12.810 Problem set 4

Need help?: Office hours Thursdays 1.30-2.30pm in 54-1712.

Collaboration is allowed, but write up the solution on your own. Show all work. Give units for all numerical results. Put axis labels and units on any graphs.

1. Latent heating associated with phase changes of water strongly affects the dynamics of the atmosphere in both the tropics and extratropics. In class we showed that the QGPV is conserved for adiabatic inviscid flow: $D_g q/Dt = 0$. Suppose now that we include a potential temperature tendency J from latent heating such that the thermodynamic equation in the QG approximation is

$$\frac{D_g \theta}{Dt} + w\frac{\partial \theta_0}{\partial z} = J. \tag{1}$$

   (a) Derive an equation for $D_g q/Dt$ that includes the effects of latent heating.

   (b) Next you will investigate the response of QGPV to an isolated region of (positive) latent heating in the mid-troposphere. Fig. 1 shows a vertical profile of latent heating versus height. Make a sketch of the resulting tendency of QGPV ($D_g q/Dt$) versus height. Explain how the changes in q are consistent with the relationship between static stability anomalies and PV anomalies that we discussed in class.

   (c) Now consider a positive QGPV anomaly in the lower troposphere in a region of westerly shear as shown in Fig. 2. What does the westerly shear imply about the tilt of potential temperature surfaces? Use conservation of potential temperature to explain whether ascent will begin to occur to the east or west of the QGPV anomaly. (Note potential temperature will not be conserved if ascent subsequently leads to latent heating, but it is ok to use conservation of potential temperature to determine where the ascent will begin to occur).

   (d) Let's assume that the ascent leads to moist convection and latent heating with a vertical profile like in Fig. 1. Show in Fig. 2 where QGPV will be generated by latent heating and indicate whether the generation is positive or negative. Does this generation of QGPV effectively cause the QGPV anomaly to propagate to the east or west? For simplicity, you may assume that the shear does not distort the QGPV anomaly on the timescale being considered.

2. The QG omega equation determines the vertical velocity for geostrophically balanced flows. In class we derived the omega equation for flows that are adiabatic and inviscid. In this problem, you will explore how diabatic heating affects vertical motion.

   (a) The QG thermodynamic equation with a diabatic term J is given by equation 1 (where now J is any diabatic heating such as latent heating or radiative heating). Show that the inviscid QG omega equation in log pressure coordinates is

$$N^2\nabla^2 w + f_0^2 \frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial}{\partial z}\rho w = f_0 \frac{\partial \mathrm{VA}}{\partial z} - f_0 \nabla^2 \mathrm{TA} + \frac{R\Pi}{H}\nabla^2 J, \tag{2}$$

   where the notation is as in class. Recall that $\mathrm{VA} = \mathbf{u}_g \cdot \nabla \zeta_g + \beta v_g$ is the vorticity advection and $\mathrm{TA} = \mathbf{u}_g \cdot \nabla(\partial \psi/\partial z)$ is the thermal advection as defined in class.

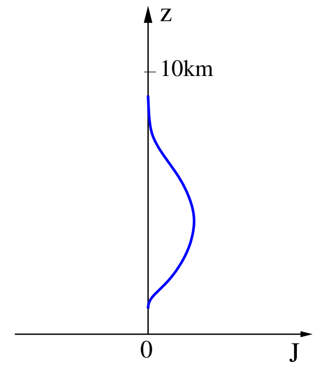

Figure 1: Idealized vertical profile of potential temperature tendency J due to latent heating.

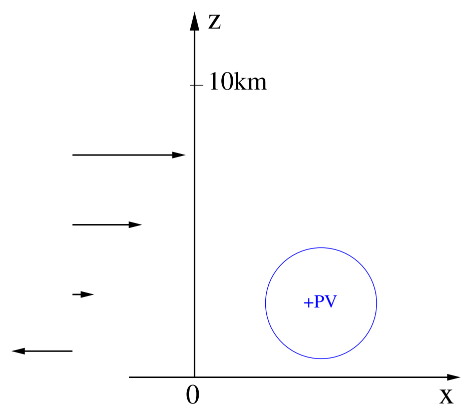

Figure 2: Positive QGPV anomaly in the lower troposphere in the Northern Hemisphere in a region of westerly shear. Background zonal winds are indicated by arrows.

(b) Suppose there is a positive diabatic heating anomaly that is localized in x and y. Use equation (2) to infer what effect this would have on vertical velocity (including the sign of w at the middle of the heating anomaly).

(c) Suppose that the omega equation is applied to estimate the vertical velocity in a disturbance in the subtropics. Assume that VA and TA are small for this disturbance. Furthermore assume that the horizontal length scale L of the disturbance is much smaller than the Rossby deformation radius ND/f_0, where D is the vertical depth of the disturbance, and D is of similar magnitude to the scale height H. What are the two terms that dominate equation (2) in this situation? Explain your reasoning.

3. In this problem you will derive some important properties of vertically propagating Rossby waves. Consider a QG Rossby wave that is propagating upwards (in the group velocity sense), with a uniform background zonal velocity u_0 and constant N^2.

(a) Starting with the dispersion relation from class, derive an expression for the vertical component of the group velocity and use it to show that the phase tilt is westward with height.

(b) For simplicity, assume that the perturbation streamfunction is of the form

$$\psi' = \psi_0 e^{z/(2H)} \sin(kx + ly + mz - \omega t). \tag{3}$$

Derive an expression for the meridional potential temperature flux, $\overline{v'\theta'}$, where the overbar denotes a zonal average in x with periodic boundary conditions in x. (Use the streamfunction expression from above and do not use results we derive in class for E-P fluxes). Use your result to show that the potential temperature flux is poleward in *both* hemispheres. Thus upward propagating Rossby waves, such as those generated by mountains and land/ocean contrasts, contribute to the poleward heat flux that helps to maintain the climate of higher latitudes.

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