14.01 Problem Set 2

Due at 5pm on October 6th, 2023
Late problem sets are **not** accepted.

1 Income and Substitution Effects [10 points]

Ben consumes only apples (x) and t-shirts (y). His preferences can be represented by the following utility function: $U(x, y) = x^2y^3$ The price of apples is p_x, the price of t-shirts is p_y, and Ben has an income of m dollars. Derive Ben's demand for apples and t-shirts as a funtion of p_x, p_y and m. Suppose that initially the prices are $p_x = p_y = 1$ and income is $m = 5$. How many apples does Ben buy? Now suppose that the price of apples increases to $p_x = 2$. How many apples will he buy now? How much of the drop in demand for apples is due to the substitution effect and how much is due to the income effect? Calculate this numerically and show it in a graph.

2 True or False (15 Points)

For each of the following statements, indicate if they are True or False. Justify your answer.

1. *(3 Points)* The short run average cost is always at least as large as the long run average cost (That is, if x is the short run cost and y is the long run cost then $x \geq y$).

2. *(3 Points)* A firm with constant returns to scale in the short run must also have constant returns to scale in the long run.

3. *(3 Points)* In a competitive market, firms maximize profits by choosing the price that equals their marginal cost.

4. *(3 Points)* If a firm's production function exhibits constant returns to scale then it has diminishing marginal returns on every factor of production.

5. *(3 Points)* A firm with increasing returns to scale has decreasing marginal costs.

3 Returns to scale (20 Points)

Consider a firm with the following production function

$$F(K, L) = L^{\alpha}K^{\beta}$$

where α and β are real numbers between 0 and 1.

1. *(3 Points)* How does the returns to scale of the firm's production function depend on α and β?

2. *(2 Points)* Find the marginal product of labor and capital and the marginal rate of technical substitution between capital and labor.

Take as given that the long run total cost curve for this production function is

$$c(q) = (\alpha + \beta) \left(\frac{w}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{r}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}}$$

3. *(5 Points)* Derive the long run marginal cost curve and the average cost curve for this producion function.

4. *(10 Points)* Plot and compare the long run marginal and average cost curves when $\alpha + \beta < 1$, $\alpha + \beta = 1$ and $\alpha + \beta > 1$. Your graph does not need to be to scale but should be qualitatively accurate in terms of comparing the marginal and average cost curves, and their dependence on q. Provide an intuition, using the concept of returns to scale.

4 Short Run and Long Run Cost (30 Points)

Consider a firm with the following production function

$$F(L, K) = L^{\frac{1}{3}} K^{\frac{1}{3}}$$

where L denotes labor and K denotes capital.

1. *(5 Points)* Find the marginal product of labor and capital and the marginal rate of technical substitution for the firm.

2. *(5 Points)* Graph and label the isoquant map.

Suppose the wage is $w = 1$ and the cost of capital is $r = 3$. Assume that in the short run, the firm has fixed capital at $\bar{K} = 1$.

3. *(5 Points)* Derive the firm's short-run total cost curve as a function of quantity q.

4. *(5 Points)* Derive the short-run marginal cost and average variable cost curves and plot them. Your graph does not need to be to scale but should be qualitatively accurate in terms of shapes and points of intersection, etc.

5. *(5 Points)* What is the firm's short-run supply curve? Over what prices will the firm produce a positive quantity?

6. *(5 Points)* As we showed in Question 3, the total long run cost curve for a production function of the type used in this question is

$$c(q) = (\alpha + \beta) \left(\frac{w}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{r}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}}$$

   Derive the total long run cost curve for $F(K, L) = L^{\frac{1}{3}} K^{\frac{1}{3}}$. Is the total cost curve in the long run larger than, smaller than or equal to the total cost curve in the short run when $\bar{K} = 1$?

5 Short Run Cost and Changes in Productivity (25 Points)

Consider a firm with the following production function

$$F(L, K) = AL^{\frac{1}{3}}K^{\frac{1}{3}}$$

where A is an exogenous variable that represents technology.

1. *(5 Points)* Find the marginal product of labor and capital, and the marginal rate of technical substitution for the firm. How does an increase technological improvement, represented by an increase in A, affect the the marginal product of labor, the marginal product of capital and the $MRTS$?

2. *(5 Points)* Draw and compare the isoquants for firm production $q = 30$ for $A = 1$ and $A' = 2$. Explain the intuition behind any changes in these isoquants.

Now consider a firm with the following production function:

$$F(K, L) = (L + A)^{\frac{1}{3}} K^{\frac{1}{3}}$$

3. *(5 Points)* Find the marginal product of labor and capital, and the marginal rate of technical substitution for the firm. How does an increase technological improvement, represented by an increase in A, affect the the marginal product of labor and capital and the $MRTS$? Compare with your result from 1.

4. *(5 Points)* Draw and compare the isoquants for firm production $q = 30$ for $A = 1$ and $A' = 2$. Explain the intuition behind any changes in these isoquants.

5. *(5 Points)* Which of the two production functions shown above is more compatible with the following situation: A car manufacturer can use robots to replace workers in the workplace. Justify your answer.