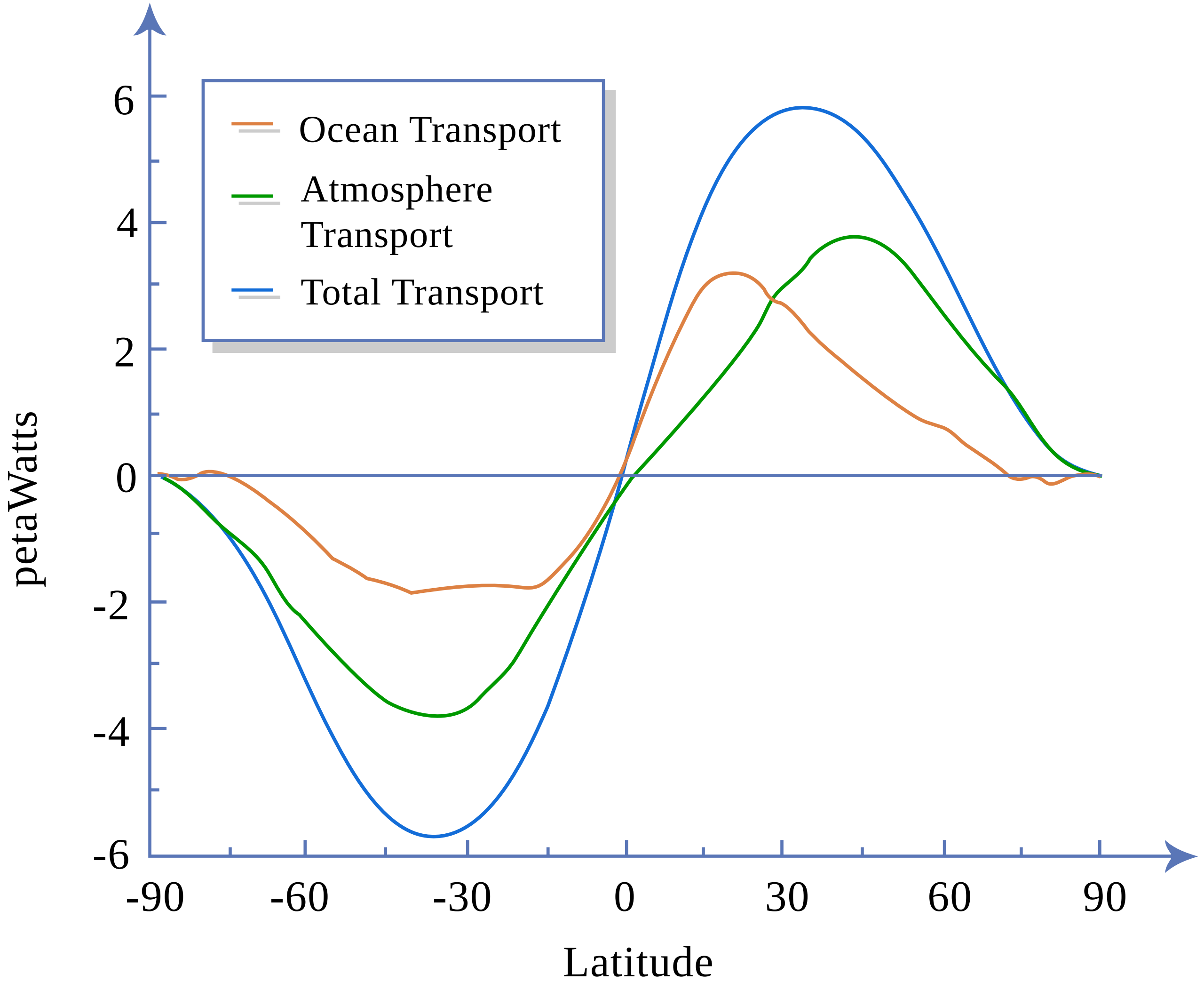
Lateral Heat Transport



Heat Transport by Oceans and Atmosphere

Figure by MIT OCW.

