Radiative Transfer

Chapter 3, Hartmann

- Shortwave Absorption:
  - Clouds, H_20, O_3, some CO_2
- Shortwave Reflection:
  - Clouds, surface, atmosphere
- Longwave Absorption:
  - Clouds, H_20, CO_2, CH_4, N_2O

Planck’s Law

- Based on assumption of local thermodynamic equilibrium
  - (Not valid at very high altitudes in atmosphere)

$$B_{\upsilon}(T) = \frac{2\hbar\upsilon^3}{c^2\left[e^{\hbar\upsilon/kT} - 1\right]}$$

$$k = Boltzmann's\ constant$$

$$\hbar = Planck's\ constant$$

$$\upsilon = \ frequency$$

$$c = speed\ of\ light$$

Stefan-Boltzmann Law is the integral of the Planck function over all frequencies and all angles in a hemisphere:

$$\pi \int_0^\infty B_\upsilon(T)\,d\upsilon = \sigma T^4$$

$$\sigma = \frac{2\pi^5 k^4}{15c^2\hbar^3}$$

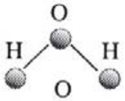
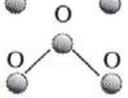
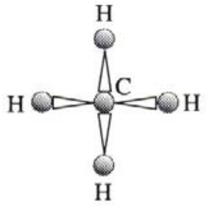
Absorption and Emission in a Gas:

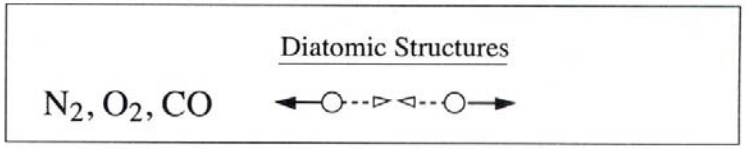
$$\textit{Photon energy} \quad E_{\upsilon} = \hbar\upsilon$$

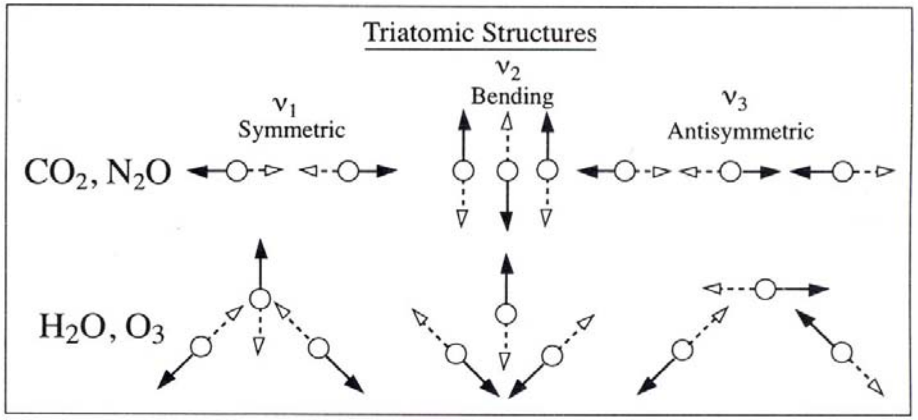
$$\textit{Atomic energy levels} \quad E_{\upsilon} = n\hbar\upsilon, \;\; n = 0,1,2,3\ldots$$

An isolated atom can absorb only those photons whose energy is equal to the difference between two atomic energy levels

Molecules have additional energy levels:

Molecule	Arrangement	Permanent Dipole Moment
N_2		No
O_2		No
CO		Yes
CO_2		No
N_2O		Yes
H_2O		Yes
O_3		Yes
CH_4		No





For molecules in a gas:

$$E_{total} = E_{atomic} + E_{vibrational} + E_{rotational} + E_{translational}$$

Translational energy is the kinetic energy of molecular motions in a gas, proportional to the gas temperature. Not quantized.

