# 14.01 Problem Set 6

SOLUTIONS

November 18, 2023

## 1 True, False, or Uncertain? (20 Points)

1. *(5 Points)* For a worker, if a rise in wages yields a lower choice of leisure that proves leisure is an inferior good.

   **Solution:** False. If the income effect is small then leisure can be an inferior good but labor supply depend positively on wages.

2. *(5 Points)* Policy interventions such as price controls can improve efficiency when markets are not competitive.

   **Solution:** True. For example, in a typical monopolistic market, a price ceiling can effectively restore perfect competition price and decrease deadweight loss.

3. *(5 Points)* In a two-period intertemporal consumption model where consumption in both periods is a normal good, an increase in the interest rate leads to an increase in consumption in period 1 of a utility maximizing household.

   **Solution:** Uncertain. It depends on the relative strength of the substitution effect vs the income effect.

4. *(5 Points*) Consider a monopsonist who only hires labor to produce a certain good. His marginal cost curve is always below the labor supply curve.

   **Solution:** False. Under monopsony, the firm internalizes that their hiring decisions affects wages. In particular, the more labor they hire the higher the wages. Then, their marginal cost of labor will be above the labor supply curve.

## 2 Monopsony and Labor Supply (45 Points)

Consider a small town where there is only one employer: a coal mine. Since the owner of the mine is the sole employer of this town they have monopsony power. The market for coal is competitive, so the mine takes the price of coal $p$ as given. We normalize the price of coal to one.

Suppose that the workers in this town have identical preferences over consumption and labor, which yields the aggregate labor supply

$$L = w^{\theta}$$

where $w$ are wages, i.e. the cost of labor. There are $N$ identical workers in this town. Suppose the coal mine has the following production function:

$$F(L) = A\sqrt{L}$$

where $A$ is constant, and recall that the firm takes the price of coal $p = 1$ as given.

1. *(5 Points)* Suppose the firm behaved competitively in the labor market, what is the equilibrium wage and labor?

   **Solution:**

   If the firm behaved competitively then from the firm's problem

$$\frac{1}{2}AL^{-\frac{1}{2}} = w = L^{\frac{1}{\theta}} \implies \boxed{L^* = \left(\frac{A}{2}\right)^{\frac{1}{\frac{1}{2}+\frac{1}{\theta}}}} \quad \boxed{w^* = \left(\frac{A}{2}\right)^{\frac{\frac{1}{\theta}}{\frac{1}{2}+\frac{1}{\theta}}}}$$

Now suppose from now on that the coal mine internalizes that its hiring decisions affect wages. When the coal mine behaves like a monopsonist it solves

$$\max \pi = F(L) - w(L) \times L$$

where $F(L)$ is the production function and $w(L)$ is the inverse labor supply

2. *(5 Points)* Derive an expression for the firm's marginal cost of labor, and compare it with the case when the firm takes wages as given. Provide an economic intuition.

   **Solution:** The monopsonist's marginal cost of labor is given by

   $$w + \frac{dw}{dL}L = w\left(1 + \frac{dw}{dL}\frac{L}{w}\right) = w\left(1 + \frac{1}{\theta}\right)$$

   In the firm behaved competitively then $\frac{dw}{dL} = 0$. Now $\frac{dw}{dL} > 0$ because the firm knows that to hire more workers they need to increase wages.

3. *(5 Points)* From the firm's optimality condition, derive a markdown relationship between the marginal product of labor and the wage. Markdown means that the firm's marginal product of labor will be above the wage. How does the markdown depend on the elasticity of labor supply? Provide an economic intuition.

   **Solution:** In an optimum the marginal product of labor is equal to the marginal cost of labor. In this case:

   $$MPL = \left(1 + \frac{1}{\theta}\right) w$$

   and the markdown is $\frac{1}{\theta}$, which depends negatively on the elasticity of labor. If labor is more elastic then the firm knows that small movements in the wage will have a large impact on labor supply, so the markdown will be smaller.

4. *(10 Points)* What is the equilibrium wage? Compare it with the case where the coal mine behaves competitively and provide an economic intuition for the difference. How does the equilibrium wage depend on $\theta$ and $A$? Provide an economic intuition.