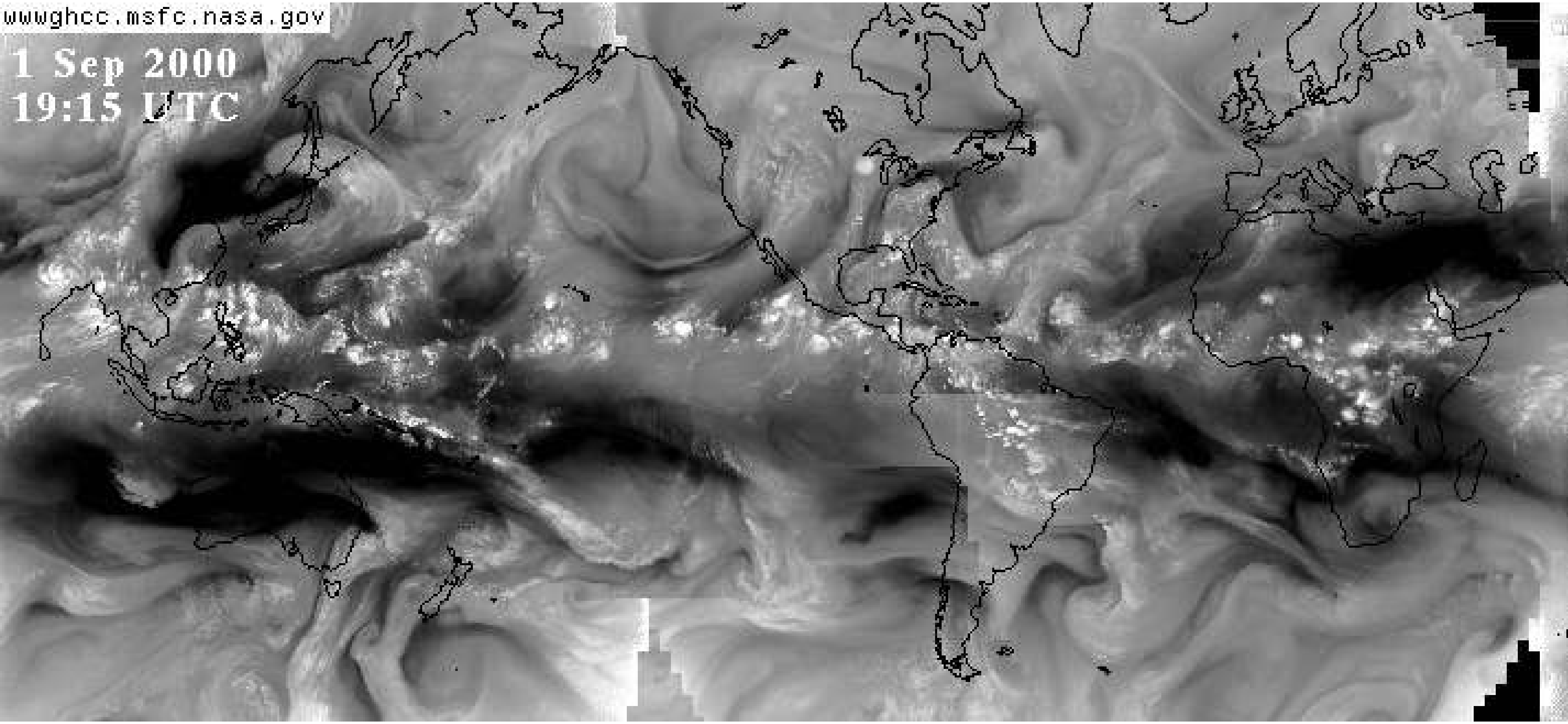


Image courtesy of NASA.

Steady Flow:

$$\nabla \bullet \left[F_{rad}\hat{k} + F_{conv}\hat{k} + \rho \mathbf{V} E \right] = 0,$$

where

$$E \equiv c_p T + gz + L_v q + \frac{1}{2} | \mathbf{V} |^2$$

Integrate from surface to top of atmosphere:

$$\nabla \bullet \overline{\rho \mathbf{V} E} + F_{rad_{TOA}} - \left(F_{rad} + F_{conv} \right)_{surface} = 0$$

What causes lateral enthalpy transport by atmosphere?

1: Large-scale, quasi-steady overturning motion in the Tropics,

2: Eddies with horizontal dimensions of ~ 3000 km in middle and high latitudes

First consider a hypothetical planet like Earth, but with no continents and no seasons and for which the only friction acting on the atmosphere is at the surface.