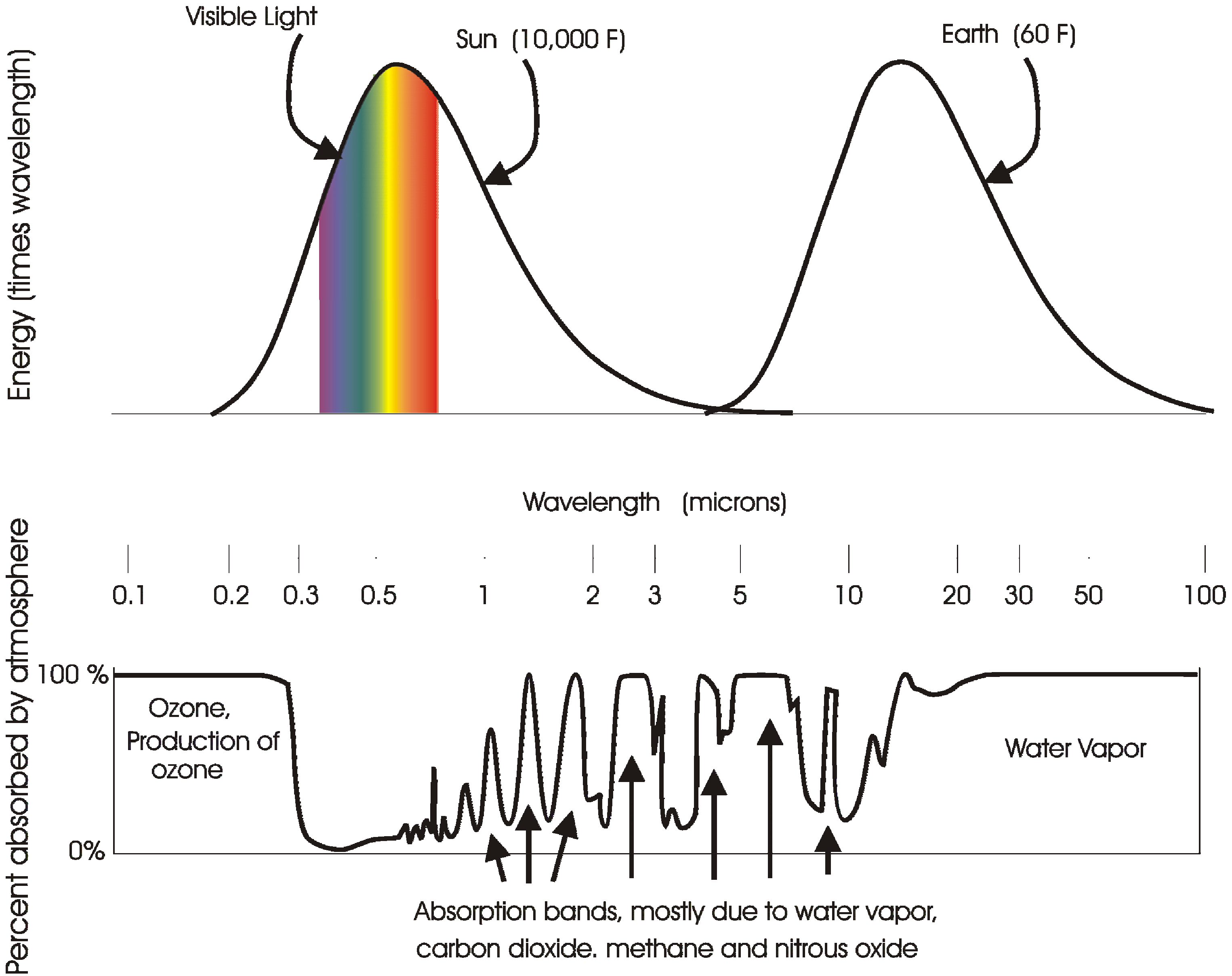
Molecules in a gas can absorb more frequencies than isolated atoms.

Collisions between molecules can carry away energy or supply energy to interactions between matter and photons.

