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JONATHAN
GRUBER:

All right, why don't we get started? Today we're going to move on to, finally, the most realistic market structure. We talked about perfectly competitive markets. Now, that was a very useful, extreme example to help us think about economic efficiency. We then flipped over to talk about the somewhat more real estate case of monopoly, but still, very few markets have only one participant. A true monopoly is rare in the private market.

What most markets are marked by are probably more features of oligopoly, which is a market with a small group of firms competing with each other, but with barriers to entry that keep out an unlimited number of firms. Think about these as markets where there are some barriers to entry, so firms just can't consciously enter and exit like they could in our IBM/Dell example, but where there's small enough barriers to entry that a few firms have gotten in, not just one. So it's not a natural monopoly. It's not like only one firm can be in there. Multiple firms are in there, but they only know they have to compete with each other, not with the big, wide world.

So for example, the classic example of an oligopoly industry would be the auto industry. Auto manufacturers clearly compete. Clearly, if you watch any sporting event and watch how much advertising that goes, they're clearly competing with each other. They're comparing to each other all the time.

But most of the cars in the world are produced by fewer than 10 auto manufacturers. The notion that we have a perfectly competitive market of thousands of sellers selling identical goods is clearly not right when it comes to buying a car. So that's the model we're going to want to focus on for the next few lectures.

Now, within an oligopoly market, whenever we think about this market, we want to start by noting that within this market, these limited sets of competitors can behave
in two ways. They can behave cooperatively or non-cooperatively.

Cooperatively means that they can form what's called a cartel. So when there's an oligopoly market and the firms cooperatively get together and make decisions, that's called the cartel, the most famous example of which is OPEC, the Organization of Petroleum Exporting Countries, which are the set of countries that control about 2/3 of the world's oil, led by Saudi Arabia, is the major player in OPEC. It's a cartel of about a dozen nations.

And what they do is they control the vast majority of the world's oil reserves. And by behaving cooperatively, they essentially turn themselves into a monopoly. OPEC acts as if they've got the monopoly in oil. Certainly they used to.

Now it's getting harder. Other non-OPEC countries are starting to produce more oil and it's breaking down. But for a long time, they were essentially the cooperative producer of oil, and act essential like a monopoly, and they made lots of profits like a monopoly. That kept prices high, they kept production inefficiently low, and they made lots of money.

However, that's a great outcome for producers, but as we'll talk about next time, it's actually a hard outcome to enforce. Turns out to be hard to keep cartels together. And so typically, oligopoly markets behave in a non-cooperative way, with the participants competing with each other, not cooperating with each other. In this case, you can actually get them driving their profits down far below the monopoly level, and indeed, perhaps even all the way to the competitive level.

So you can think about markets as a competitive as one extreme and the monopoly as the other extreme, oligopoly in between. A cooperative oligopoly market, like cartel, will end up close to the monopoly outcome. A non-cooperative market will end up somewhere in between, and we're going to model today's where in between do they end up.

Now, to think about this, we're going to have to turn to a tool, which has really become a dominant tool in economics over the last 40 years, which is the tool of game theory. Game theory. So basically, we're going to think of oligopoly firms as engaging in a game.

And as with any game, you need to know two things. One is you need to what's the strategy, and the second is you need to know when is the game over. What's the equilibrium? And that's, essentially, what you do with any game.

And so basically, the key with game theory is that we are going to find the equilibrium, and that's going to yield for us the strategy that players are going to use. However, equilibrium in a game is not well-defined. It's not like a set of rules that are printed out, like monopoly.

In a non-cooperative oligopoly market, the equilibrium, you have to actually come up with different concepts of what equilibrium is. There's not a hard and fast scientific rule. And the typical one that's used is called the Nash equilibrium, the Nash equilibrium, named for John Nash, the famous mathematician, who economists have claimed as their own, even though he was really a mathematician.

But we gave him the Nobel Prize anyway. And if you think of economists, probably one of the most famous, you all know about the movie and book Beautiful Mind. He is based on the father of game theory.