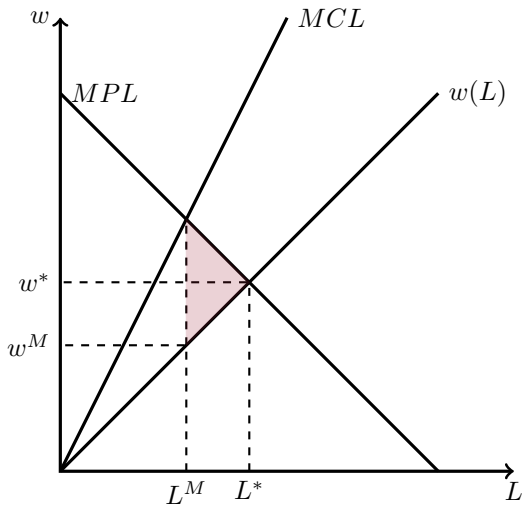
   **Solution:** From the markdown condition:

   $$\frac{1}{2}AL^{-\frac{1}{2}} = \left(1 + \frac{1}{\theta}\right) w \implies \boxed{L^M = \left(\frac{\theta}{\theta+1}\frac{A}{2}\right)^{\frac{1}{\frac{1}{2}+\frac{1}{\theta}}}} \quad \boxed{w^M = \left(\frac{\theta}{\theta+1}\frac{A}{2}\right)^{\frac{\frac{1}{\theta}}{\frac{1}{2}+\frac{1}{\theta}}}}$$

   Notice that both equilibrium wages and labor are smaller. Intuitively, the employer internalizes that by reducing employment they are also reducing their marginal costs. If $A$ is higher then both increase because the marginal product of labor increases and therefore workers are more productive and will be paid more.

5. *(10 Points)* Draw in a general graph (that is, not only for the functional form assumptions of this exercise) with labeled axes the labor supply curve, the marginal product of labor, and the firm's marginal cost. Show the equilibrium labor and wage when the labor market is competitive and under monopsony. Are there any losses in efficiency due to monopsony? If so, show them in the graph and provide an economic intuition.

   **Solution:**




   The area shaded in red represents the loss in efficiency due to the monopsony power. There are workers whose value added is larger than their cost who are not hired because the firm wants to decrease wages.

6. *(5 Points)* Consider a government trying to set a minimum wage to maximize total surplus in the labor market. What minimum wage should it set?

   **Solution:** The government should try to set a minimum wage equal to the equilibrium wage under competitive markets.

7. *(5 Points)* True or False? Provide an economic intuition: a minimum wage always reduces employment.

   **Solution:** False. In the case of monopsony, a minimum wage increases employment.

# 3 Labor Supply(35 Points)

Suppose that workers in Cambridge have identical preferences over consumption an leisure, given by

$$U(c, l) = \log c + \log l$$

Where $c$ is consumption, and $l$ is leisure. We normalize a worker's total hours to one. Let $p$ denote the price of the consumption good and $w$ denote the wage. Cambridge workers get a welfare transfer from the government equal to $p \times T$.

1. *(5 Points)* Write down and graph the worker's budget constraint.

   **Solution:**

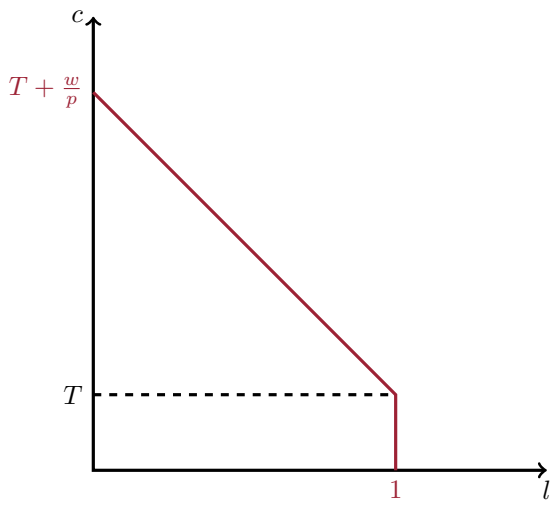
$$pc = w(1 - l) + pT$$

2. *(5 Points)* Find the optimal level of consumption and hours of work for a worker in Cambridge, as a function of $w$, $p$ and $T$.

   **Solution:** At optimium:

$$\frac{MU_c}{p} = \frac{MU_l}{w}$$

$$pc = lw$$




With the budget constraint:

$$w(1-l) + pT = lw$$

which gives:

$$l = \frac{pT + w}{2w}$$

$$h = 1 - l = \frac{w - pT}{2w}$$

$$c = \frac{pT + w}{2p}$$

We still need to check that this is on the interior.

$$l \leq 1 \iff \frac{pT + w}{2w} \leq 1 \iff pT \leq w$$

. If this is not fulfilled, then $l = 1$ and $c = T$.

3. *(5 Points)* How does the worker labor supply depend the welfare transfer $T$? Provide an economic intuition

   **Solution:** The worker's labor supply decreases as $T$ increases. Increasing $T$ increases the worker's wealth, which has a positive income effect on leisure since it is a normal good but has no substitution effect. Therefore, hours worked decreases.