Natural, pressure and Doppler broadening

Principal Atmospheric Absorbers

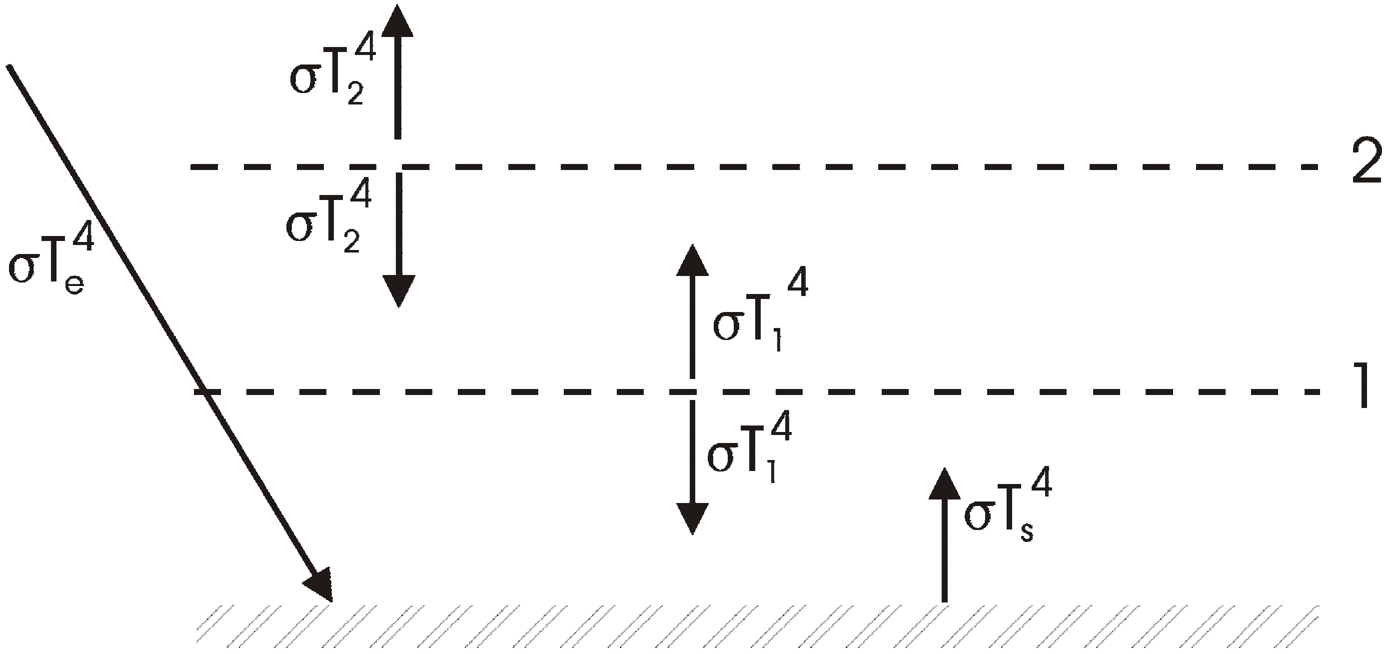
- H_2O: Bent triatomic, with permanent dipole moment and pure rotational bands as well as rotation-vibration transitions
- O_3: Like water, but also involved in photodissociation
- CO_2: No permanent dipole moment, so no pure rotational transitions, but temporary dipole during vibrational transitions
- Other gases: N_2O, CH_4


Visible Light
Sun (10,000 F)
Earth (60 F)
Energy (times wavelength)
Wavelength (microns)
0.1
0.2
0.3
0.5
1
2
3
5
10
20
30
50
100
Percent absorbed by atmosphere
100 %
0%
Ozone,
Production of
ozone
Water Vapor
Absorption bands, mostly due to water vapor,
carbon dioxide. methane and nitrous oxide

Radiative Equilibrium

- Equilibrium state of atmosphere and surface in the absence of non-radiative enthalpy fluxes
- Radiative heating drives actual state toward state of radiative equilibrium

Extended Layer Models



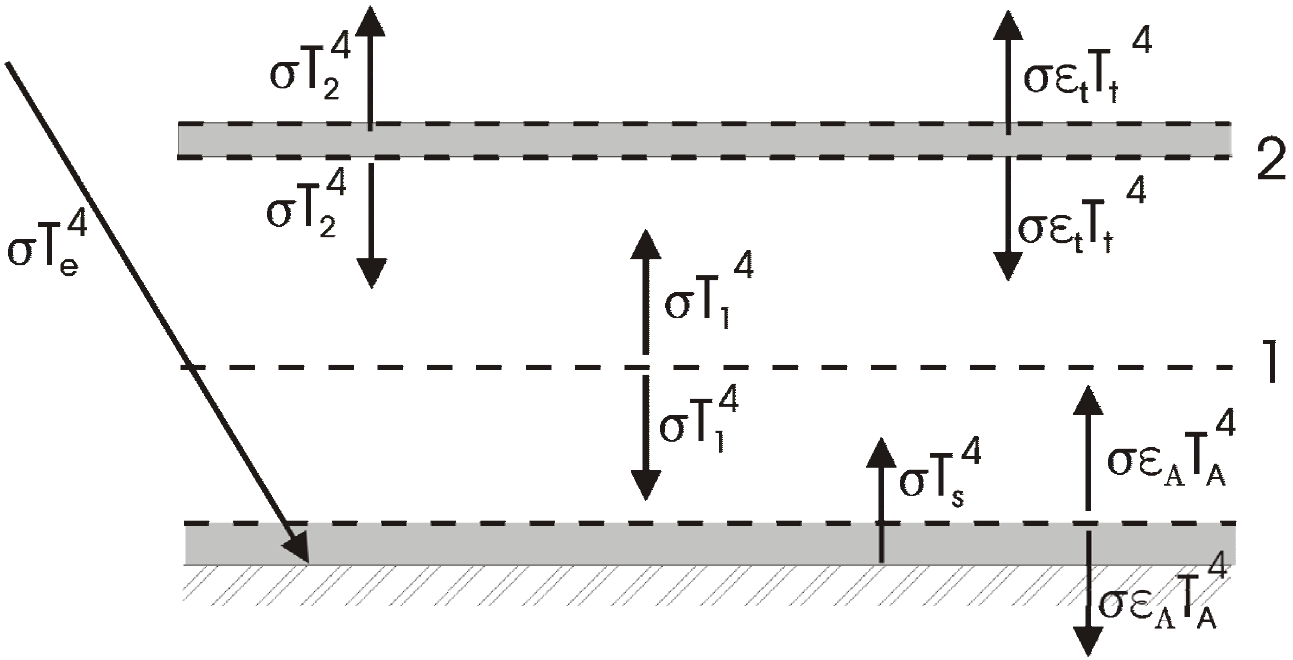
$$TOA: \quad \sigma T_2{}^4 = \sigma T_e{}^4 \rightarrow T_2 = T_e$$

