12.810 Problem set 5

Need help?: Office hours Thursdays 1.30-2.30pm in 54-1712.

Collaboration is allowed, but write up the solution on your own. Show all work. Give units for all numerical results. Put axis labels and units on any graphs.

1. Consider small-amplitude plane QG Rossby waves with wavevector (k, l, m) and frequency ω on a beta plane. Show that the Eliassen Palm flux $\mathbf{F} = (F_y, F_z)$ is related to the group velocity $\mathbf{c}_g = (c_{gy}, c_{gz})$ and wave activity density A by $F_y = c_{gy}A$ and $F_z = c_{gz}A$. In your answer, use the log-pressure vertical coordinate and assume that the vertical wavelength is small such that $m \gg H^{-1}$, that the wave streamfunction has the form

$$\psi' = \Psi_0 \exp(\frac{z}{2H}) \sin(kx + ly + mz - \omega t), \tag{1}$$

   where Ψ_0 is real, and that the background zonal flow U_0 and buoyancy frequency N are constant.

2. In class we derived the condition for stability for a baroclinic zonal flow (the Charney-Stern criterion). In this problem you will apply the Charney Stern criterion to a particular flow. The flow has the form of an upper-level zonal jet with $U_0(z) = B \exp\left(-\frac{(z-z_J)^2}{2L^2}\right)$ where z_J controls the vertical position of jet, L controls its vertical extent, and B controls its amplitude and whether it is easterly or westerly. For simplicity, we will suppose there is no lower or upper boundary, and we will work with the log-pressure QGPV but neglect variations of ρ and N^2. Over what range of values of B (including negative B) could the flow be unstable? Does a jet of this form become unstable at a lower absolute speed if it is easterly or westerly?

3. In class we analyzed the Eady model of baroclinic instability in which normal modes grow or decay exponentially in time. But rapid transient growth can occur for disturbances that are not normal modes and whose spatial structure changes over time. To investigate this possibility, we will use the log-pressure QGPV, but neglect the vertical variations of ρ and N^2 and set $\beta = 0$, such that

$$q = f + \nabla^2 \psi + \frac{f^2}{N^2}\frac{\partial^2 \psi}{\partial z^2}.$$

   We will assume that the perturbation QGPV is given by $q'(x, z, t) = Q \sin(kx + m(t)z)$, where Q is a constant, $k > 0$ is the wavenumber in the x direction, and the vertical wavenumber $m(t)$ varies with time with initial condition $m(0) = m_0 > 0$. We assume that the basic state zonal velocity is $U_0 = \Gamma z$ where $\Gamma > 0$ is a constant vertical shear, and there are no upper or lower boundaries.

   (a) Show that $m = m_0 - \Gamma k t$, and write an expression for the slope of lines of constant q' (which we will refer to as the phase lines). How does the slope of phase lines vary as a function of time? Assuming $m_0 > 0$, $k > 0$ and $\Gamma > 0$, make sketches of the phase lines at $t = 0$, $t = m_0/(\Gamma k)$, and $t = 2m_0/(\Gamma k)$ to illustrate the development of q'.

(b) We will measure the magnitude of the disturbance by its kinetic energy per unit volume. First show that the perturbation streamfunction ψ' is given by

$$\psi' = -\frac{Q}{k^2 + \frac{f^2}{N^2} m^2} \sin(kx + m(t)z). \tag{2}$$

Then show that the kinetic energy spatially averaged over one horizontal wavelength is given by

$$K = \frac{\rho k^2 Q^2}{4(k^2 + \frac{f^2}{N^2} m^2)^2}. \tag{3}$$

Make a sketch of the spatially averaged kinetic energy with respect to time.

(c) The QGPV perturbations are advected but do not change in magnitude over time, yet the kinetic energy can grow and decay. (In fact, this type of transient growth is in some cases faster than the normal mode instability that we considered in class.) Give a physical explanation, using QGPV and QGPV inversion, for the mechanism by which the kinetic energy changes over time. Be sure to include growth and decay phases in your explanation. Hint: You may find it helpful to draw phase lines of q' and the associated induced velocities.

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