14.01 Problem Set 1

SOLUTIONS

September 29, 2023

1 Positive and Normative Statements [10 points]

Please classify the following statements as either positive or normative. Briefly explain the reasoning for your choice.

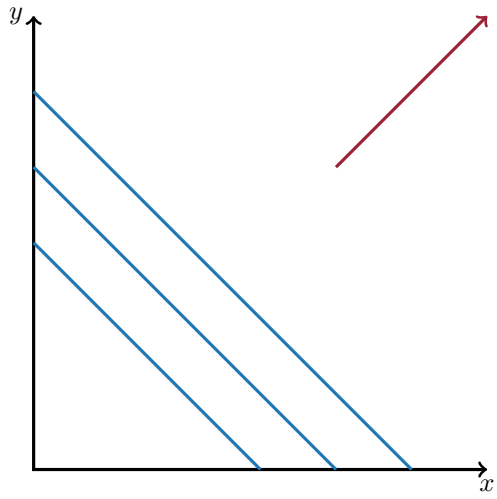
1. *(2 points)* The unemployment rate in the US is currently 4 percent.
   Solution: Positive. It describes the state of the US economy.
2. *(2 points)* The government should increase the minimum wage.
   Solution: Normative. This statement is about what the government should do, which is a value judgement.
3. *(2 points)* Inflation is always and everywhere caused by an increase in government spending.
   Solution: Positive. It establishes a causal relationship between inflation and government spending.
4. *(2 points)* Tax cuts for businesses stimulate growth.
   Solution: Positive. It establishes a causal relationship between taxes and growth, which can be true or false.
5. *(2 points)* Tax cuts for businesses are good for the people of the US.
   Solution: Normative. It is a prescription on what the government should do.

2 Indifference Curves [20 points]

In each of the following examples, the consumer consumes only two goods, x and y. Based on the information given in each statement, sketch a plausible set of indifference curves (draw at least two curves on a set of labeled axes and indicate the direction of higher utility). Then, write down a possible form of the utility function $u(x, y)$ that is consistent with your graph.

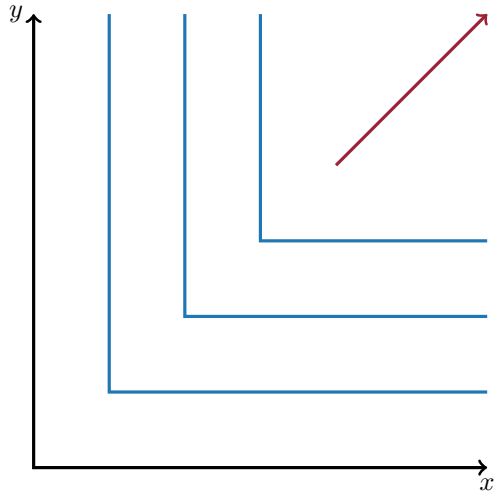
1. *(5 points)* Pedro is thirsty can choose between buying store-brand bottled water (x) or premium-brand bottled water (y). For him, these two types of bottled water are perfect substitutes, and he always chooses the cheaper option without any consideration for the brand name.

   Solution: Pedro only cares about the price, so he chooses the cheapest one. His preferences can be represented by $U(x, y) = x + y$.



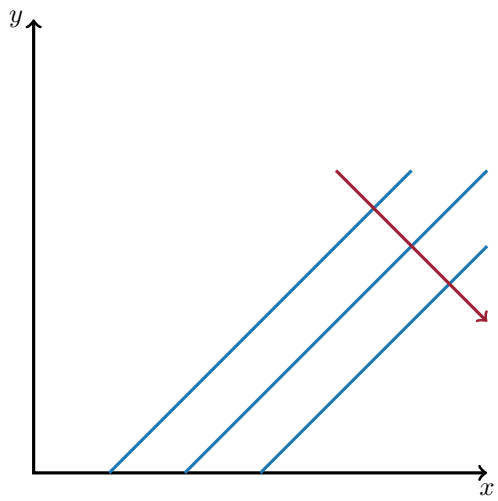
2. *(5 points)* Andrew is a well-known collector of Air Jordans. When considering buying a new model, he needs one right shoe (x) and one left shoe (y) of that model. Just one without the other one is useless to him.

   Solution: Andrew only values having both shoes. His preferences can be represented by $U(x, y) = min\{x, y\}$.