$$Middle\ Layer: \quad 2\sigma T_1{}^4 = \sigma T_2{}^4 + \sigma T_s{}^4 = \sigma T_e{}^4 + \sigma T_s{}^4$$

$$Surface: \quad \sigma T_s{}^4 = \sigma T_e{}^4 + \sigma T_1{}^4$$

$$\rightarrow \quad T_s = 3^{1/4} T_e \qquad\qquad T_1 = 2^{1/4} T_e$$

Effects of emissivity<1



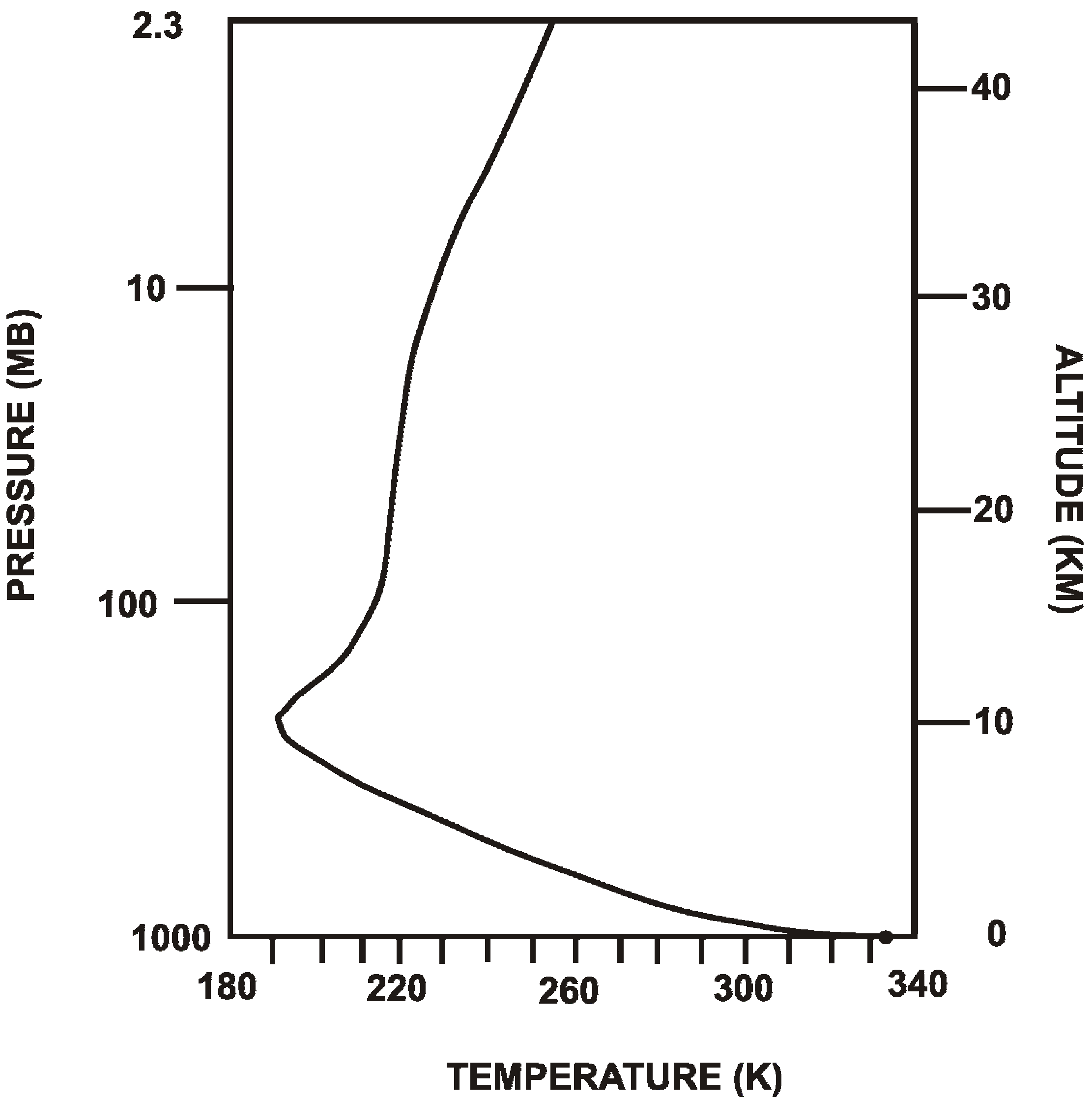
$$\mathit{Surface}: \quad 2\varepsilon_A \sigma T_A{}^4 = \varepsilon_A \sigma T_1{}^4 + \varepsilon_A \sigma T_s{}^4$$

