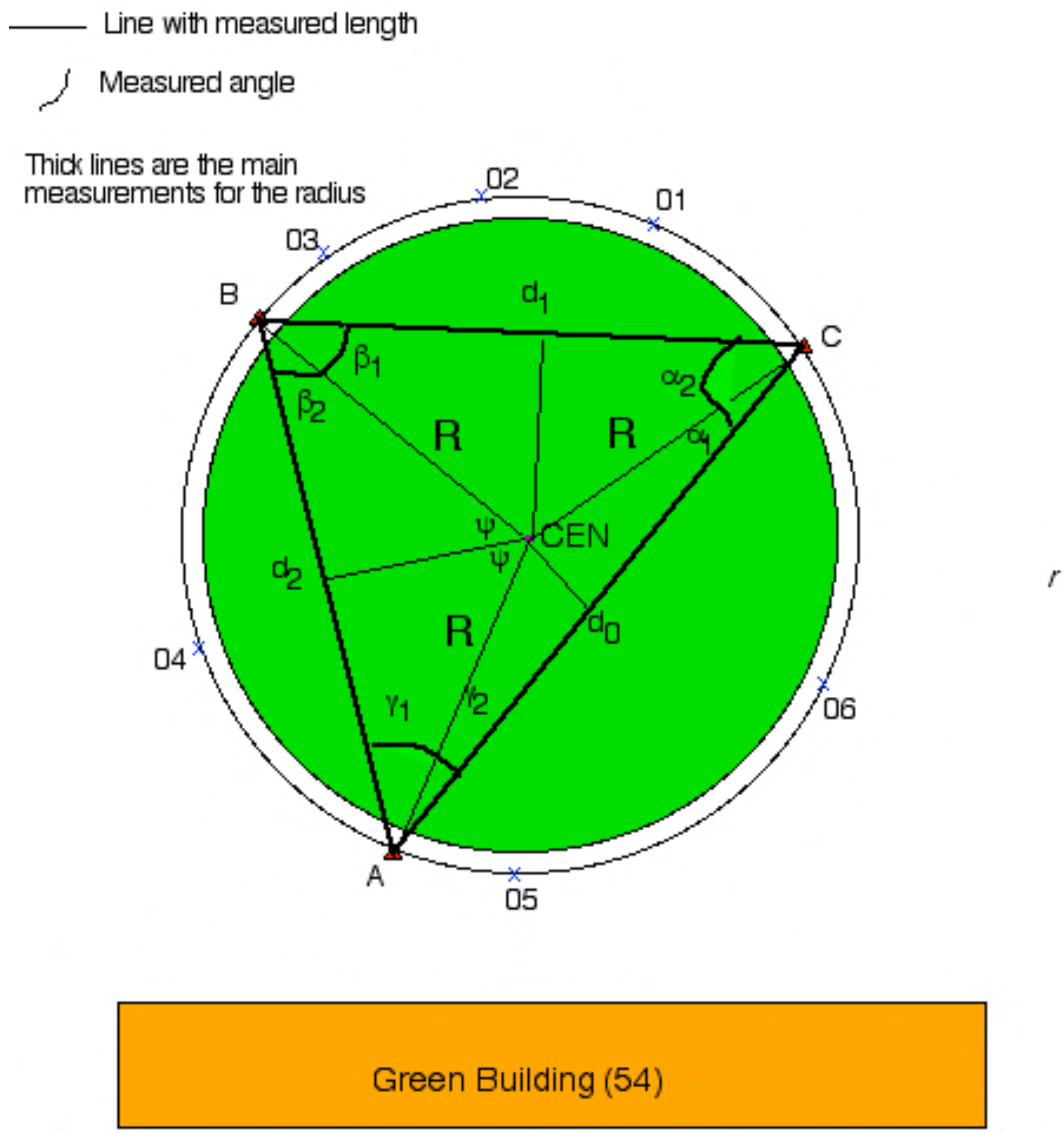
12S56 Project 3 **Monday November 21, 2005.**

From the measurements made you are to determine:
(a) Estimates of the radius from the main tripod measurements
(b) Estimates of the radius for each rod measurements. Plot the radius as a function of angle around the center of the circle.
(c) The distance from the sprinkler in the center of the circle to the center of the center of circle.

