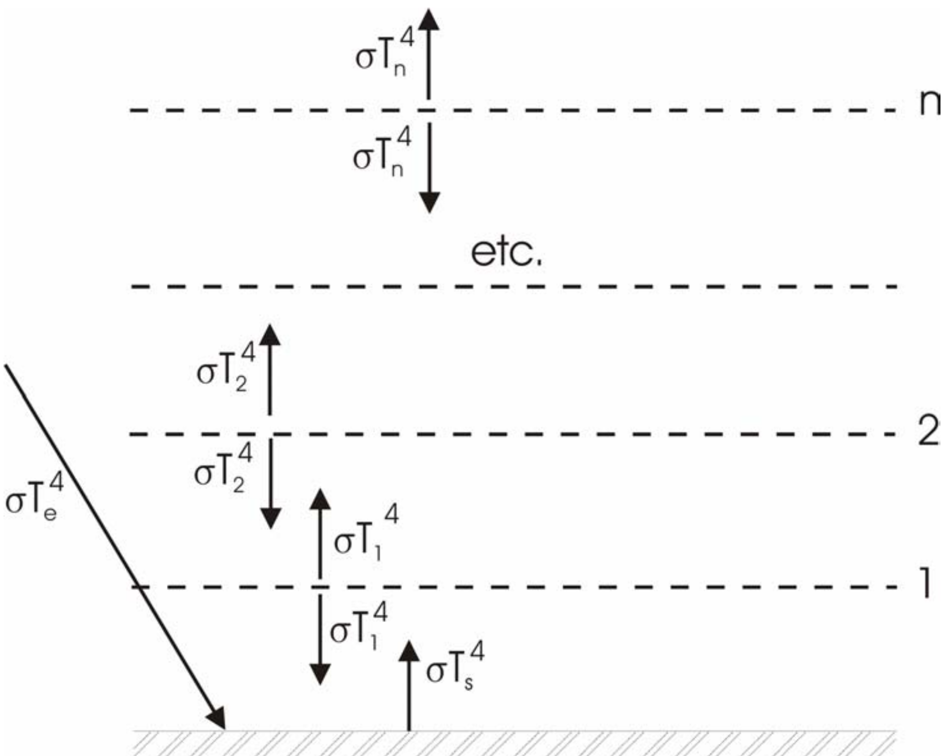
Climate Physics

Problem Set 1
Solutions

Generalize the simple radiative equilibrium problem described in class to an arbitrary number of layers (n), as shown below.



Solve this system to find the temperature of each layer as a function of T_e.

<u>Solution:</u>

First define and index *i* corresponding to the layer number, counting downward. (So *i=1* is the top layer, *i=2* is the next layer down, etc.) Then the radiative balance of each interior layer *i* is

$$2\sigma T_i^4 = \sigma T_{i+1}{}^4 + \sigma T_{i-1}{}^4.$$

This can be written instead

$$\sigma T_{i+1}{}^4 - \sigma T_i^4 = \sigma T_i^4 - \sigma T_{i-1}{}^4.$$

This implies that

$$\Delta T^4 = a,$$

where *a* is a constant. Integrating this difference equation gives

$$T_i^4 = ai + b, \tag{1}$$

where *b* is another constant. Now energy balance at the top of the atmosphere gives

$$T_1 = T_e,$$

while solution of the problem with just two layers gives

$$T_2 = 2^{1/4} T_e$$

This implies that in equation (1), $a = T_e^{\,4}$ and $b = 0$. So the solution (1) may be written

$$T_i = i^{1/4} T_e \tag{2}$$

Now the surface has to be treated as a special case, since all solar radiation is absorbed there. The surface energy balance is

$$\sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_n^{\,4}.$$

Using the solution (2) for T_n in the above gives

$$T_s = (n+1)^{1/4} T_e.$$

This together with (2) constitutes the solution to the problem.