This planet has an exact nonlinear equilibrium solution for the flow of the atmosphere, characterized by

1. Every column is in radiative-convective equilibrium,

2. Wind vanishes at planet's surface

3. Horizontal pressure gradients balanced by Coriolis accelerations

Hydrostatic balance:

$$\frac{\partial p}{\partial z} = -\rho g$$

In pressure coordinates:

$$\frac{\partial \phi}{\partial p} = -\alpha,$$

$$where$$

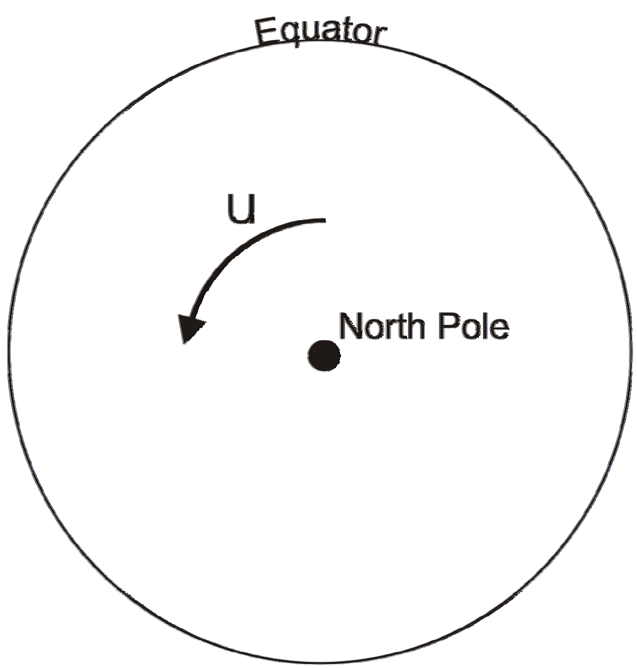
$$\alpha = {}^{1}\!/_{\rho} \equiv specific\ volume,$$

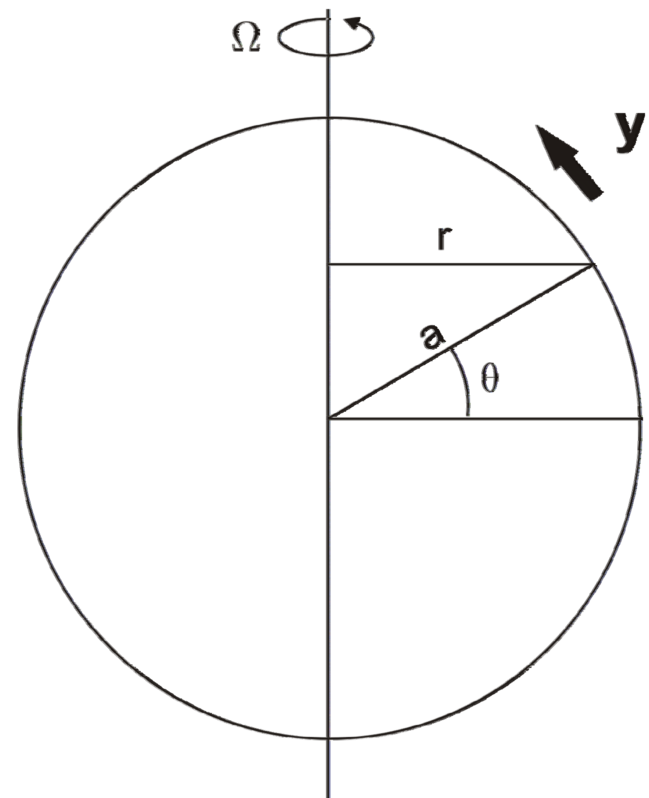
$$\phi = gz \equiv geopotential$$

Horizontal force balance in *inertial* reference frame:

$$\frac{du}{dt} = -\alpha \frac{\partial p}{\partial x}, \qquad \frac{dv}{dt} = -\alpha \frac{\partial p}{\partial y}$$

Rotating reference frame of Earth:





$$\frac{dv}{dt} = -\alpha\frac{\partial p}{\partial y} - \frac{u^2}{r}\sin\theta$$

$$u = \Omega a\cos\theta + u_{rel}, \qquad r = a\cos\theta$$

$$\rightarrow \frac{dv}{dt} = -\alpha\frac{\partial p}{\partial y} - \underbrace{\Omega^2 a\cos\theta\sin\theta} - 2\Omega\sin\theta u_{rel} - \frac{u_{rel}^2}{a}\tan\theta$$