$$\rightarrow \quad T_A = \left(5/2\right)^{1/4} T_e \simeq 321K \quad < T_s$$

$$\mathit{Stratosphere}: \quad 2\varepsilon_t \sigma T_t{}^4 = \varepsilon_A \sigma T_2{}^4$$

$$\rightarrow \quad T_t = \left(1/2\right)^{1/4} T_e \simeq 214K \quad < T_e$$

Full calculation of radiative equilibrium:



Problems with radiative equilibrium solution:

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)

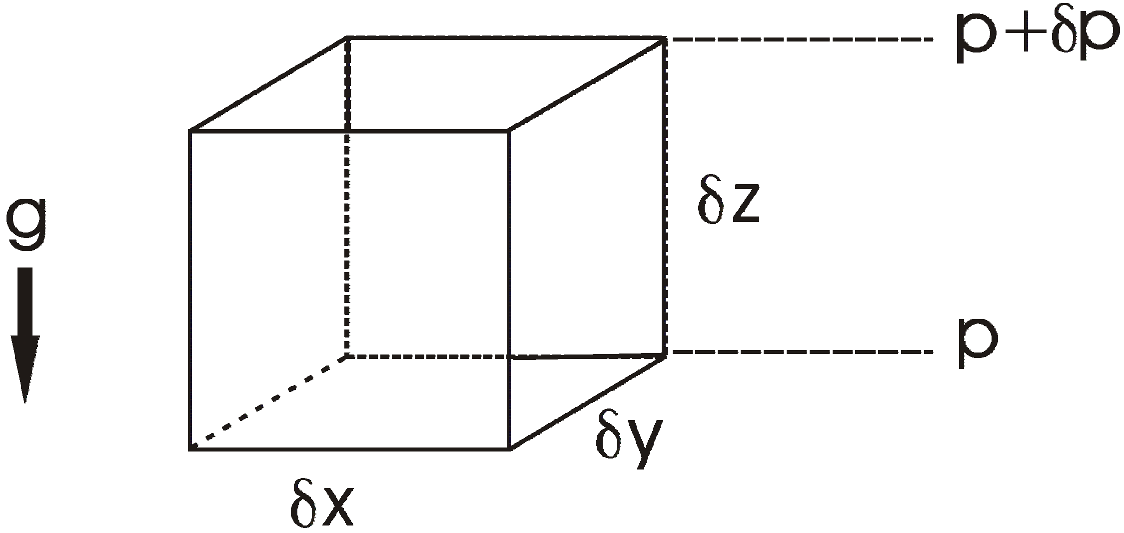
Missing ingredient: Convection

- As important as radiation in transporting enthalpy in the vertical
- Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer

When is a fluid unstable to convection?

- Pressure and hydrostatic equilibrium
- Buoyancy
- Stability

Hydrostatic equilibrium:



$$Weight: \quad -g\rho\delta x\delta y\delta z$$

$$Pressure: \quad p\delta x\delta y-\left(p+\delta p\right)\delta x\delta y$$

$$F=MA: \quad \rho\delta x\delta y\delta z\frac{dw}{dt}=-g\rho\delta x\delta y\delta z-\delta p\delta x\delta y$$

$$\frac{dw}{dt}=-g-\alpha\frac{\partial p}{\partial z}, \qquad \alpha = {}^{1}\!\big/_{\rho} = specific\ volume$$

Pressure distribution in atmosphere at rest:

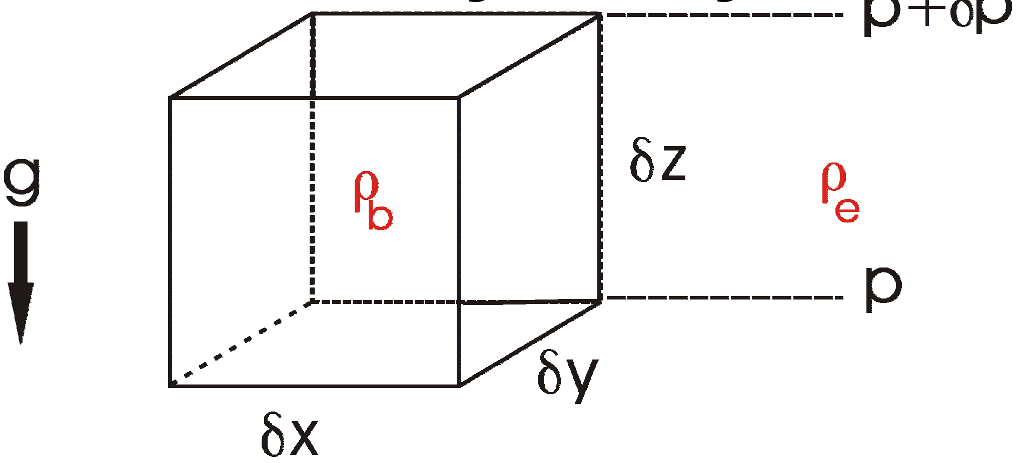
$$Ideal\ gas: \quad \alpha = \frac{RT}{p}, \qquad R \equiv \frac{R^*}{\bar{m}}$$