So basically, what is the Nash equilibrium? The Nash equilibrium is defined as the point at which no player wants to change their strategy, given what the other players are doing. So the point at which no player wants to change its strategy, given what the other players are doing.

So in other words, every player is happy with where they are. Given what every other player's decided, I'm happy to do what I've decided. So I've got a strategy, and given the strategy other players are using, if I'm happy with my strategy, then that's in equilibrium.

So this is a super abstract concept, so let's illustrate it with an example. And the classic example of game theory is the prisoner's dilemma, which many of you, I'm sure, know about, maybe the most of you, but let's just go through it. This is the thing from the old cop movies you see, where they arrest two guys and they put them in separate rooms and they basically interrogate them separately.

They're put in separate rooms, and let's say that these guys get told the following. They each get told separately the following thing. They get told that right now, if
nothing else happens, there's enough evidence to send them each away for one year.

However, they're told, if they turned on their friend and say their friend's guilty, then they go free and their friend gets five years. If their friend turns on them, then the friend goes free and they get five years. But if they both turn on each other, they both get two years.

Set up as if they both stay silent, they both get one year. If one turns, then that person gets to leave and the person gets five years. But if they both turn, then they each get two years. So how do we think about decision-making in that context? The way we do that is we write down, we call, a payoff matrix. We write down in matrix form this decision.

So let's think about what a playoff matrix looks like. Up here is prisoner B, and here you have prisoner A. Prisoner A. And prisoner A can remain silent or they can talk, and prisoner $B$ can remain silent or they can talk.

And then we just write down, what are the outcomes, or the payoffs, from these different strategies? So prisoner A says nothing and prisoner B says nothing, then A gets one year and $B$ gets one year. If prisoner $A$ says nothing and prisoner $B$ says, oh yeah, prisoner A is definitely guilty, then prisoner A gets five years and prisoner gets zero years.

If the opposite happens, if prison A says, yeah, B's guilty, and B doesn't say anything about $A$, then $A$ gets zero years and $B$ gets five years. But if they both say the other one's guilty, then they each get two years. OK, that's the payoff matrix.

And now we want to ask, given this payoff matrix, what is the right strategy for each prisoner to pursue? And the way we do this in the Nash equilibrium concept is we look for a dominant strategy. Is there a strategy that I would pursue regardless of what the other person does? And if there is, I'll pursue that.

Because remember, the Nash equilibrium concept is, what do I want to do, given what the other person is doing? If I have a strategy I want to do no matter what the other person is doing, then I'll do it. So when asked, is there a dominant strategy? Is there a strategy that is the best thing to do, no matter what the other guy does?

Well, clearly, if they're cooperating, if these were stupid police and they sat them in the same room, told them and then left, the two guys could cooperate. Well, clearly, the dominant cooperative strategy is for both of us to remain silent. That's the dominant cooperate strategy. And as a team, we only get two total years in jail, where everything gets many more years in jail. So if they're buddies and they trust each other and they cooperate, then that's clearly the right strategy.

But let's say the police are smart and put them in separate rooms. Well, what's the dominant non-cooperative strategy? What is the strategy that A or B, say A, should produce? Yeah? Why?

AUDIENCE:

JONATHAN GRUBER:

Either way, you're going to get less years. Like if you're the only person silent and you talk, you get zero, and if they talk and you talk, you only get two versus five.

Exactly. For prisoner A in this first row-- compare the first column. We'll say prisoner $B$ is silent. Then clearly, you're better off talking than not talking, zero rather than one. Let's say prisoner B talks. Then you're still better off talking than not talking. So no matter what B does, you should talk. Likewise, prisoner B, no matter what A does, $B$ should talk.

So the non-cooperative equilibrium is actually this outcome. They both end up talking. You get sort of a race to the bottom. The non-cooperative outcome is much worse than if they could have cooperated. So basically, what you get is that the non-cooperative equilibrium is always worse for the players than the cooperative equilibrium.