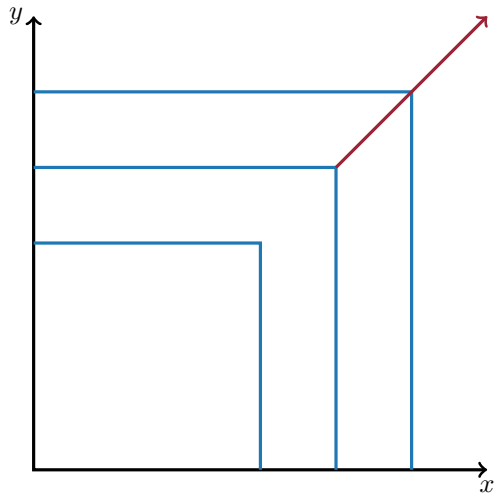
3. *(5 points)* Joshua wants good grades (x) but does not like studying (y). He will trade one extra hour of study for ten more points in the exam.

   Solution: Joshua is indifferent between one extra hour of study if he has ten more points in the exam. His preferences can be represented by $U(x, y) = x - 10y$.



4. *(5 points)* Jon likes consuming apples (x) and bananas (y), but only cares about the one he consumes the greatest quantity.

   Solution: Jon only cares about the greater quantity of apples or bananas. His preferences can be represented by $U(x, y) = max\{x, y\}$.



3 Utility Maximization [30 points]

Tai consumes only wine (x) and cheese (y). His preferences can be expressed by the following utility function: $U(x,y) = xy^3$. The price of wine is p_x, the price of cheese is p_y, and Tai has an income of m dollars.

1. *(2 points)* Write down Tai's budget constraint.

   Solution: Tai's budget constraint is

   $$p_x x + p_y y = m$$

2. *(4 points)* Calculate the Marginal Rate of Substitution (MRS) at an arbitrary bundle (x,y).

   Solution: The marginal rate of substitution between wine and cheese is

   $$MRS = -\frac{\frac{\partial U(x,y)}{\partial x}}{\frac{\partial U(x,y)}{\partial y}} = -\frac{y^3}{3xy^2} = -\frac{1}{3}\frac{y}{x}$$

3. *(4 points)* Compute Tai's demand for wine and cheese as a function of p_x, p_y, and m.

   Solution: The marginal rate of substitution between wine and cheese is

   $$MRS = -\frac{\frac{\partial U(x,y)}{\partial x}}{\frac{\partial U(x,y)}{\partial y}} = -\frac{y^3}{3xy^2} = -\frac{1}{3}\frac{y}{x}$$

   In an optimum

   $$MRS = -\frac{p_x}{p_y} \implies \frac{1}{3}\frac{y}{x} = \frac{p_x}{p_y} \implies y = 3\frac{p_x}{p_y}x$$

   Replacing in the budget constraint

   $$p_x x + p_y 3\frac{p_x}{p_y}x = m \implies 4p_x x = m \implies \boxed{x = \frac{1}{4}\frac{m}{p_x}} \quad \boxed{y = \frac{3}{4}\frac{m}{p_y}}$$

4. *(4 points)* Repeat parts 2 and 3, but this time assuming Tai's preferences are $U(x,y) = x + 3y$.

   Solution: If preferences are $U(x,y)$ then the MRS is constant and equal to $MRS = -\frac{1}{3}$. This means that Tai is willing to trade three units of wine for a unit of cheese. Put it other way, he is willing to pay three times more for a unit of cheese than a unit of wine. Then, there are three cases:

   $$x = \begin{cases} 0 & 3p_x > p_y \\ \in \left[0, \frac{m}{p_x}\right] & 3p_x = p_y \\ \frac{m}{p_x} & 3p_x < p_y \end{cases} \qquad y = \begin{cases} \frac{m}{p_y} & 3p_x > p_y \\ \in \left[0, \frac{m}{p_y}\right] & 3p_x = p_y \\ 0 & 3p_x < p_y \end{cases}$$

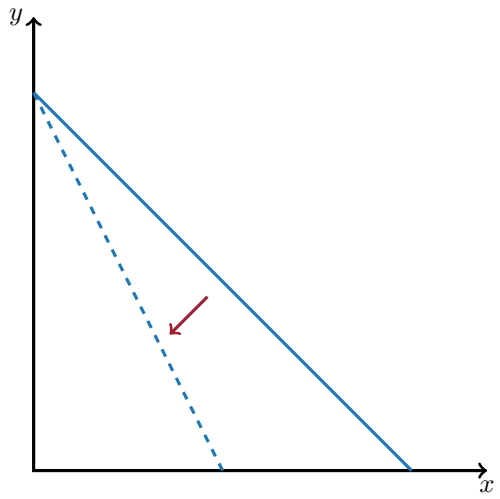
Return to the original preferences given by $U(x, y) = xy^3$. Suppose that the government imposes a tax on wine, such that if the price of a wine is p_x, the consumer must pay $(1 + \tau)p_x$.

