12.810 Problem set 2

Need help?: Office hours Thursdays 1.30-2.30pm in 54-1712.

Collaboration is allowed, but write up the solution on your own. Show all work. Give units for all numerical results. Put axis labels and units on any graphs.

1. In this problem you will use a script (available as lee_wave_810.py or lee_wave_810.m) to solve for a mountain wave. The script uses the Boussinesq approximation and assumes constant buoyancy frequency ($N = 0.01\text{s}^{-1}$) and basic-state horizontal wind u_0. The domain is 2-dimensional (x-z) of width 70km and height 15km. The ridge is a single Gaussian, centered at $x = 35$km, of amplitude 500m and of standard deviation 3km. Note that a Gaussian ridge includes contributions from a wide range of horizontal wavelengths. Running the script plots the vertical velocity and streamlines.

    (a) Run the script for basic-state wind $u_0 = 5$, 20, and 120 m s^{-1} and discuss the resulting wave in each case. Include vertical and horizontal propagation in your discussion. Indicate on the graphs of streamlines the positions where clouds might be seen if the atmosphere is close to saturation (e.g. upstream, downstream, over the ridge).

    (b) Using the analytic results from class, calculate the vertical wavelength in the *hydrostatic* limit for u_0=5 m s^{-1}. Compare this vertical wavelength with what you find from running the script for this u_0. Do you think the wave is close to the hydrostatic limit?

    (c) The width of the Gaussian ridge as measured by its standard deviation is 3km. For sinusoidal topography of wavelength 6km, use the analytic results from class to calculate the critical u_0 at which the wave doesn't propagate in the vertical (becomes evanescent). Then run the script for the Gaussian ridge for a range of values of u_0 and estimate the critical u_0 for an evanescent wave. Is the critical value for the Gaussian ridge well predicted by the result for the sinusoidal topography of wavelength 6km? Can you explain what sets the critical speed for the Gaussian ridge in this domain?

    (d) The script assumes that the waves are linear. By considering the magnitude of u', assess the validity of the linear approximation for each of $u_0 = 5$, 20, and 120 m s^{-1} (the disturbance wind u' is given by the variable uxz in the script).

2. In this problem, you will calculate the vertical flux of x-momentum for the case of a stationary mountain wave. For simplicity use the Boussinesq approximation with reference density ρ. Assume that the height of the topography is given by $h = h_0 \cos(kx)$ and that the amplitude is small. Assume also that the basic-state horizontal velocity u_0 and buoyancy frequency N are constant. In this case, we found in class that the vertical velocity of the wind in a vertically propagating wave is

$$w' = -ku_0 h_0 \sin(kx + mz)$$

with vertical component of the group velocity

$$c_{gz} = \frac{ku_0 m}{k^2 + m^2},$$

where m is the vertical wavenumber. The vertical velocity for an evanescent wave was

$$w' = -ku_0h_0 \sin(kx)\exp(-rz),$$

where r is real and positive.

(a) For the vertically propagating wave, calculate the mean vertical momentum flux $\rho\overline{u'w'}$ and show that it is constant with respect to height. For $u_0 > 0$, what is the sign of the vertical momentum flux? (Explain your reasoning) The constancy of the vertical momentum flux is an important result that occurs also in more general cases as we will discuss further in class. You should interpret the mean $\overline{(\cdot)}$ as a mean over one wavelength in x. For example, $\overline{w} = L^{-1}\int_0^L w dx$ where L is the wavelength in x.

(b) What is the vertical momentum flux for the evanescent wave?

(c) The vertical flux of x-momentum carried vertically by the wave must be balanced by a drag force on the mountain. This drag force, known as mountain drag, is in the x direction and it arises due to variations in pressure on the mountain. Derive an expression for the normalized pressure perturbation ϕ' of the wave. Then make sketches versus x of the topography h and ϕ' at the surface ($z = 0$) for (i) the propagating wave and (ii) the evanescent wave. Explain how the pressure variations on the different sides of the mountains are consistent with the fluxes of momentum found in parts (a) and (b). Include the sign of the momentum flux in your explanation.

3. This problem gives you practice thinking about log-pressure coordinates which are often used in atmospheric dynamics. You will show how the buoyancy frequency in log-pressure coordinates is related to buoyancy frequency in height coordinates. Let z be the log-pressure coordinate defined in class and N the corresponding buoyancy frequency where

$$N^2 = \frac{R\,\Pi}{H}\frac{d\theta}{dz}.$$

Here R is the gas constant, $\Pi = (p/p_0)^\kappa$ is the Exner function, and $H = RT_*/g$ is the scale height based on a reference temperature T_*. The buoyancy frequency in regular height coordinates (denoted $\tilde{z}$) is

$$\tilde{N}^2 = \frac{g}{\theta}\frac{d\theta}{d\tilde{z}}.$$

Show that (assuming hydrostatic balance) these two frequencies are related by

$$N^2 = \left(\frac{T^2}{T_*^2}\right)\tilde{N}^2.$$

If you use the regular density in your derivation, be sure to denote it $\tilde{\rho}$ to avoid confusion.