14.01 – Principles of Microeconomics

Midterm Exam

SOLUTIONS

Fall 2023

1 True or False (10 Points)

True or False? Justify your answer. Correct answers without the proper justification will not receive credit.

1. *(5 Points)* Suppose a consumer's preferences over goods x and y is $U(x, y) = min\{x, y\}$. Then, if $p_x > p_y$ he does not consume any of y and spends all of his income on good x.

   Solution: False. These preferences correspond to perfect compliments, so the consumer always purchases positive quantities of both goods.

2. *(5 Points)* Suppose the market for shoes is composed by identical firms with production function given by $q = L$, where L stands for labor. Suppose in the short run there are J firms. Then, the number of firms in the long run is also equal to J.

   Solution: True. Since the firm has constant returns to scale then profits are zero, so there is no firm exit or entry in the long run.

2 Consumer Theory (35 Points)

Pedro has preferences over two goods: food x and attending the Boston Celtics' games y. His preferences are given by

$$U(c,l) = x^{\frac{3}{4}} y^{\frac{1}{4}}$$

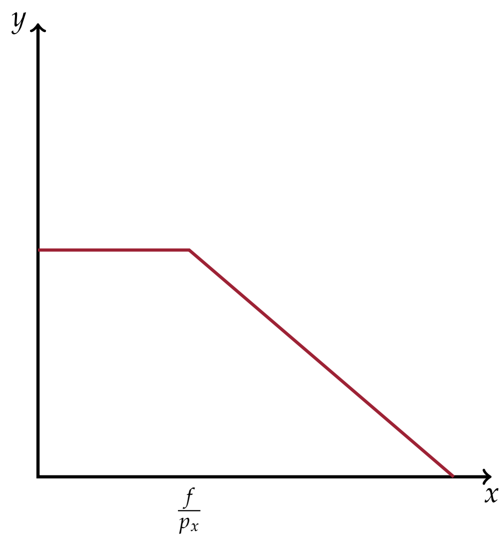
Food has a price p_x and a ticket to attend a Boston Celtics' game costs p_y dollars. Pedro's income is equal to m dollars. Pedro is also eligible for f dollars in Food Stamps. Food stamps is additional income that can only be spent in food. Throughout this exercise assume that $f < 3m$

1. *(3 Points)* Write down and graph Pedro's budget constraint.

   Solution: Pedro's budget constraint is given by

$$\begin{cases} p_y y = m & p_x x \leq f \\ p_x x + p_y y = m + f & p_x x > f \end{cases}$$

   Graphically



2. *(5 Points)* Calculate Pedro's demand for food and tickets, as a function of the prices p_x and p_y, his income m and the value of food stamps f.

   Solution: Conjecture that $x > \frac{f}{p_x}$ because those first units are for free thanks to food stamps.

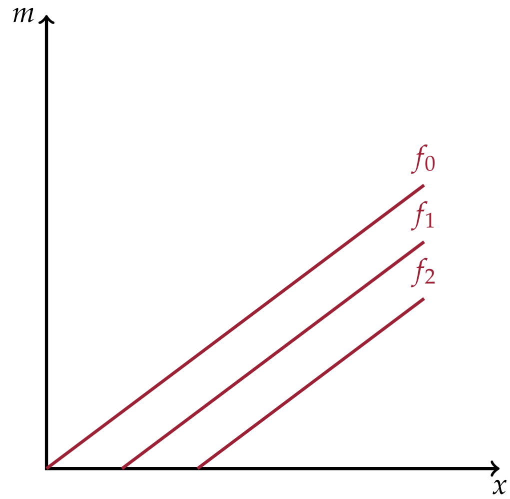
$$x = \frac{3}{4}\frac{m+f}{p_x} \qquad y = \frac{1}{4}\frac{m+f}{p_y}$$

   Now we check that effectively $p_x x > f$:

$$p_x x = \frac{3}{4}(m+f) > f \Leftrightarrow 3m > f$$

   which occurs by assumption.