And this was like an unbelievable insight of Nash. Before Nash, we always thought competition was always and everywhere good. We always thought more competition is better, for the reasons we talked in the first 10 or 12 lectures of this class.

Nash was the first one to say, no, actually, competition can be bad. Cooperation can be better. I don't know if you remember the scene in a Beautiful Mind where they're picking up girls in the bar. And he described basically a Nash strategy, how competition will lead to the worst outcome. And basically, that's what you see here, that competition can actually lead to a worse outcome than cooperation, and that
was really Nash's brilliant insight.

Now, this is a cute example with prisoners, but actually-- well first, two points. First of all, this generally shows you how you do gain favor with Nash equilibrium. Basically, you look at the payoff matrix, you find the dominant strategy, and then you find where those dominant strategies intersect. And here, the dominant strategies intersect at this cell, therefore that's the equilibrium.

So that's basically how you do game theory in a game theory kindergarten level. You look at the matrix. You find each player's dominant strategy. And you find the point at which those dominant strategies intersect, and at that point, that's the equilibrium.

Now, that's all well and good for a simple example like this, but let's actually apply to an economics example. Let's think about advertising. So think about Coke and Pepsi. Right now, let's think about their decision to advertise.

Now, obviously it's a simple problem. Obviously Pepsi should just be illegal because Coke is way better. But sadly, it's not, and sometimes I have to drink Pepsi and I'm very sad. But nonetheless, in the real world, we have Coke and Pepsi and they have to decide how much to advertise.

Now, the dominant cooperative strategy would be to say, look, advertising costs us a ton of money. Let's just split the market. Let's have a monopoly market and just split it. We're close to splitting it anyway. Coke's got some more of it. We're close to splitting it. Let's just split it. Yeah?

AUDIENCE:

JONATHAN

## GRUBER:

JONATHAN GRUBER:

AUDIENCE: Can you actually do that? Because I remember, there were places that you get where you aren't allowed to sell in the same place.
Can you actually do that?

What? where you arentallowed to sell in the same place.

OK, but that's different than the cooperative. That's imposed not by Coke and Pepsi jointly. That's imposed by Pepsi saying to a university campus, for example, we will cut you a better deal if you'll agree not to sell Coke. That's not cooperation. That's
competition.

So there's cooperative strategy. What if they don't cooperate? Well, let's imagine we have the following payoff matrix. You've got Pepsi up here, and they can advertise or not advertise. And you've got Coke here, and they can advertise or not advertise.

And let's say the payoff matrix is the following. Let's say the total amount of profit to be made is 16 whatever, billion, whatever units you want to make it, $\$ 16$ billion. And let's say if there's no advertising, Coke gets 8 and Pepsi gets 8 .

But let's say advertising costs money. It costs $5, \$ 5$ billion. So let's say if they both advertise, then they still end up splitting the market, but they only make 3. C equals 3, P equals 3. I'm sorry, advertise. yes, you're right. C equals 3, P equals 3. And here $C$ equals $8, P$ equals 8 .

So basically, you have a situation where they both end of splitting the market either way, but they just split a smaller net profit if they advertise. So clearly, they'd rather be here than here. But what happens in the off diagonal elements? Well, let's say also that if Coke advertises but Pepsi does not, then let's say Coke ends up making $\$ 13$ billion and Pepsi ends up making minus-- I'm sorry, if Coke advertises and Pepsi does not, they split money.

And Pepsi makes negative 2. They actually lose money because they have fixed costs and they don't sell anything. Nobody buys Pepsi. It'll lose money. And let's say if Pepsi advertises and Coke doesn't, then Coke makes negative 2 and Pepsi makes 13. So actually, if you don't advertise, you're really screwed, and the other guy is really screwed. Yeah?

AUDIENCE: Does this include the cost of advertising?

JONATHAN This does include the cost of advertising. But it's just Coke gets a huge market, GRUBER: expands its market. So now let's play the game. Well, now let's say you're Coke.