$$Hydrostatic: \quad \frac{1}{p}\frac{\partial p}{\partial z} = -\frac{g}{RT}$$

$$Isothermal\ case: \quad p = p_0 e^{-z/H}, \qquad H \equiv \frac{RT}{g} = "scale\ height"$$

Earth: H~ 8 Km

Buoyancy:



$$\text{Weight}: \quad -g\rho_b \delta x \delta y \delta z$$

$$\text{Pressure}: \quad p\delta x\delta y-(p+\delta p)\delta x\delta y$$

$$F=MA: \quad \rho_b \delta x\delta y\delta z\frac{dw}{dt}=-g\rho_b\delta x\delta y\delta z-\delta p\delta x\delta y$$

$$\frac{dw}{dt}=-g-\alpha_b\frac{\partial p}{\partial z} \qquad \text{but} \quad \frac{\partial p}{\partial z}=-g/\alpha_e$$

$$\rightarrow \quad \frac{dw}{dt}=g\frac{\alpha_b-\alpha_e}{\alpha_e}\equiv B$$

Buoyancy and Entropy

Specific Volume: $\alpha = {}^{1}\!/_{\rho}$

Specific Entropy: s

$$\alpha = \alpha(p,s$$

$$\left(\delta\alpha\right)_p = \left(\frac{\partial\alpha}{\partial s}\right)_p \delta s = \left(\frac{\partial T}{\partial p}\right)_s \delta s$$

$$B = g\frac{\left(\delta\alpha\right)_p}{\alpha} = \frac{g}{\alpha}\left(\frac{\partial T}{\partial p}\right)_s \delta s = -\left(\frac{\partial T}{\partial z}\right)_s \delta s \equiv \Gamma\,\delta s$$

The adiabatic lapse rate:

$$First\ Law\ of\ Thermodynamics:$$

$$\dot{Q} = T\frac{ds_{rev}}{dt} = c_v\frac{dT}{dt} + p\frac{d\alpha}{dt}$$

$$= c_v\frac{dT}{dt} + \frac{d\left(\alpha p\right)}{dt} - \alpha\frac{dp}{dt}$$

$$= \left(c_v + R\right)\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

$$= c_p\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

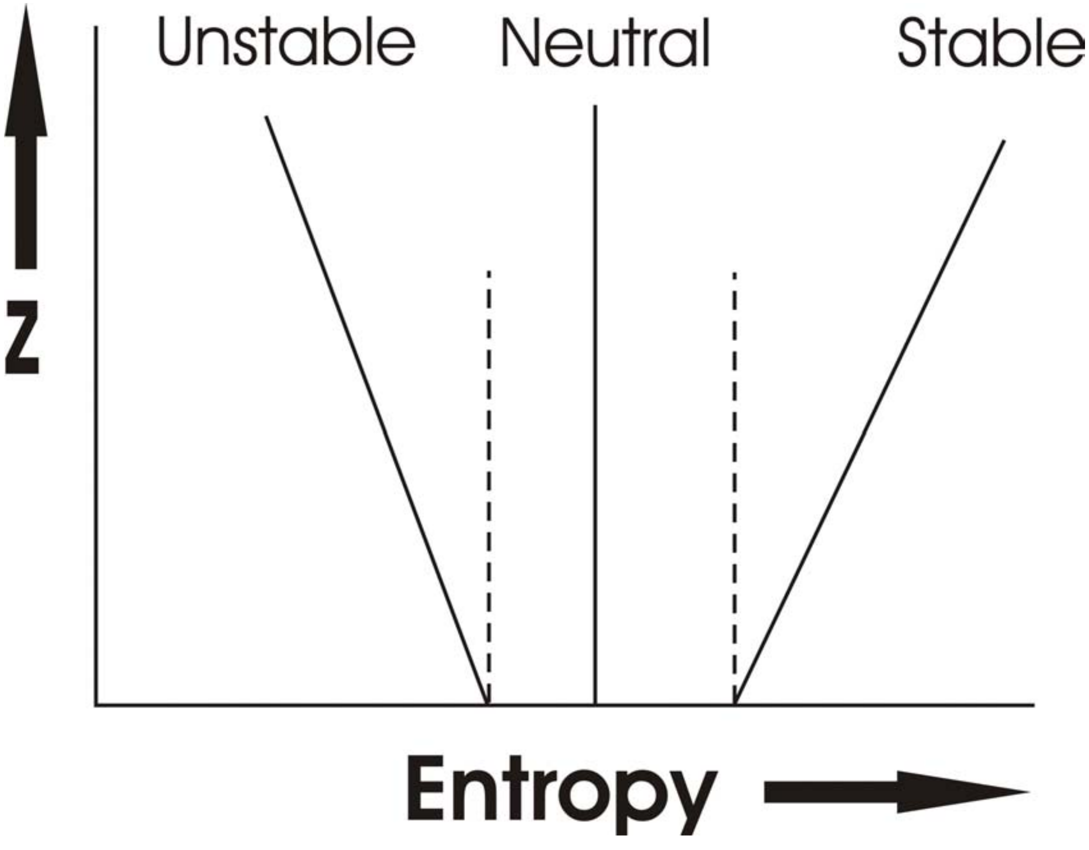
$$Adiabatic: \quad c_p dT - \alpha dp = 0$$

