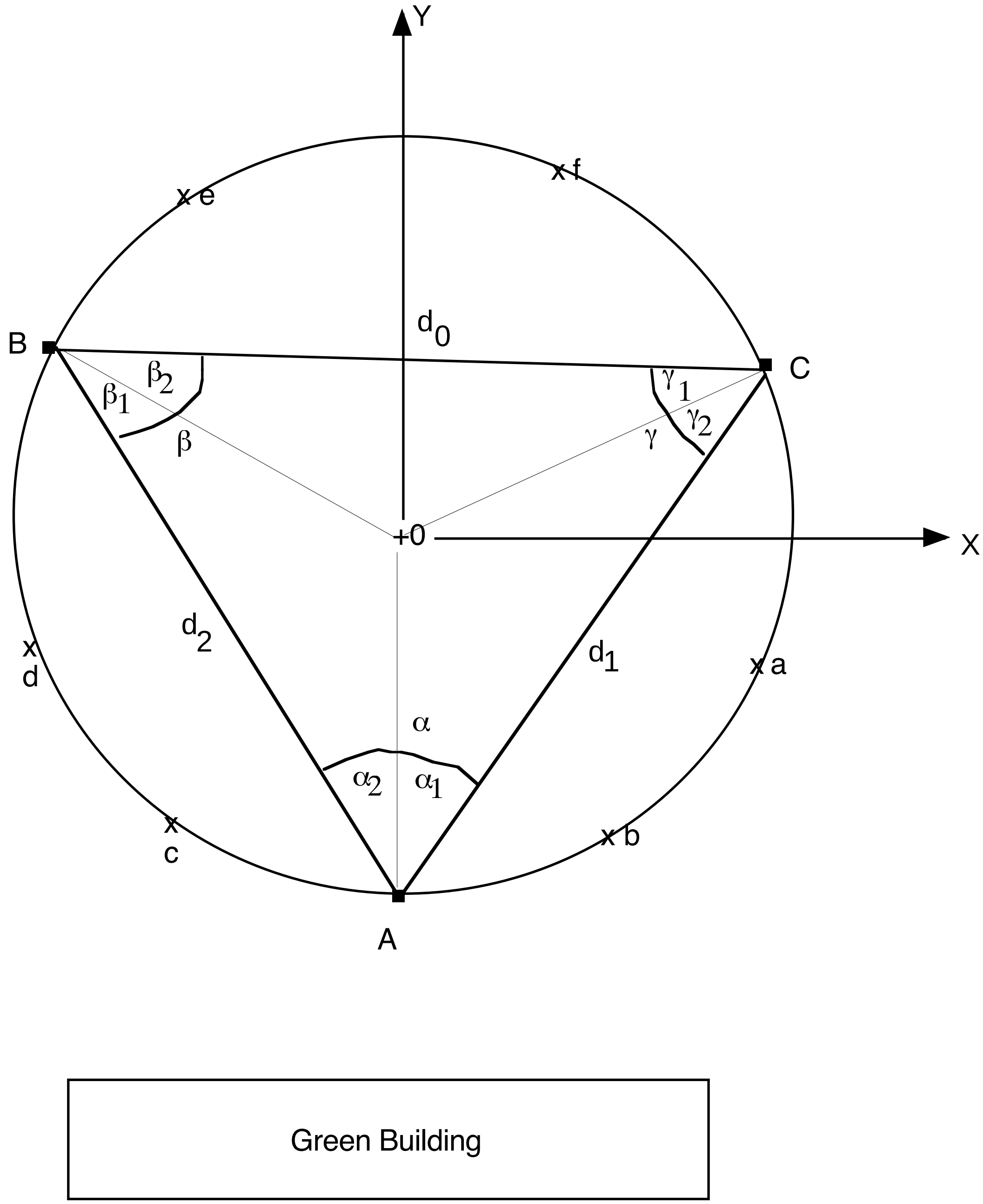
12.S56 GPS: Where Are You?
Fall 2008

**12S56 Circle data    Collected 11/17/2008**




**Data**

| **AT** | **TO** | **Angles (deg)** | **Distance (m)** | |
|---|---|---|---|---|
| A | B | 359.9999 | 33.476 | d2 |
| | C | 74.6273 | 32.221 | d1 |
| B | C | 0.0001 | 39.838 | d0 |
| | A | 51.2465 | 33.476 | d2 |
| C | A | 0.0001 | 32.220 | d1 |
| | B | 54.1232 | 39.838 | d0 |