4. *(5 Points)* How does an increase in wages impact the labor supply compared to a decrease in prices? Provide an economic intuition.

   **Solution:** They have the similar effect. The worker only cares about their real wage, that is $\frac{w}{p}$, because they care about how much they can consume given a certain wage.

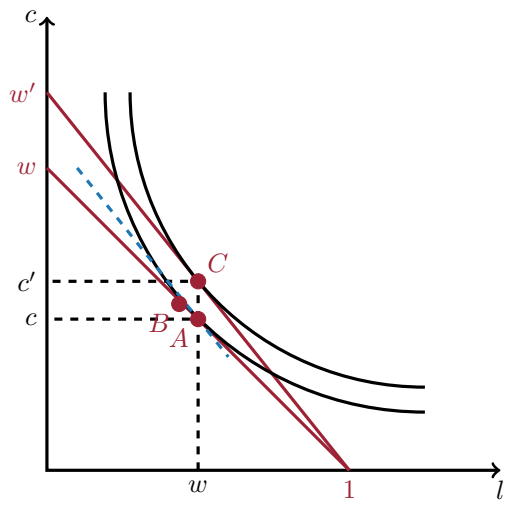
5. *(10 Points)* Suppose initially $T = 0$, $w = 1$ and $p = 1$ and suddenly wages increase to $w' = 2$. Calculate the worker's optimal choice of consumption and leisure before and after the change in wages. In a graph, show how we can decompose this change into a substitution and income effect, and provide an economic intuition of both effects. You **are not** required to quantify the size of the income and substitution effects.

   **Solution:** The choice of leisure is the same before and after the change, $l = \frac{1}{2}$.

   Substitution effect: As wage increases, the opportunity cost of leisure increases and the worker chooses to work more. Income effect: As wage increases, the worker is relatively wealthier and consumes more leisure (a normal good). These effects cancel out for this utility function.

   For consumption, $c = w/2$. So before, $c = 1/2$ and after, $c = 1$. This is because the substitution and income effect are both acting in the same direction.

   Graphically




- Substitution effect: $A \to B$
- Income effect: $B \to C$

Suppose from now on that $T = 0$, $p = 1$, and there are $N$ identical workers in Cambridge, and labor demand is given by

$$L = N - w$$

6. *(5 Points)* Find the equilibrium wage and aggregate labor. How do they depend on the quantity of workers, $N$? Provide an economic intuition.

   **Solution:** Each worker always supplies $\frac{1}{2}$ hours of labor (from previous parts) so the aggregate labor is $\frac{N}{2}$. Plugging this equilibrium quantity to the labor demand, we find $w = \frac{N}{2}$.

   Both vary linearly with $N$. This is expected for the aggregate labor since the labor supply is perfectly inelastic. The wage also varies linearly with $N$ since the labor demand is always constant, so an increase in $N$ must be accompanied by an increase in $w$.

# 4 Intertemporal Consumption (30 Points)

Consider a two-period model of consumption choice by an individual. Suppose she receives an income $I_1$ in the first period and $I_2$ in the second period (in addition to her savings). Let $c_1$ denote this individual's consumption in period one and $c_2$ her consumption in period two. The interest rate is $r$, so if she receives $1 + r$ dollars in period two for every dollar saved in period one. Assume that the price of consumption in both periods is equal to one.

1. *(5 Points)* Let $s$ denote this consumer's savings in period one. Write down the consumer's budget constraint in period one and period two, and combine them to obtain the intertemporal budget constraint. Draw the intertemporal budget constraint in a graph with $c_1$ on the $x$ axis and $c_2$ on the $y$ axis.

   **Solution:** The per-period budget constraints are

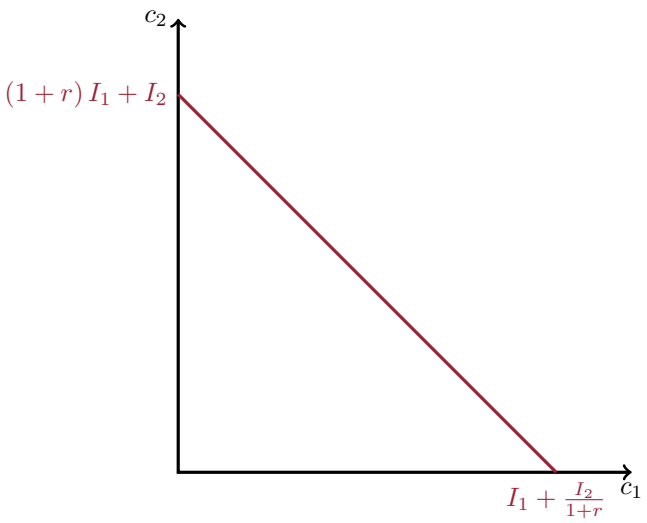
   $$c_1 + s = I_1 c_2 = I_2 + (1 + r)s$$

   The intertemporal budget constraint is

   $$c_1 + \frac{c_2}{1 + r} = I_1 + \frac{I_2}{1 + r}$$

2. *(5 Points)* Does an increase in the interest rate make this consumer richer or poorer? Provide an economic intuition.