$$Hydrostatic: \quad c_p dT + g dz = 0$$

$$\rightarrow \quad \left({}^{dT}\!/_{dz}\right)_s = -\,{}^{g}\!/_{c_p} \equiv -\Gamma_d$$

$$\Gamma = g/c_p$$

Earth’s atmosphere: $\Gamma = 1\,K/100\,m$



Model Aircraft Measurements (Renno and Williams, 1995)

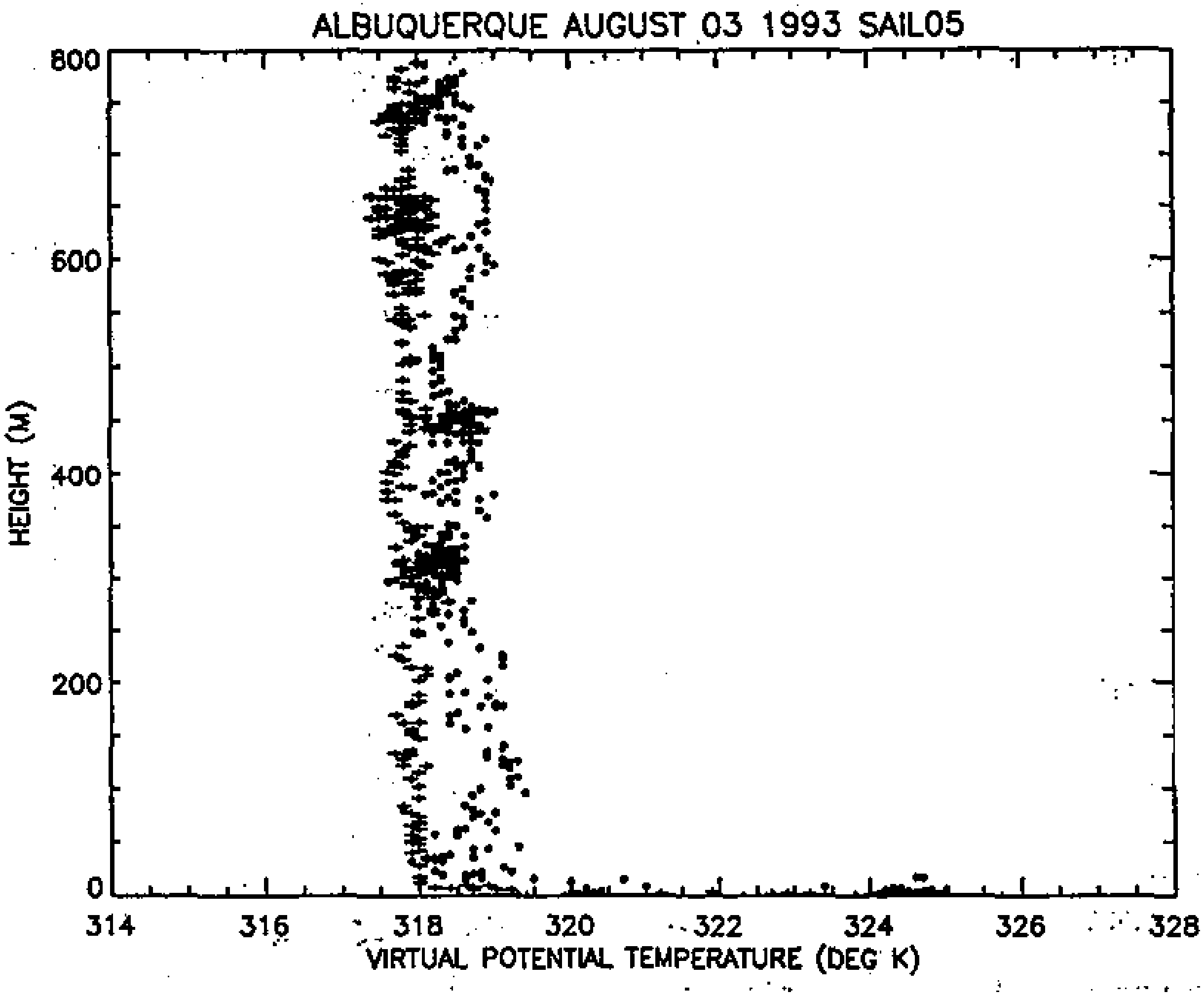