Bracketed term absorbed into defintion of gravity:

$$\frac{dv}{dt} = -\alpha\frac{\partial p}{\partial y} - 2\Omega\sin\theta u_{rel} - \frac{u_{rel}^2}{a}\tan\theta$$

$$\cong -\alpha\frac{\partial p}{\partial y} - 2\Omega\sin\theta u_{rel}$$

$$\equiv -\alpha\frac{\partial p}{\partial y} - fu_{rel}, \quad where\ f \equiv 2\Omega\sin\theta$$

Phrase in pressure coordinates:

$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz = 0$$

$$\rightarrow \frac{\partial p}{\partial y} = -\frac{\partial p}{\partial z}\left(\frac{\partial z}{\partial y}\right)_p = \rho g\left(\frac{\partial z}{\partial y}\right)_p = \rho\left(\frac{\partial \phi}{\partial y}\right)_p$$

$$\frac{dv}{dt} = -\left(\frac{\partial \phi}{\partial y}\right)_p - fu_{rel}$$

Force balance:

$$\left(\frac{\partial \phi}{\partial y}\right)_p = -fu_{rel} \qquad \textit{Geostrophic balance}$$

$$\left(\frac{\partial \phi}{\partial y}\right)_p = -fu_{rel}$$

$$\left(\frac{\partial \phi}{\partial p}\right) = -\alpha$$

$$Eliminate\ \phi:$$

$$f\frac{\partial u}{\partial p} = \left(\frac{\partial \alpha}{\partial y}\right)_p = \frac{R}{p}\left(\frac{\partial T}{\partial y}\right)_p \qquad Thermal\ wind$$

Zonal wind increases with altitude if temperature decreases toward pole

Moist adiabatic atmosphere:

$$s^* = constant$$

$$\alpha = \alpha\left(s^*, p\right)$$

$$\rightarrow \left(\frac{\partial \alpha}{\partial y}\right)_p = \left(\frac{\partial \alpha}{\partial s^*}\right)_p \frac{\partial s^*}{\partial y}$$

$$Maxwell: \quad \left(\frac{\partial \alpha}{\partial s^*}\right)_p = \left(\frac{\partial T}{\partial p}\right)_{s^*}$$

$$\rightarrow f\,\frac{\partial u}{\partial p} = \left(\frac{\partial T}{\partial p}\right)_{s^*} \frac{\partial s^*}{\partial y}$$

Integrate from surface to tropopause, taking u=0 at surface:

$$fu_T = -\left(T_s - T_T\right)\frac{\partial s^*}{\partial y} = -\left(T_s - T_T\right)\frac{\partial s_b}{\partial y}$$

$$\boxed{u_T = -\frac{\left(T_s - T_T\right)}{2\Omega \sin\theta}\frac{\partial s_b}{\partial y}}$$

Implies strongest west-east winds where entropy gradient is strongest, weighted toward equator

Two potential problems with this solution:

1. Not enough angular momentum available for required west-east wind,

2. Equilibrium solution may be unstable

Angular momentum per unit mass:

$$M = a\cos\theta\left(\Omega a\cos\theta + u\right)$$

At tropopause:

$$M_T = a\cos\theta\left(\Omega a\cos\theta + u_T\right)$$

$$= a\cos\theta\left(\Omega a\cos\theta - \frac{\left(T_s - T_T\right)}{2\Omega a\sin\theta}\frac{\partial s_b}{\partial\theta}\right)$$

Maximum possible value of M is its resting value at equator:

$$M_{\max} = \Omega a^2$$

We require that $M \leq M_{\max}$:

$$\frac{\partial s_b}{\partial \theta} > -\frac{2\Omega^2 a^2}{T_s - T_T} \sin^2 \theta \tan \theta$$

Violated in much of Tropics

Violation results in large-scale overturning circulation, known as the Hadley Circulation, that transports heat poleward and drives surface entropy gradient back toward its critical value