| | | | |
|---|---|---|---|
| O | 38.8351 | 20.656 | Center |
| a | 322.6759 | 10.032 | |
| b | 340.1612 | 21.625 | |
| c | 16.7340 | 38.323 | |
| d | 37.6851 | 41.375 | |
| e | 69.1422 | 35.558 | |
| f | 103.3052 | 17.551 | |

**Solution:**
Solution adjustment. The first step in the analysis is to make the angles consistent (i.e., sum to 180 degrees). These adjustments are usually made by distributing the "misclose" (the difference from 180 deg), into each angle inversely proportional to the line lengths. In our case the lengths are all about the same length so we add 0.001 deg to each angle. (This corresponds to mis-pointing by ~0.5 mm over the 33-39 meter distances). The distance measurements all agree in the forward and back directions except for one 1 mm difference. The first measurement was adopted.
(a) Using the geometry from the figure above at site 00, we can write two equations for the radius:

$$R\cos\alpha_2 = d_2/2$$
$$R\cos\alpha_1 = d_1/2 \quad \text{where } \alpha_1 + \alpha_2 = \alpha$$

The division of these two equations results in the R being canceled and using the expansion of $\cos(\alpha - \alpha_1) = \cos\alpha\cos\alpha_1 + \sin\alpha\sin\alpha_1$ we can write

$$\frac{\cos\alpha\cos\alpha_1 + \sin\alpha\sin\alpha_1}{\cos\alpha_1} = \frac{d_2}{d_1}$$

By expansion, this equation reduces to:

$$\tan\alpha_1 = (\frac{d_2}{d_1} - \cos\alpha)/\sin\alpha$$

Using the estimate of $\alpha_1$, we can then solve for the radius R.
For each corner point the results are:

$$\tan\alpha_1 = (\frac{d_2}{d_1} - \cos\alpha)/\sin\alpha \Rightarrow \alpha_1 = 38.7499\,\text{deg} \Rightarrow R = 20.658\ m$$

$$\tan\beta_1 = (\frac{d_0}{d_2} - \cos\beta)/\sin\beta \Rightarrow \beta_1 = 35.8791\,\text{deg} \Rightarrow R = 20.658\ m$$

$$\tan\gamma_1 = (\frac{d_1}{d_0} - \cos\gamma)/\sin\gamma \Rightarrow \gamma_1 = 15.3724\,\text{deg} \Rightarrow R = 20.658\ m$$