3. *(5 Points)* Graph Pedro's Engel curve for food for different values of food stamps f and assuming $p_x = 1$. Provide an intuition on how the Engel curve depends on the value of food stamps.



Solution: The demand when $p_x = 1$ is

$$x = \frac{3}{4}(m+f) \implies m = \frac{4}{3}x - f$$

Graphically

where $f_0 < f_1 < f_2$. The Engel curve is decreasing in the value of food stamps because because the higher the their value the lower income needed to consume good x.

Suppose Pedro's income is $m = 1000$ dollars and he receives food stamps for $f = 200$ dollars. Prices are initially $p_x = 1$ and $p_y = 1$. Suddenly, the price of food increases to $p'_x = 2$.

4. *(10 Points)* Calculate Pedro's demand for food and tickets before and after the change in prices. Decompose the effect of a change in prices into substitution and income effect. Graph Pedro's consumption choices, as well as the decomposition into substitution and income effect.

   Solution: Let (x, y) and (x'', y'') denote the consumption bundles before and after the price change. We solve for the intermediate bundle (x', y') by using the conditions $MRS = -\frac{p'_x}{p_y}$ and $u(x', y') = u(x, y)$. Then we can solve for the substitution and income effects.

   Pedro's demand as a funtion of p_x, p_y, m, and f is

$$(x, y) = (\frac{3(m+f)}{4p_x}, \frac{m+f}{4p_y})$$

   Initially $p_x = 1$ and $y = 1$, so $U_i = 653.85$. The initial consumption (x, y) and the consumption after price change (x'', y'').

$$(x, y) = (900, 300)) \qquad (x'', y'') = (450, 300)$$

   We use the following system to find the income needed to attain the level of utility U_i at the new prices:

$$MRS = -\frac{p_x}{p_y} \qquad u(x, y) = U_i$$

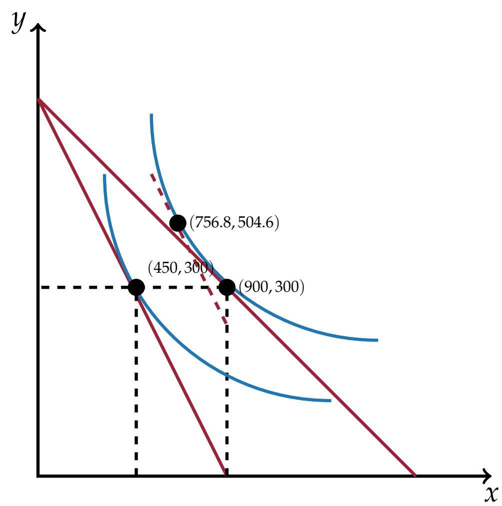
   Replacing our values

$$-\frac{3y}{x} = -2 \qquad x^{3/4}y^{1/4} = 683.85$$

From the first equation we have $3y = 2x$. In the second equation

$$(\frac{3y}{2})y^{1/4} = 683.85 \implies x \approx 756.8 \qquad y \approx 504.6$$

Then, the drop in the demand for food due to the substitution effect is equal to $900 - 756.8 = 143.2$, and the drop due to the income effect is equal to $756.8 - 450 = 306.8$.



5. *(5 Points)* Calculate the increase in food stamps necessary for Pedro's utility to be unchanged after the change in prices.

   Solution: Pedro's welfare is unchanged at the new prices p'_x and P_y when he consumes x' and y'. Then he

   $$p'_x x' + p_y y'$$

   So the additional income in food stamps needed is

   $$\Delta f = p'_x x' + p_y y' - m - f$$

6. *(2 Points)* Suppose instead of food stamps Pedro receives a stimulus check from the Federal Government. Let s denote the size of the stimulus check. What are the new demands for food and tickets as a function of prices p_x and p_y, his income m and the stimulus check s

   Solution: Now the budget constraint is equal to

   $$p_x x + p_y y = m + s$$

   And the new demands are

   $$x = \frac{3}{4}\frac{m+s}{p_x} \qquad y = \frac{1}{4}\frac{m+s}{p_y}$$

7. *(5 Points)* What should be the size of the stimulus check s for Pedro to be indifferent between receiving food stamps or the stimulus check?