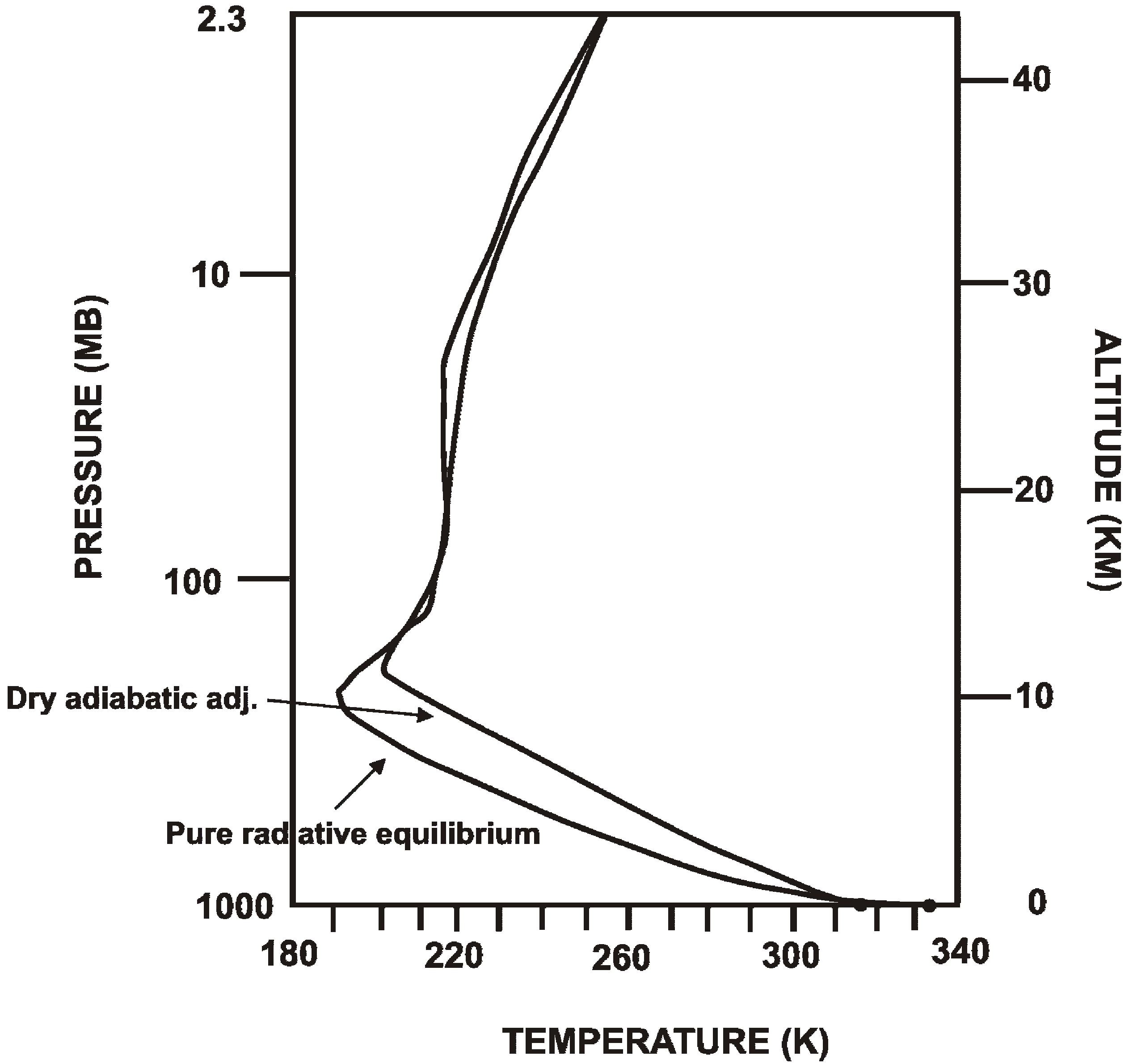


Image courtesy of AMS.

Radiative equilibrium is unstable in the troposphere

Re-calculate equilibrium assuming that tropospheric stability is rendered neutral by convection:

Radiative-Convective Equilibrium



Better, but still too hot at surface, too cold at tropopause