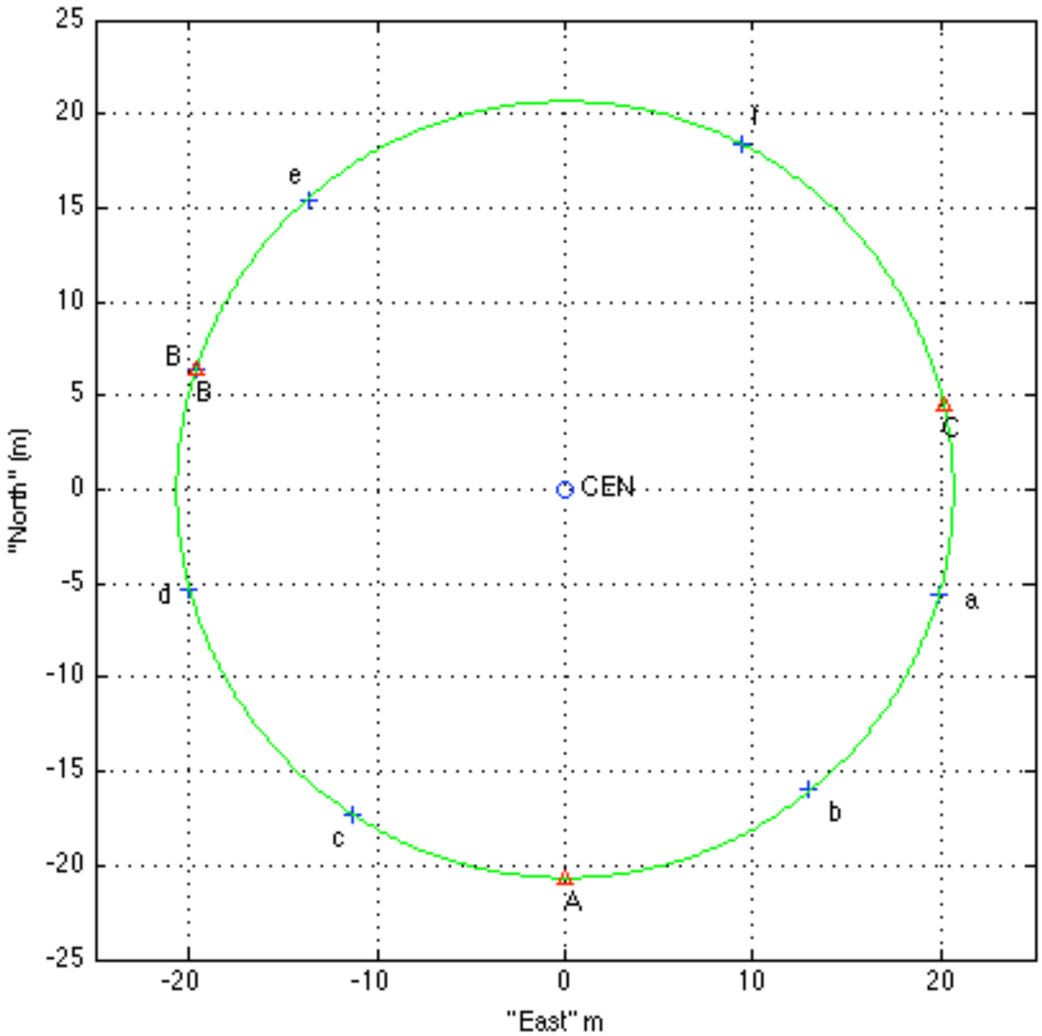
(b) To find the radius to each of the intermediate points, we use the data from site C. The cosine rule is used to solve for *r* and the sine rule to solve for ψ. To solve these equations we use:

$$\tan\alpha'_1 = (\frac{d}{d_1} - \cos\alpha')/\sin\alpha' \Rightarrow r = d'/(2\cos\alpha'_1)$$