Data Set:

At	To	Angle (deg)	Distance (m)	
A	B	0.001	33.358	d2
A	C	68.102	35.182	d0
B	C	0.002	38.350	d0
B	C	58.336	33.258	d1
B	A	0.002	35.185	d1
B	C	53.562	38.350	d2
B	01	109.547	8.624	
B	02	83.564	25.466	
B	03	68.219	33.152	
B	04	31.690	41.405	
B	05	356.853	33.921	
B	06	323.546	15.456	
B	CEN	31.859	20.676	

For the solution this figure has now been labeled with angles and sides. R is the radius of circle computed from the 3 main sites, and r is the radius computed to each of the intermediate points. Angle ψ is the center angle to each of the intermediate points.



Solution:

(a) Using the geometry from the figure above at site 00, we can write two equations for the radius:

$$R\cos\alpha_2 = d_2/2$$

$$R\cos\alpha_1 = d_1/2 \quad \text{where } \alpha_1 + \alpha_2 = \alpha$$

The division of these two equations results in the R being canceled and using the expansion of $\cos(\alpha - \alpha_1) = \cos\alpha\cos\alpha_1 + \sin\alpha\sin\alpha_1$ we can write

$$\frac{\cos\alpha\cos\alpha_1 + \sin\alpha\sin\alpha_1}{\cos\alpha_1} = \frac{d_2}{d_1}$$

By expansion, this equation reduces to:

$$\tan\alpha_1 = (\frac{d_2}{d_1} - \cos\alpha)/\sin\alpha$$

Using the estimate of α_1, we can then solve for the radius R.
For each corner point the results are:

$$\tan\alpha_1 = (\frac{d_2}{d_1} - \cos\alpha)/\sin\alpha \Rightarrow \alpha_1 = 31.6584\,\text{deg} \Rightarrow R = 20.667\ m$$

$$\tan\beta_1 = (\frac{d_0}{d_2} - \cos\beta)/\sin\beta \Rightarrow \beta_1 = 21.9075\ \text{deg} \Rightarrow R = 20.667\ m$$

$$\tan\gamma_1 = (\frac{d_1}{d_0} - \cos\gamma)/\sin\gamma \Rightarrow \gamma_1 = 31.6678\ \text{deg} \Rightarrow R = 20.669\ m$$