   **Solution:** An increase in the interest rate makes this consumer poorer because it reduces the present value of future income. Now, a dollar in the second period is worth less from a period-one perspective.




Suppose this consumer's preferences are given by

$$U(c_1, c_2) = \log c_1 + \beta \log c_2$$

where $\beta \in (0, 1)$ is the discount factor, which represents that consumers do not like postponing consumption. From now on, assume $I_2 = 0$.

3. *(5 Points)* Write down the consumer's intertemporal choice problem, and solve for the optimal choice of $c_1$ and $c_2$.

   **Solution:** The consumer's problem is

   $$\max U(c_1, c_2) = \log c_1 + \beta \log c_2$$

   subject to

   $$c_1 + \frac{c_2}{1+r} = I_1$$

   The optimality condition is

   $$MRS = -\frac{\frac{dU}{dc_1}}{\frac{dU}{dc_2}} = -\frac{c_2}{\beta c_1} = -(1+r) \implies c_2 = \beta(1+r)c_1$$

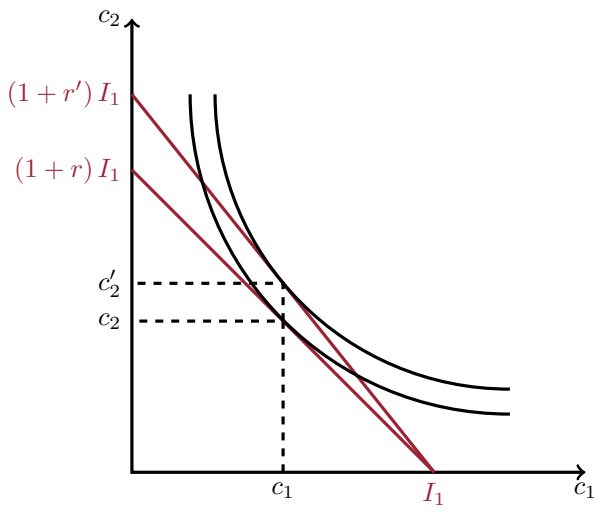
   Into the intertemporal budget constraint

   $$c_1 = \frac{1}{1+\beta} I_1$$

   $$c_2 = \frac{\beta(1+r)}{1+\beta} I_1$$

4. *(10 Points)* Now suppose the interest rate $r$ increases to $r'$. How does this consumer's choices change? Provide an economic intuition. In the same graph, draw the intertemporal budget constraint, the indifference curve tangent to the budget constraint and show the consumer's optimal choices. Plot the new budget constraint and the indifference curve that is tangent to the new budget constraint, as well as the consumer's new choices.

   **Solution:** Consumption on period one does not change, and consumption on period two increases. Intuitively: since $I_2 = 0$ then a change in the interest rate does not affect the present value of income. Thus, a change in the interest rate is only a change in the relative price of consumption on the second period, which now becomes relatively cheaper. From the substitution effect the consumer substitutes from consumption in period one to consumption in period two by increasing their savings. From the income effect it consumes more of both goods because their purchasing power increased. If we add both effects, they cancel out for consumption on the first period and increase consumption on the second period.




5. *(5 Points)* Using the consumer's optimal choices, find an expression for consumption growth, that is $\frac{c_2}{c_1}$, as a function of the interest rate $r$ and the discount factor $\beta$. When is consumption growing? That is, provide a condition so that $\frac{c_2}{c_1} > 1$. Provide an economic intuition.

**Solution:** Consumption growth is given by

$$\frac{c_2}{c_1} = \beta(1+r)$$

Notice that if $\beta(1+r) > 1$ then consumption grows over time. Intuitively, $\beta$ represents how much this consumer dislikes postponing consumption, whereas $1+r$ is how much the market is willing to pay to postpone consumption. If the market is willing to pay more than the consumer's dislike, then they will choose to postpone consumption and there will be positive growth.

# 14.01 Principles of Microeconomics
Fall 2023