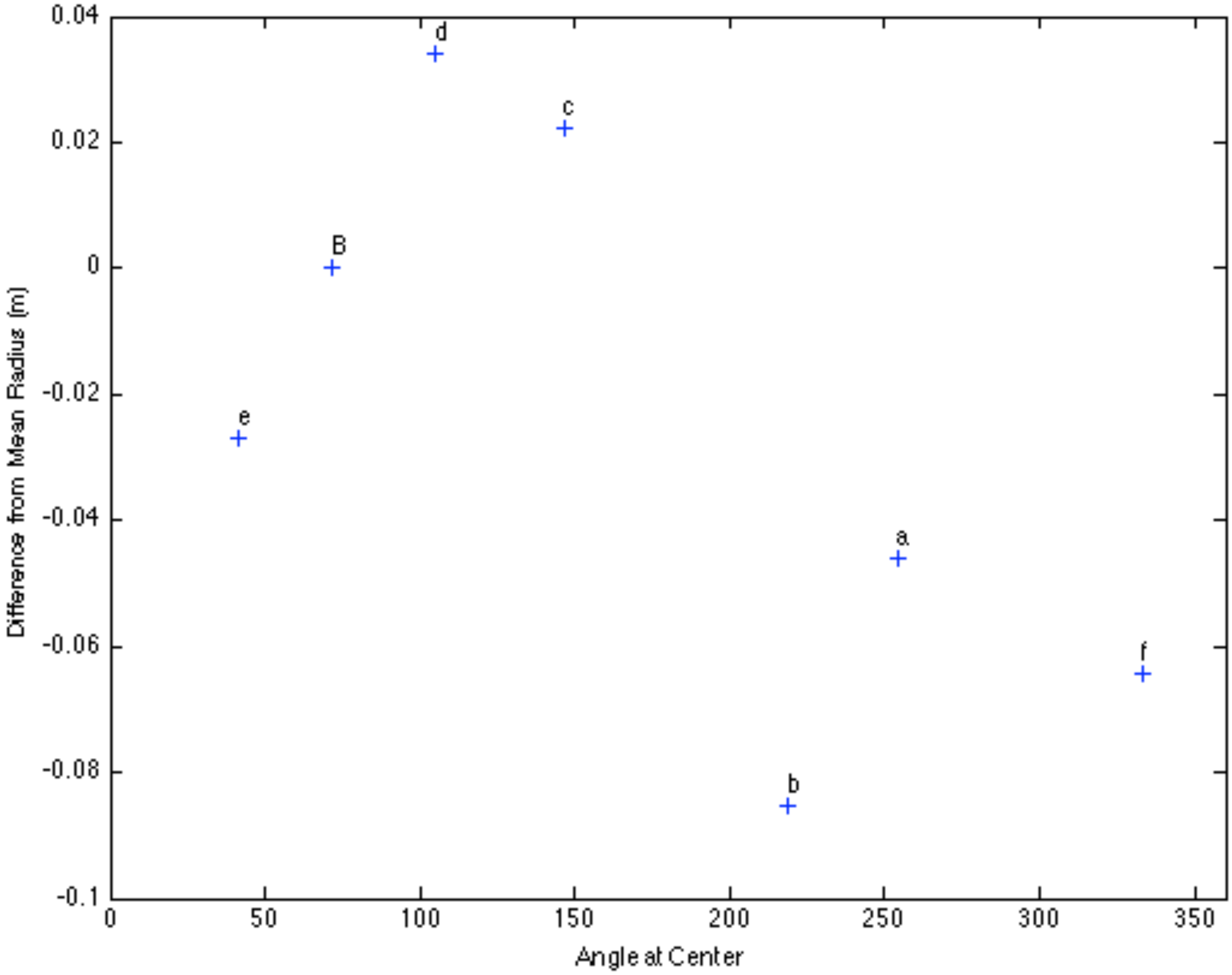
$$\psi = 2[90 - (\alpha' - \alpha'_1)]$$

(c) The position of the sprinkler at the center (CEN) and computed by geometry. If the spigot had been exactly at the center, the distance to it would have been 20.658 m (compared to the measured value of 20.656 m). The difference in position places the spigot 0.031 m from the center at ψ =-93 deg.

The total results are shown in the figure below. ("South" is the direction from the center of the circle to point A, "East" at right angles to this direction.




The residuals to the mean radius and a function of the angle at the center are in the figure below:




This project was solved using Matlab code Proj_3_08.m. The output of the code (in addition to the figures above is:

```
12S56 Project Number 3
Sum of angles in triangle is 179.9971 deg, adding 0.0010 to
each angle
-------12S56 2008------------------------
Results for each angle/distance pair
Alpha    1  38.7499  Radius 1 20.658
Beta      1  35.8791  Radius 2 20.658
Gamma 1  15.3724  Radius 2 20.658
Mean radius 20.658
-----------------------------------
 Point      Radius     Drad  Angle
    B    20.658     0.000  71.7568
```

```
    a    20.612    -0.046  254.3308
    b    20.572    -0.085  219.0855
    c    20.680     0.022  146.6871
    d    20.692     0.034  104.8668
    e    20.631    -0.027  41.5311
    f    20.593    -0.064  332.9446
Sprinkler Postion  0.031 (m) at -93.32 deg
>>
```