You say, well, if I advertise and Pepsi advertises, I make 3. But if I don't advertise and Pepsi advertises, I make negative 2. So I should advertise. If I advertise and Pepsi doesn't advertise, I make 13. If I don't advertise and Pepsi doesn't advertise, I make 8.

So either way, my dominant strategy is to advertise. And likewise, Pepsi does the same thing. I screwed up writing this compared to my notes, but it's good because it shows you-- I flipped the matrix, but the logic is the same. It helps you not just memorize cells of the matrix but learn the logic.

The point is either way, the dominant strategy is to advertise, so they both advertise. So real world example of how you can end up. Now, so much of Pepsi and Coke do this. Actually, there was an industry that did this. So when I was a kid, you never, ever saw ads for liquor on TV. There were beer ads and wine ads, but no hard alcohol ad. No bourbon, no whiskey, no nothing, gin.

All these Captain Morgan's ads we see now, they didn't exist when I was a kid. But it wasn't because the law. It was because the hard liquor industry cooperatively agreed none of them would advertise. So they actually imposed the cooperative equilibrium, and then that broke down. I don't know the story of how it broke down. But it broke down.

Now they all advertise, and they're probably all worse off than they were when they didn't advertise. We'll talk next time about why it probably broke down. I don't know the stories. I have a rough sense, and we'll talk about that next time. But this is the point of how a non-cooperative equilibrium can drive you to a bad outcome.

Now, basically, this doesn't just apply to prisoners or businesses. It applies to people, too. So let's say poor Hector back there has had a fight with his girlfriend. And they've had a big fight. They've going a little while. They've had a big fight. And Hector has got to decide, do I apologize or do I wait for her to apologize?

Well, the last thing Hector wants is to go up there and apologize and have her say, forget it. I'm breaking up with you. That'd be the worst. If he knows she's going to be like, oh, I'm sorry, too, then he'd be happy to do it. But what if he goes, no I'm breaking up with you, and she's thinking the same thing.

So what happens, they break up. We've been through this many times in our lives. This is the non-cooperative strategy. Basically, if you know what the other person is going to do, your dominant strategies to be an asshole, and basically that happens a lot in the context of the real world.

So now we have this sad-sounding outcome, that basically game theory leads to bad outcomes for producers, at least. But this is what's exciting about game theory. So when I went to grad school, back when dinosaurs roamed the earth, game theory was taught barely in the sequence. It was like an extra course, taught a little bit. Now it dominates the teaching of microeconomics, in economics. And it doesn't dominate, but it's a whole like component of our core microeconomics education, because it's given such a cool set of tools to think about these decisions.

Now, I can't give you even 1\% of the flavor of game theory. If you want to learn more, I highly suggest you take 1412 , which our game theory class, and you can learn a ton. But let me show you one interesting wrinkle of the things game theory can do, to go beyond this.

And that's to imagine that Coke and Pepsi are not playing a one shot game, but a repeated game. Repeated game. So now imagine that Coke says to Pepsi the following, I promise to not advertise as long as you don't advertise. But if you ever advertise, I will advertise forever.

Coke says to Pepsi, I promise not to advertise as long as you don't advertise, but if I ever catch you advertising, I'm going to advertise forever. So think about Pepsi choice in period 1. Pepsi's choice in period 1. In period 1, they could say, ha, stupid Coke. I'm going to jump on and advertise. They promised not to advertise.

So if Pepsi advertises, they're going to make 13 in period 1 because Coke's taken themselves off to the side. But after period 1, they're going to make 8 forever. No, I'm sorry. They're going to make 3 forever. Because Coke's going to advertise. They're going to advertise. They break down to the non-cooperative equilibrium, if Pepsi advertises.

Now, what if Pepsi doesn't advertise? As long as it doesn't advertise, then it gets to deal with Coke, so it makes 8 forever. We'll talk later in the course about how you combine numbers that happen at different times, but trust me, 8 forever is a way better deal than 13, than 3 forever.