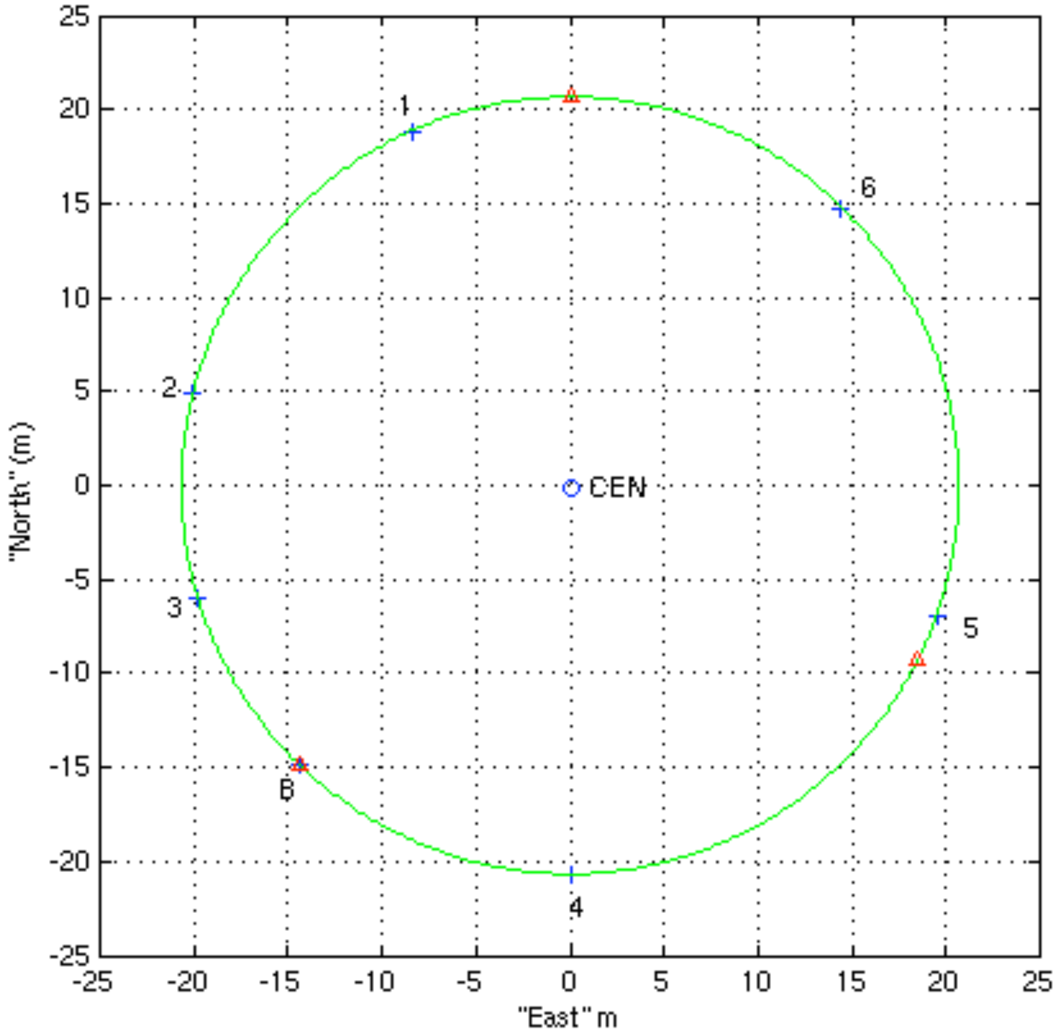
(b) To find the radius to each of the intermediate points, we use the data from site C. The cosine rule is used to solve for r and the sine rule to solve for ψ. To solve these equations we use:

$$\tan\alpha'_1 = (\frac{d}{d_1} - \cos\alpha')/\sin\alpha' \Rightarrow r = d'/(2\cos\alpha'_1)$$

