

## Fall 2017 14.01 Problem Set 7

### Problem 1: True or False (20 points)

Determine whether the following statements are True or False. Explain your answer.

1. Consider the Cournot duopoly model with linear demand and asymmetric costs, that is, the cost functions are  $C(q_1) = c_1q_1$  for firm 1 and  $C(q_2) = c_2q_2$  for firm 2 (where  $c_1$  and  $c_2$  are constants). In the equilibrium of the game,
  - (a) (5 points) Claim 1: both firms will be producing strictly positive quantities.
  - (b) (5 points) Claim 2: the firm with the lowest cost will be producing a higher quantity than its competitor.
2. (5 points) In a competitive labor market, the market labor supply curve is always upward sloping.
3. (5 points) Consider a price-taking firm in the short run (capital is fixed). If the product produced by a firm becomes more valuable (i.e. its price increases), then the firm will respond by increasing its labor demand.

### Problem 2: Labor Demand (30 points)

Suppose that a representative, perfectly competitive, firm in the market has production function

$$F(K, L) = K^{\frac{1}{2}}L^{\frac{1}{2}}$$

the price of the firm's product is equal to  $p$ , the price per unit of capital is  $r$ , and the cost per unit of labor is  $w$ .

1. (10 points) Consider the short run case when capital is fixed at some level  $\bar{K}$ .
  - (a) What is the firm's choice of labor as a function of  $r, w, \bar{K}, p$ ?
  - (b) Taking parameters  $r, \bar{K}, p$  as fixed, graph the firm's labor demand as a function of  $w$ .

- (c) Now, taking  $r, \bar{K}, w$  as fixed, graph the firm's supply curve as a function of  $p$ . How does the firm supply curve change when  $w$  increases?
2. (8 points) Suppose that  $\bar{K} = 180, r = 4$  and the demand for the firm's product is given by  $Q^D = 10 - p$ .
- (a) Solve for the equilibrium quantity and price in this market as a function of  $w$ . How does the wage affect the equilibrium price and quantity?
- (b) Find the elasticity of labor demand with respect to the wage in this market, taking into account the effect that wages have on equilibrium price and quantity.
3. (8 points) Now consider the long run, with the same demand curve  $Q^D = 10 - p$  for the firm's product and price of capital is still  $r = 4$ , but the firm may freely adjust capital.
- (a) Now solve for the equilibrium price and quantity in the long run as a function of wage. How does wage affect the equilibrium price and quantity?
- (b) Find the elasticity of labor demand with respect to wage in the long run, taking into account the effect that wages have on equilibrium price and quantity.
4. (4 points) Compare your answers in the short run and long run. How does the ratio of the elasticity in the short run to the elasticity in the long run change as  $w$  increases? Explain.

### Problem 3: Taxes and the Labor Market (20 points)

Suppose a worker has preferences over consumption and leisure that can be represented by the following utility function:

$$U = \ln(c) + \ln(l)$$

There are 16 hours per day available for leisure ( $l$ ) and work ( $L$ ) (the other 8 hours in the day are for sleeping). The hourly wage is  $w$ , and assume that the price of each unit of consumption is \$1. The only source of income for this worker is the wage.

- (4 points) Write down the worker's budget constraint in terms of  $c$  and  $L$ .
- (6 points) Find the optimal consumption and work as a function of  $w$ .
- (6 points) Now suppose there are 100 workers identical to the one we analyzed. There are also 100 firms, each one with a production function  $y = \sqrt{L}$ . Suppose that the price of the firms' output is \$1 per unit. Find the labor supply and labor demand curves, and use them to find the equilibrium wage and labor.

4. (4 points) Suppose that the government sets a proportional tax on wages, so for every \$1 paid by the firm, the worker receives  $$(1 - \tau)$ .
- Write down the new budget constraint for the worker and find the optimal consumption and labor.
  - Find the demand for labor by each firm as a function of  $w$ .
  - Find the new equilibrium wage and labor of the labor market

(Note: before solving the problem, try to think about the intuition of how the tax will affect labor supply and demand so you can check your understanding of the economics of the problem, not just the math).

## Problem 4: Firm, Labor Market Equilibrium (30 points)

Suppose that there are two types of workers in the economy: domestic workers and immigrant workers. All workers have preferences over consumption ( $c$ ) and labor ( $\ell$ ) that can be represented by the following utility function:

$$U = \ln(c) - \frac{\ell^{1+\varepsilon}}{1+\varepsilon}$$

where the cost of one unit of consumption is equal to 1. A representative firm under perfect competition, hires the two workers for production according to the production function

$$F(L_I, L_D) = 120(L_I^r + L_D^r)^{\frac{1}{2r}}$$

where  $L_I$  is the total immigrant labor used and  $L_D$  is the total domestic labor. The price of one unit the firm's output is equal to 1 and immigrant and domestic workers are paid wages  $w_I$  and  $w_D$  respectively. The only source of income for a worker is his wage.

- (6 points) Solve for the optimal labor choice of domestic and immigrant workers, taking as given the wage and the price of the consumption good.
- (6 points) Suppose that immigrant and domestic labor are perfect substitutes ( $r = 1$ ), and the economy has 4 immigrant workers and 12 domestic laborers. Find the equilibrium wage paid to workers. (In equilibrium, the total amount of labor supplied by workers must be equal to the total amount of labor used by the firm).
- (6 points) Suppose there is an influx of immigrants so that the total number of immigrant workers increases to 13. What is the equilibrium wage now for domestic workers and has the wage increased or decreased? Explain why this has happened.

4. (6 points) Now consider the case when immigrant labor and domestic labor are complementary ( $r = -1$ ) and suppose that there are 4 immigrant workers and 12 domestic laborers. What is the equilibrium wage now for immigrant and domestic workers?
5. (6 points) Again, suppose that the total number of immigrant workers increases to 13. What is the equilibrium wage for domestic and immigrant workers? How does this compare to your answer in part (4)? Explain why the effect of the additional immigrant workers is different from your answer in part (3).

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