   Solution: Since $3m > f$ the size of the necessary size of the stimulus check is $s = f$.

3 Short-Run Producer Theory (15 Points)

You own an ice cream factory and employ both capital K and labor L to produce ice cream. Capital is fixed at $0 < \bar{K} < 1$. Your production function is

$$F(L, \bar{K}) = \log(1 + L) + \log(1 + K)$$

where log is the natural logarithm i.e., $\log x = y \iff x = e^y$. Recall that $\log(1) = 0$.

Suppose that you have no capital in the short run (so $\bar{K} = 0$).

1. *(3 Points)* Find the short-run cost function, as a function of w, r, $\bar{K}$ and q.

   Solution: The demand for labor in the short run is $L^{SR}(q) = e^q - 1$, so the short-run cost function is given by

   $$C^{SR}(q) = r\bar{K} + w(e^q - 1) = w(e^q - 1)$$

2. *(5 Points)* Find the short-run marginal cost and average cost. If the firm chooses to produce, how much will it produce as a function of the price?

   Solution: The marginal cost and average cost curves are

   $$MC(q) = \frac{dC^{SR}(q)}{dq} = we^q$$

   $$AC(q) = \frac{C^{SR}(q)}{q} = w\frac{e^q - 1}{q}$$

   If the firm chooses to produce it will produce

   $$p = we^q$$

3. *(7 Points)* Now suppose that $\bar{K}$ can be chosen to minimize cost for given prices and quantity produced. Suppose that $w = r = 1$. Solve for the firm's long run cost function and long-run supply curve.

   Solution: When the firm minimizes costs

   $$MRTS = -\frac{\frac{1}{1+L}}{\frac{1}{1+K}} = -\frac{w}{r} \implies 1 + K = \frac{w}{r}(1 + L)$$

   Replacing in the production function

   $$\log(1 + L) + \log\left(\frac{w}{r}(1 + L)\right) = q \implies 2\log(1 + L) = q + \log\left(\frac{r}{w}\right)$$

   $$\implies \boxed{L = \sqrt{\frac{r}{w}}e^{\frac{1}{2}q} - 1} \quad \boxed{K = \sqrt{\frac{w}{r}}e^{\frac{1}{2}} - 1}$$

   Replacing, the cost function is

   $$C(q) = 2\sqrt{wr}e^{\frac{1}{2}q} - (w + r)$$

   When $w = r = 1$ then

   $$C(q) = 2\left(e^{\frac{1}{2}q} - 1\right)$$

   PLOT HERE

4 Long-Run Production Theory (30 Points)

Now suppose that instead there is a different ice cream factory that the following long-run cost function

$$C(q) = \begin{cases} q^2 + 9 & q > 0 \\ 0 & q = 0 \end{cases}$$

Suppose that the market for ice cream is competitive, and firms take prices as given.

1. *(3 Points)* Calculate the firm's long run supply curve. Does the firm choose to produce a positive quantity at any price?

   Solution: The long run supply curve is given by

   $$p = MC(q) = 2q$$

   The firm will produce as long as $p \geq AVC(q)$. This occurs as long as the price is at least as large as the minimum average variable cost. Solving

   $$2q \geq q + \frac{9}{q} \iff q \geq 3$$

   Then the firm produces as long as $p \geq 6$.

Suppose there are 4 identical firms producing ice-cream in the town. These firms hold ice-cream production licenses so that the number of firms is at most 4. The market demand for ice-cream is

$$Q_D = 30 - p$$

2. *(2 Points)* Write down the aggregate supply of ice cream.

