Climate Physics and Chemistry

Entropy Problem

In this simple model atmosphere, the rates of radiative and convective heating cancel when the atmosphere is in energy equilibrium. The energy balance for each layer is

$$\text{Surface}: \quad \sigma T_s^4 + F_s = \sigma T_e^4 + \sigma T_1^4,$$
$$\text{Layer1}: \quad 2\sigma T_1^4 + F_c = \sigma T_2^4 + \sigma T_s^4 + F_s,$$

and

$$\text{Layer2}: \quad 2\sigma T_e^4 = \sigma T_1^4 + F_c.$$

In these equations the convective fluxes, F_s and F_c must be positive to be physically consistent. Also, the assumption of convective neutrality that we impose is

$$T_1 = T_2 + \Delta T,$$

and

$$T_s = T_1 + \Delta T.$$

Top of the atmosphere energy balance requires that $T_2 = T_e$.

1. It follows from convective neutrality and the energy balance that the *radiative* heating of each layer and the surface is given by

$$\dot{Q}_s = \sigma T_e^4[1 + (1+x)^4 - (1+2x)^4] = F_s,$$
$$\dot{Q}_1 = \sigma T_e^4[1 + (1+2x)^4 - 2(1+x)^4] = F_c - F_s,$$

and

$$\dot{Q}_2 = \sigma T_e^4[(1+x)^4 - 2] = -F_c,$$

where $x \equiv \Delta T/T_e$.

2. From the preceding radiative heating rates, the entropy production owing to radiation is

$$\begin{aligned}\dot{s}_{rev} =& \frac{\dot{Q}_s}{T_e + 2\Delta T} + \frac{\dot{Q}_1}{T_e + \Delta T} + \frac{\dot{Q}_2}{T_e} = \frac{F_s}{T_e + 2\Delta T} + \frac{F_c - F_s}{T_e + \Delta T} - \frac{F_c}{T_e} \\ =& \frac{F_s}{T_e}\left(\frac{1}{1+2x} - \frac{1}{1+x}\right) + \frac{F_c}{T_e}\left(\frac{1}{1+x} - 1\right) \\ =& -\frac{F_s}{T_e}\left(\frac{x}{(1+x)(1+2x)}\right) - \frac{F_c}{T_e}\left(\frac{x}{1+x}\right).\end{aligned}$$

Clearly, the radiative entropy production is negative since the convective fluxes are positive.

3. Since there must not be any *net* entropy source in equilibrium, a positive entropy source is required to offset the radiative entropy sink found in 2. Clearly, this process is convection, which is a turbulent, irreversible process. In a dry atmosphere, almost all of the actual irreversible entropy source is owing to frictional dissipation, while it turns out that in our atmosphere, the main source is mixing of moist and dry air in association with cumulus convection.