So actually, by having this be a repeated game, Coke has solved the prisoner's dilemma. It's essentially imposed a cooperative equilibrium on the problem. So that's how repeated game can fix this. But-- this is where the game gets really
exciting-- that only works if this game never ends, because once Coke or Pepsi thinks there's an end to the game, the entire thing breaks down.

So imagine, for example, that Coke makes the offer to Pepsi, but Pepsi is worried that in 10 years, the government is going to outlaw soda. The government said, look, we're heading that direction. Soda is going to be illegal in 10 years, so I don't want to do this.

I'm sorry, I have that in my mind. So Coke offers the deal now, what do I think? Well, let's think about Pepsi's decision in the ninth year. They've made 8, 8, 8, 8, 8, and they get to year 9. Now in year 9, they know that next year there's no more game. So what should they do? Advertise. Grab the 13 in the last period, because Coke can't punish them because the game's over.

But Coke knows this. So what's Coke going to do in the ninth year? Advertise. It's going to advertise, so they're both going to make 3. Well, if Pepsi knows Coke's going to advertise in the ninth year no matter what, what should Pepsi in the eighth year? Advertise. And if Coke knows Pepsi is going to advertise in the eighth year, what should Coke do? And so on, and it ends up that they both advertise all the way through.

So the game breaks down if it's an end. This is really kind of neat, and this is what game theory is all about, is how do you think through these more complicated scenarios that are much more complicated than the prisoner's dilemma, and actually think about how firms and individuals might actually behave? Yeah?

AUDIENCE:

JONATHAN GRUBER:

## AUDIENCE:

Wouldn't it also be advantageous if they just advertised the first year instead of these contracts, kind of what we were talking about earlier?

Sure. And once again, that's what you cover in a field course like game theory. What about alternative forms of contracting, with exclusionary contracting, what we call tying in contracting? That's great, and they would. But that's why you got to take 1412, OK? Yeah?

Would it be a better outcome if they cooperated and switched periods of advertising? Like for the first period, they get 13, they get minus 2 .

Yeah, the way I've set this problem up, if they could commit to that, that would be right. But you'd have to commit to it. Because then the period that Pepsi promised to take off, if they actually advertised that period, then Coke's screwed. So that would work as a repeated game solution, but it wouldn't work as a non-repeating game. It would work as an infinite repeated game but not a non-infinite repeated game. Good question. OK, other questions?

All right, so that's the basis of game theory. That's just a taste for the excitement that you can learn with game theory. But in fact, in economics, we like to write those as fun examples, but we really prefer to do math. So let's actually think about the math of how we take game theory concepts and put them in practice.

And the way we do that is through the concept of the Cournot model. The Cournot model of non-cooperative oligopolies. So the Cournot model of non-cooperative oligopoly is the standard workhorse model. It takes this intuition and puts it into the optimizing math we've been doing so far in this class.

Now let's imagine non-cooperative case, but now let's imagine not just two choices, but realistically, there's a whole set of choices. Then how would you behave in that case? So let's imagine that there's two airlines, United and American. So we have an oligopolistic two-firm airline industry.

Obviously, the math can extend to more firms, but just to start, and I'll talk about that next lecture. But for now, imagine a two-firm industry, United and American. And because the hub and spoke system we discussed last time, let's imagine that they're the only two folks that go from Boston to Chicago.

Because it's hub and spoke system. The only folks that go from Boston to Chicago are United and American, and they do, in fact, dominate that line. So let's imagine they're the only folks, and say no other firms can compete on this route because they can't get slots at the airport.

So the question is, how do these firms decide how many flights to run? It's not just advertise, don't. It's literally a continuous decision of how many flights to run every day and how much to charge. They've got to make that decision.

And the Nash equilibrium here, the subset of Nash, for this example, is called the

Cournot equilibrium. And the Cournot equilibrium exists when a firm chooses a quantity such that, given the quantity chosen by the other firm, they don't want to change.

So a firm chooses, essentially, a profit-maximizing quantity, given the quantity chosen by the other firm. And that profit-maximizing quantity, then you're in Cournot equilibrium, if you have chosen a quantity that is profit-maximizing, given what the other firm is doing.