5. *(2 points)* What is the new budget constraint? In an xy graph, plot the budget constraint before and after the implementation of the tax on wine.

   Solution: The new budget constraint is

   $$(1 + \tau)p_x x + p_y y \leq m$$

   Graphically

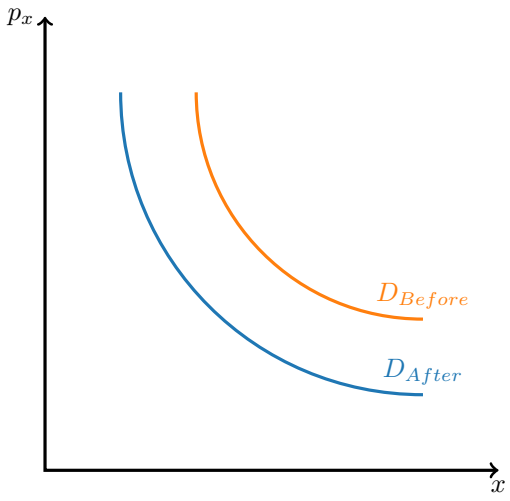


6. *(4 points)* What is the new demand function for wine? Draw the demand curve before and after the tax and briefly explain intuitively the difference between them (You don't need to plot specific values of m, τ, but make sure your graph is qualitatively accurate).

   Solution: The new demand curve is

   $$x = \frac{1}{4}\frac{m}{(1 + \tau)\, p_x}$$

   Graphically The tax works as an increase in the price consumers need to pay for wine. For each price p_x now



   consumers need to pay $(1 + \tau)p_x$, thus from the law of demand we now the quantity demanded will be lower at any price p_x.

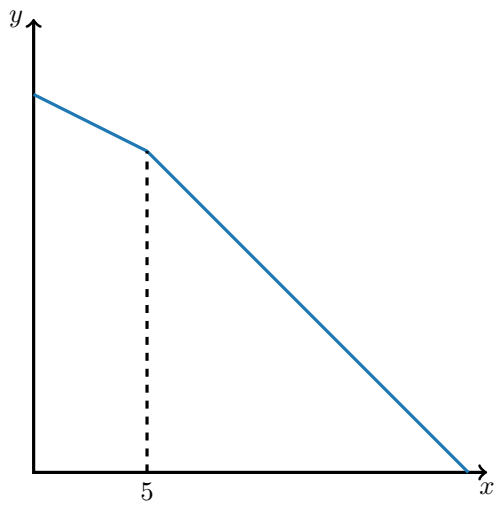
Return to the case without taxes and recall preferences are given by $U(x, y) = xy^3$. Suppose Tai has a 50 percent discount coupon for the first 5 bottles of wine he purchases.

7. *(5 points)* Assume that $\frac{m}{p_x} > 5$. Write down Tai's new budget set. Draw the new budget set on an xy graph.

Solution: The new budget set is

$$m = \begin{cases} \frac{1}{2}p_x x + p_y y & x \leq 5 \\ \frac{1}{2}5p_x + p_x(x-5) + p_y y & x > 5 \end{cases}$$

Graphically



8. *(5 points)* Let $p_x = 2$, $p_y = 1$ and $m = 40$. How many wine and cheese will Tai consume?

Solution: Conjecture Tai consumes more than 5 bottles of wine. If he consumes more than 5 bottles of wine then the relative price of an additional bottle of wine is equal to $\frac{p_x}{p_y}$, whereas if he consumed less than five bottles it would have been $\frac{1}{2}\frac{p_x}{p_y}$ because of the discount. From $MRS = -\frac{p_x}{p_y}$ we have $y = 3\frac{p_x}{p_y}x$. Replacing in the budget set:

$$\begin{aligned} \frac{1}{2}5p_x + p_x(x-5) + p_y 3\frac{p_x}{p_y}x &= m \\ 4p_x x - \frac{5}{2}p_x &= m \end{aligned}$$

Then

$$\boxed{x = \frac{1}{4}\frac{m + \frac{5}{2}p_x}{p_x}} \qquad \boxed{y = \frac{3}{4}\frac{m + \frac{5}{2}p_x}{p_y}}$$

Plugging in $p_x = 2$, $p_y = 1$ and $m = 40$, Tai consumes $x = \frac{45}{8}$ and $y = \frac{135}{4}$.

