow to onset at all. This represents the stabilizing effect of transport.
2. There now exists some minimum  $Q$ , below which the climate will unstably proceed to an ice/snow covered earth. This represents the destabilizing effect of transport.

Both the above effects result very generally from the sharing of heat between low and high latitudes. Clearly, heat transport from low latitudes initially inhibits the onset of ice/snow at the poles. However, as the ice/snow line advances, the transport of heat out of warmer regions cools these regions

to such an extent that  $Q$  must actually be increased to keep up with further advances. This situation is clearly unstable. We shall refer to the percentage  $Q$  must be reduced from its present value to reach instability as the ‘global stability’. The results in Figure 2.4 correspond to a global stability of only  $\sim 2\%$ . To be sure, the solar constant might not vary this much. However,  $Q$  can be viewed as a general measure of global heating. Changes in  $Q$  can be simulated by changes in  $I$  and/or  $A$ , for example.

As will be seen in the exercise, the above estimate of *global stability* is hardly firm (even in the context of this model), but our only interest at this point is in the general rôle of heat transport.