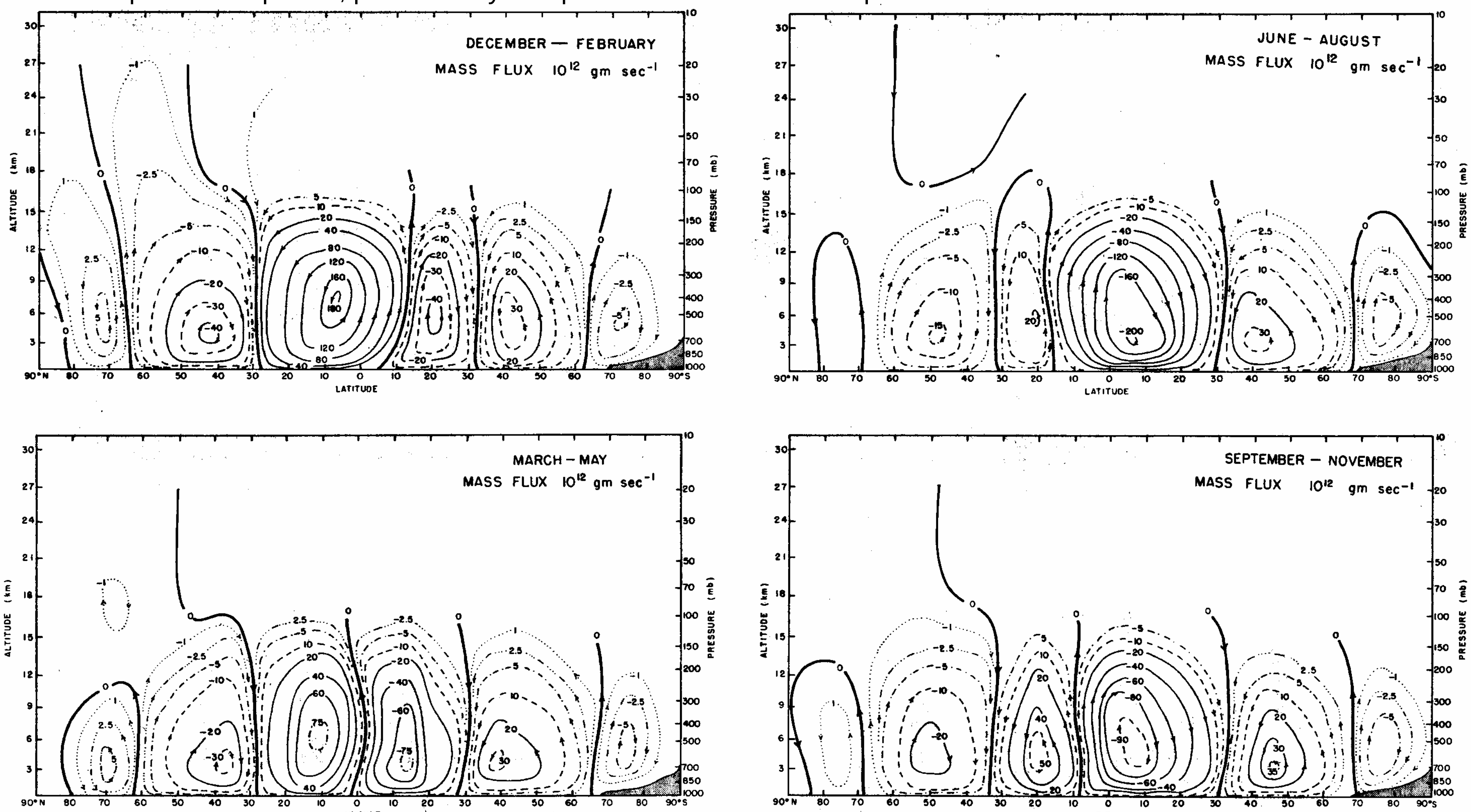


Figure 3.19

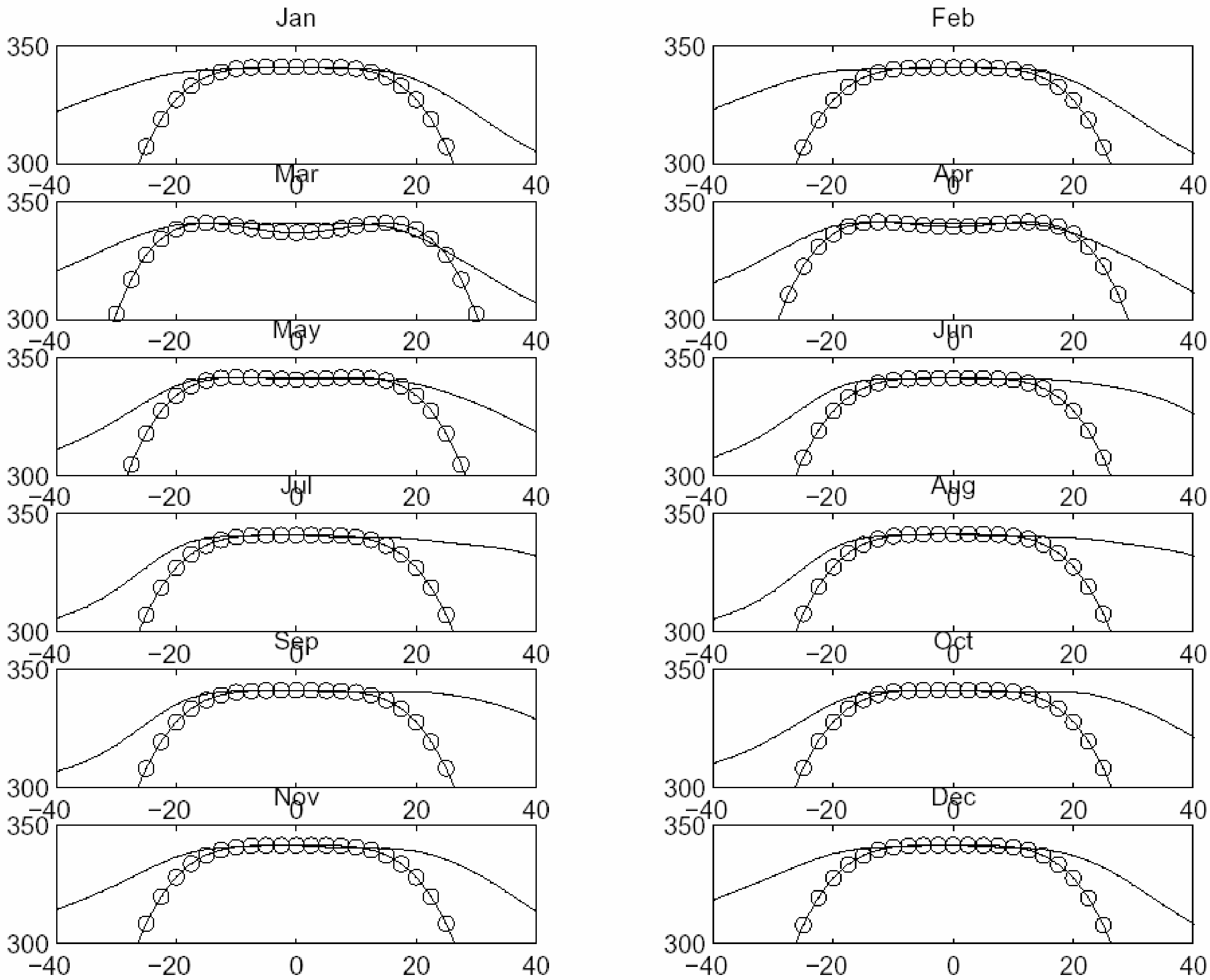


Figure 2: Critical (open circles) and observed (solid line) distributions of θ_{es} for every month at the 600 mb level as a function of latitude

Concept of eddy fluxes:

$$\nabla \bullet \overline{\rho \mathbf{V} E} + F_{rad_{TOA}} - \left(F_{rad} + F_{conv} \right)_{surface} = 0$$

$$\rho \mathbf{V} = \left\{ \rho \mathbf{V} \right\} + \rho \mathbf{V}',$$

$$E = \left\{ E \right\} + E',$$

$$where \ \left\{ X \right\} \equiv \frac{1}{2\pi} \int_0^{2\pi} X d\lambda$$

$$\rightarrow \nabla \bullet \left[\overline{\left\{ \rho \mathbf{V}' E' \right\}} + \overline{\left\{ \rho \mathbf{V} \right\} \left\{ E \right\}} \right] + F_{rad_{TOA}} - \left(F_{rad} + F_{conv} \right)_{surface} = 0$$

Image removed due to copyright restrictions.

Citation: See image of observed eddy heat flux. Oort, A. H., and J. P. Peixoto. "Global Angular Momentum and Energy Balance Requirements from Observations." *Adv Geophys* 25 (1983): 355-490.

Observed annual mean eddy heat flux, from Oort and Peixoto, 1983

Eddy heat fluxes not efficient enough to prevent temperature gradients from developing

Fluxes broadly down-gradient, but not related in a simple way to temperature gradients

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See Figure 1.6 in Hartmann Dennis L. *Global Physical Climatology*. Reading, MA: Academic Press, p. 411. ISBN: 0123285305.