So basically, how do we actually carry this out? Let's talk about the steps. So the first step, I'm going to talk intuitively about the math, what we're going to do, and then I'm going to talk mathematically and graph what we actually do.

There's essentially three steps in solving for the Cournot equilibrium. The first is ask how your demand changes when some of it's absorbed by other firms. So the first is solve for your residual demand function. What does your demand curve look like, given what the other firm does? That's step 1.

Step 2 is then you develop a marginal revenue, which is a function of the other firm's quantity. Little q. It's multiple firms. The other firm's-- that's really bad, hard to read-- the other firm's quality. So your marginal revenue is a function. Typically, it's a function-- I'm sorry-- of both your quantity and the other firm's quantity. A function of both your quantity and the other firm's quantity.

We develop marginal revenue as a function of your own quantity. We know how to do that. Now we develop a margin as a function of your quantity and the other firm's quantity. Then you simply set this marginal revenue equal to marginal cost, and that delivers you a conditional answer. That delivers you your optimal quantity as a function of the other firm's quantity.

Well, that doesn't do us a whole lot of good, except there's two firms. So the fourth step is we do the same thing for the other firm and get the same kind of equation. Then what do we have? Two equations and two unknowns, so we solve.

So what we do here is essentially the same thing we did before, but now your marginal revenue is not just a function of your own quantity, it's a function of the other guy's quantity. Same with the other guy. That gives you two equations, two
unknowns. We solve. And the point at which both firms are happy is the Cournot equilibrium.

That's confusing, so let's actually look at that. We'll do this both graphically and mathematically. Let's start with figure 13.1. To make things easy, let's start by imagining that American Airlines is a monopoly. Let's start with the world with an American Airlines monopoly.

And let's say that the demand function is $P$ equals 339 minus Q . That's the demand for flights from Boston to Chicago. And let's say that the marginal cost, to make life easier-- it could have a cost function and make your life difficult, and maybe someday I'll do that. But for now, to make your life easy, let's just say it's a flat marginal cost of \$147.

I'm not going to make life difficult with solving for marginal cost functions. For now, it's just a flat marginal cost of \$147. No matter how many flights they do, it's \$147 per passenger.

So if you're a monopolist, how do you solve this problem? Well, first you derive your marginal revenue function. Well, what's marginal revenue function? Well, revenues is $P$ times $Q$, which is 339 minus $q$ squared. So your marginal revenue function is 339 minus 2Q. That's your marginal revenue function, if you're the monopolist.

What's your marginal cost? Well, I just said it's $\$ 147$. And then you just solve. And when you solve that, you get that Q , the optimal quantity, is 96 flights. And then how do you get the price? How do you get the price of monopoly problem? How do we know what the price is? Yeah?

AUDIENCE: Where the quantity intersects the demand curve.

JONATHAN You've got to plug it back into the demand curve. Take that quantity, plug it back GRUBER: into the demand curve. So the price is 339 minus 96 , or 243 . So I just solved the monopoly problem quickly, but that's what we've done already in this class.

And you could see that in the graph here. In figure 13.1, you've got demand curve, which is P equals 339 minus Q. You've got a supply curve, which is the flat marginal cost of $\$ 147$. You develop a marginal revenue function, which is 339 minus 2 Q . As in our previous example, that's just basically an inward shift of the demand function.

We have 1,000 passengers per quarter. It doesn't really matter. It's just all standardization. And then to get the price, you read it off the demand curve. You say 96 flights means the price of $\$ 243$ per flight. OK, that's what we do if American was a monopolist.

Now, however, American is not a monopolist. American deals with United, and American doesn't know what United is going to do. So what does American do? Well, let's say American has to deal with the fact-- it now has to recognize that it's got its own demand function, qa, which is the total quantity in the market minus qu. So it has a residual demand function, which is the total demand in the market minus what United sells.