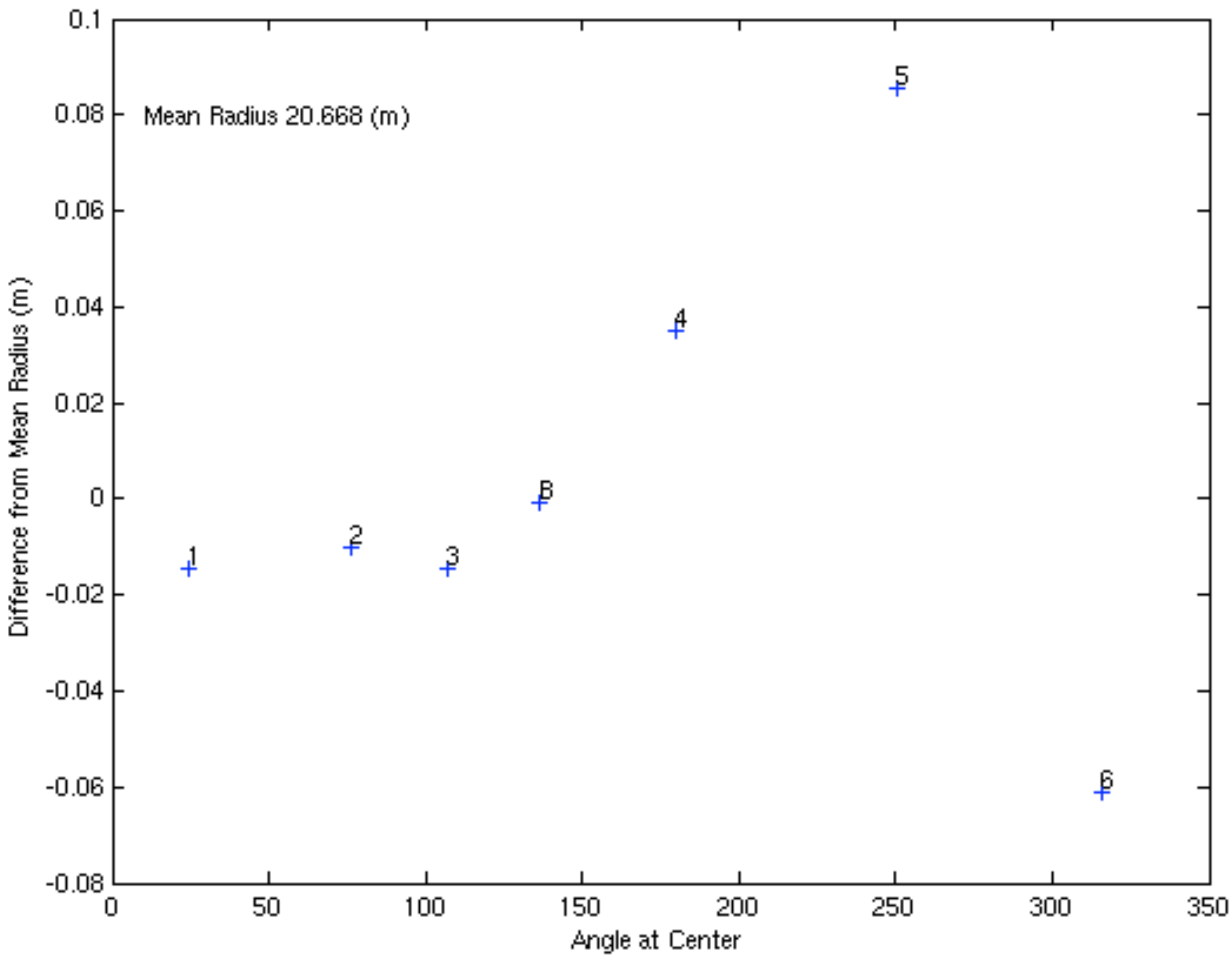
$$\psi = 2[90 - (\alpha' - \alpha'_1)]$$

(c) The position of the sprinkler at the center (CEN) and computed by geometry. If the spigot had been exactly at the center, the distance to it would have been 20.668 m (compared to the measured value of 20.676 m). The difference in position places the spigot 0.073 m from the center at ψ =-84 deg.

The total results are shown in the figure below. ("North" is the direction from the center of the circle to point C, "East" at right angles to this direction.



The residuals to the mean radius and a function of the angle at the center are in the figure below:



This project was solved using Matlab code Proj_3_05.m. The output of the code (in addition to the figures above is:

```
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Results for each angle/distance pair
Alpha     1  31.6584  Radius 1 20.667
Beta      1  21.9075  Radius 2 20.667
Gamma     1  31.6678  Radius 2 20.669
Mean radius 20.668
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 Point       Radius     Drad   Angle
    B     20.667    -0.001  136.1928
    1     20.653    -0.014  24.1016
    2     20.658    -0.010  76.1051
    3     20.653    -0.015  106.7557
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    4    20.703      0.035  180.2562
    5    20.753      0.086  250.3804
    6    20.606     -0.061  315.6779
Sprinkler Position  0.073 (m) at -83.61 deg
```