   Solution: Aggregate supply of ice cream is

   $$Q_S = 4q = \begin{cases} 4\frac{1}{2}p = 2p & p \geq 6 \\ 0 & p < 6 \end{cases}$$

3. *(5 Points)* Find the equilibrium price and quantity, as well as the profit made by each firm.

   Solution: In equilibrium $Q_D = Q_S$ so

   $$30 - p = 2p \implies \boxed{p^* = 10} \quad \boxed{Q^* = 20}$$

   Individual profits are

   $$\pi^* = \frac{Q^*}{4}\left(p^* - AVC\left(\frac{Q^*}{4}\right)\right) = 5\left(10 - \left(5 + \frac{9}{5}\right)\right) = 5\left(5 - \frac{9}{5}\right) = 5\frac{16}{5} = 16$$

Suppose that a firm has discovered a new recipe for ice cream without lactose. Since most of the town is lactose intolerant the government decides to shut down the production of ice cream with lactose. Since this ice cream is healthier, people demand it more. In particular, the new demand is

$$Q_D = (30 + A) - p$$

where $A > 0$ is an exogenous constant that reflects the higher popularity of lactose-free ice cream. Only one firm has the patent for this new recipe, and has the same production technology stated above, namely $C(q) = q^2 + 9$ if $q > 0$ and 0 if $q = 0$.

4. *(5 Points)* Solve for the monopolist's optimal choice of quantity and price as a function of A.

   Solution: Revenue is
$$R(q) = qp(q) = q(30 + A - q).$$
   The monopolist equates marginal cost $2Q^*$ with marginal revenue $30 + A - 2Q^*$. The optimal quantity is $Q^* = \frac{30+A}{4}$, yielding $p^* = \frac{90+3A}{4}$.

5. *(5 Points)* Compute the consumer and producer surplus in this market. Is it socially efficient? If not, compute the deadweight loss.

   Solution: The consumer surplus is
$$\frac{1}{2}\left(30 + A - \frac{90+3A}{4}\right)\frac{30+A}{4} = \frac{(30+A)^2}{32}.$$
   The producer surplus is
$$\frac{1}{2}\left(\frac{90+3A}{4} + \frac{30+A}{4}\right)\frac{30+A}{4} = \frac{(30+A)^2}{8}.$$
   The situation is not socially efficient due to a deadweight loss of:
$$\frac{1}{2}\left(\frac{30+A}{3} - \frac{30+A}{4}\right)\frac{30+A}{4} = \frac{(30+A)^2}{96}.$$

6. *(7 Points)* Are consumers better off consuming lactose-free ice cream even though there is a monopolist compared to a situation where there is competition but ice cream is not lactose-free? How does this depend on A? Provide an economic intuition. For full credit your answer must have an algebraic expression as well as an economic intuition. Comparing consumer surpluses without proper intuition will not result in full credit

   Solution: The consumer surplus in a competitive market is
$$\frac{1}{2}(30 - p^*)Q^* = \frac{1}{2}(30 - 10)20 = 200.$$
   Consumers will be better off with lactose-free ice cream if the following inequality holds
$$\frac{(30+A)^2}{32} > 200 \implies A > 50.$$
   One would typically expect consumers to be better off in a competitive market. However, in this case, if the value of A exceeds 50, the higher demand for lactose-free ice cream compensates for the inefficiencies introduced by a monopoly. Therefore, a sufficiently large A value indicates that the consumer preference for lactose-free ice cream is strong enough to outweigh the downsides of a monopolistic market structure.

Now consider a government who wishes to regulate the monopolist.

7. *(3 Points)* If the government can put a cap on the price of ice cream, what price cap should the government choose? Provide an economic intuition.

   Solution: The government aims to eliminate deadweight loss by setting a price cap that makes the monopolist operate like a competitive market. By equating demand $D(q)$ with marginal cost $MC(q)$, we find the socially optimal quantity $q = \frac{30+A}{3}$. The price cap should then be $\frac{60+2A}{3}$ to ensure this quantity is produced, maximizing social welfare.