So suppose, for example, American thinks-- American's got a spy inside United-- and American says, ha, I think United is going to fly 64 flights. So imagine American thinks United's going to fly 64 flights. Well, in that case, if they're going to fly 64 flights, then my demand function is p sub a equals 339 minus $q$ sub a minus 64, because the big quantity is little qa plus little qu.

So my demand function is 339 minus q sub a minus 64. Or in other words, my residual demand function is that p sub a equals 339 equals 275 minus qa. So if I think United's going to fly 64 flights, then my effective demand function is 275 minus q. And then I'm done. Then I just solve for, what would I do as a monopolist, given the other guy's flying 64 flights?

So you can see that in figure 13.2. So I have a demand function. I say, well, if United's going to fly 64 flights, that demand function gets shifted in by 64. And then I'm going to do the same thing I did before. I solve for marginal revenue. I'm going to solve for marginal revenue and I intersect that with marginal cost. That's going to happen at 64 flights and a price of $\$ 211$.

So basically, it's the same exercise. It's not that hard. You just first take out what United is going to do. The problem is American doesn't have a spy. They don't really know what United's going to do. They have to essentially develop a strategy, given the possibilities of what United might do. They have to say, look, I don't know what q
sub $u$ is, so I have to devise my optimal strategy given q sub $u$. In other words, I have to simultaneously solve for what I would do at every possible quantity United would sell. I have to solve what I would do for every possible quantity United would sell.

And we call this developing your reaction, or best response curve. Your reaction curve or your best response curve, which is, what is the best thing to do, given what the other guy's doing? What is the best thing to do, given what the other guys doing?

You could see that in figure 13.3, we show how that works. That shows best response curves. So for example, look at the intersection on the $y$-axis, where the red line hits the $y$-axis. That was our monopoly equilibrium.

I'm sorry, where the blue line hits the x-axis, my bad. We're doing American. Look at where the blue line hits the x-axis. That is assuming zero United flights. Where the blue line is the $x$-axis is where there's zero United flights. Well, we know what American would do there. They would fly 96 flights. We already solved that.

Now look at the point where United is flying 64 flights. Well, we also know what American would do then. We know that we solved, in the previous figure, they would then do 64 flights. And in general, what that blue line is is for every quantity that United flies, what does American want to fly?

So meanwhile, United is doing the same mathematics. Imagine, to make life easier-we'll almost always do this to make life easy-- imagine United has the same marginal costs as American, and obviously faces the same market demand curve. Well then, literally, their math is totally symmetric.

If American wasn't in the market, you'd have where the red line intersects the vertical axis. If American was flying zero, United would flight 96 flights, because their problem is identical to American's monopoly problem. So the red line is United's best response curve.

So we've graphed, for every possible amount of flights that United does, what's American's optimal amount of flights. We've solved for every amount of flights that American does with United's also amount of flights. Where those lines intersect is the Cournot equilibrium.

Why is that the Cournot equilibrium? Because at that point, both firms are doing the best they can, given what the other firm's chosen. Or in other words, to say this is given what the other firm's doing, neither firm wants to deviate. The profitmaximizing choice is to be where they are, given the other firm's behavior.

So basically, the Cournot equilibrium is the only equilibrium that's possible in this market. And why is that? So for example, imagine that American came in and said, look, I like doing 96 flights. I love being a monopolist. I'm just going to do 96 flights. I'm going to do 96 flights, I'm going to charge \$243.

Well, in that case, American-- United, I'm sorry-- would happily come in at $\$ 242$ and undercut them and sell lots of flights, because that's still well above marginal cost. So that's not an equilibrium because United and American are choosing different outcomes. It's only equilibrium if they're both to the point where the same outcome makes them both happy.

So that's the graphics. Let's do the math here. Let's do the Cournot math. In general, the residual demand for American is that p equals 339 minus qa minus qu. Remember, big Q is qa plus qu. Since the demand function is 339 minus big Q, I simply broke big Q into qa and qu. Stop me if this is all unclear. Simply broke the big Q into those two components.

So that means that American's revenue function-- it's called revenue A, revenue for American-- is 339 times qa minus qa squared minus qaqu. This is a new term. This was the old revenue function we had when they were monopolists. Now we've got this new term that didn't exist before.

So that means the marginal revenue for American is now 339 minus 2qa minus qu. So now their marginal revenue is actually a function now of their own behavior, but their competitor's behavior. That's the new margin revenue function.

But the profit maximization rule is the same. We just set that equal to marginal cost. We set it equal to 147 , and you solve. And what you end up getting is that $q$ sub a star-- the outcome of q sub a is 96 minus $1 / 2$ qu star, or qu. qa star is 96 minus $1 / 2 q u$. If you solve this equation, that's what you get. That's $1 / 2,1 / 2$ qu.

So now we have the optimal quantity, but it's a function of what the other guy does.

That's a problem, except that we use the same math for United. Now, if the problem's symmetric, you don't have to do the math again. You know the best response function will be symmetric, but that won't always be the case.

So I'm going to shortcut here of saying the best response function for qu is q star u equals 96 minus 1/2qa. So I've just written down the best response function. This corresponds to the graph. So q star a, that's the blue line. It's 96 minus $1 / 2 \mathrm{u}$. q star $u$, that's the red line, 96 minus $1 / 2 q a$. That's their best response function.

Now once again, to remind you, I could simply skip to this step, but normally you'd have to solve through for both firms. They might not have identical best response functions, or symmetrical best response function.

Well now we're golden. We have two equations and two unknowns. We know how to deal with that. And you solve them and you get the qa star equals qu star equals 64. You solve those two equations and two unknowns. So 64 is the solution of that system.

What's the price? Someone raise their hand and tell me. What's the price? Without looking at the graph. Yeah?

## AUDIENCE: [INAUDIBLE]

JONATHAN And how did you get that?

## GRUBER:

## AUDIENCE: [INAUDIBLE]

JONATHAN GRUBER:

You got to plug in 64 twice. A lot of people get this wrong. They'll say, oh, 339 minus 64. But no, it's 339 minus 128, because they're each flying 64 and the price comes from the total demand in the market. So the price is $\$ 211$.

That's an important mistake to avoid. A lot of people get here. They'll be super excited. They're tired. They throw the 64 back at the demand equation. But remember, demand's a function of the total market. If symmetrically they're each doing 64 , then the price is going to be $\$ 211$.
a price of $\$ 211$. Neither firm wants to deviate. And you know that because you've maximized their profits. When United is flying 64, the profits of American are maximized at flying 64. When American's flying 64, the profits for United are maximized at flying 64. Therefore, that is the Nash or Cournot equilibrium.

Now, when we get to reality, things might not always work out so neatly. Things might not be symmetric. You might also not have an equilibrium. How could you not have an equilibrium here? How could that happen graphically? What would that mean? Yeah?

## AUDIENCE: The curves don't intersect.

JONATHAN
GRUBER:

AUDIENCE:

JONATHAN
GRUBER:

If the Nash equilibrium is always worse than when they're cooperating, why is it so hard to maintain a [INAUDIBLE]?

Look, go back to the prisoner's dilemma. All we're doing here was creating best response functions. It's just there wasn't a line. It was just a point. The best response function was what we laid out here. All we did with these United and American examples was go to a continuum and develop best response functions around the best response point. Yeah?
Yeah. The best response curves might not intersect. You might not get an equilibrium. We don't know what the hell to do then. All chaos breaks loose. But you might not get an equilibrium in this market because the best response curves might not intersect.

In reality, in life, you could have funky best response curves that are non-linear or you could have multiple intersections. We call it multiple equilibria. And then it becomes an indeterminate problem and you have to figure out which equilibrium they settle at, and that involves higher order mathematics that you talk about in more advanced classes.

So this is the simplest, easiest cases. Symmetric case where linear best response functions intersect is your easiest case. But in general, the general way to solve this is the same, which is use the principle of game theory. We'll talk about that next time. Other questions? OK, let's stop there. We'll come back. Next time we'll talk about, why don't we all just get along with Mr. Rogers
once?