4 Market for Gas [20 points]

Consider the following demand function for gas among Cambridge residents

$$Q_d = \alpha + \beta P_g + \gamma P_t$$

where P_g is the price for gas, and P_t is the price of a one-way subway ticket.

1. *(2 points)* What sign would you expect γ to have? Provide an economic intuition.

   Solution: γ should be positive because subway rides and gas are substitutes: if it is cheaper to take the T then fewer people will ride their car and therefore decrease the demand for gas.

2. *(3 points)* Suppose $\alpha = 10$, $\beta = -2$, $\gamma = 1$, and $P_t = 1$ and supply is given by

   $$Q_s = 10 + P_g$$

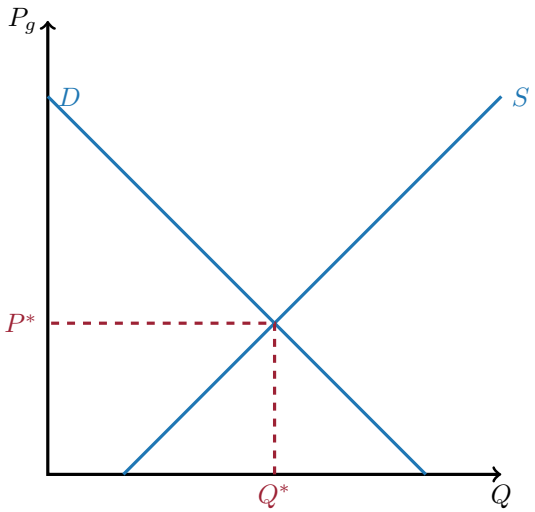
   Solve for the equilibrium price P^* and quantity Q^*. In an xy axis plot the demand and supply curves, and show where the market equilibrium is.

   Solution: Supply and demand are given by

   $$\begin{aligned} Q_d &= 11 - 2P_g \\ Q_s &= 10 + P_g \end{aligned}$$

   In equilibrium

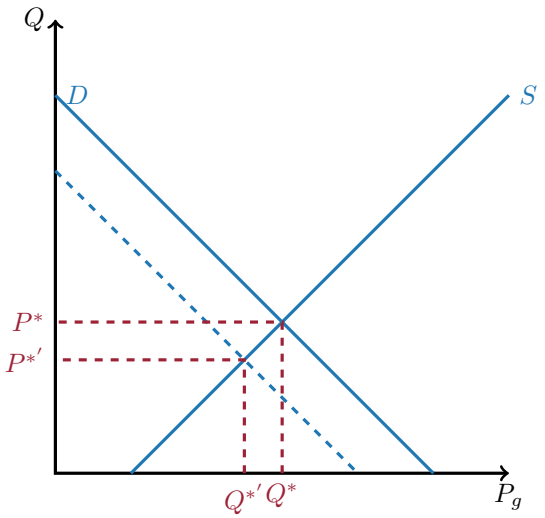
   $$Q_d = Q_s \implies 11 - 2P_g = 10 + P_g \implies \boxed{P^* = \frac{1}{3}} \quad \boxed{Q^* = \frac{31}{3}}$$



3. *(5 points)* Suppose the price of the subway ticket decreases to $P_t' = 0.5$. Solve for the new equilibrium price $P^{*'}$ and quantity $Q^{*'}$. How do they compare with their previous values? Provide an economic intuition. In an xy graph, plot the old and new demand curves, and show the old and new equilibria.

   Solution: Supply and demand are given by

   $$\begin{aligned} Q_d &= 10.5 - 2P_g \\ Q_s &= 10 + P_g \end{aligned}$$



In equilibrium

$$Q_d = Q_s \implies 10.5 - 2P_g = 10 + P_g \implies \boxed{P^{*'} = \frac{1}{6}} \quad \boxed{Q^{*'} = \frac{61}{6}}$$

A decrease in the price of the subway ticket decreases the demand for gas. At the old equilibrium price P^* there is excess supply because of the decrease in demand. Therefore, the market adjusts by a decrease in the price until demand matches supply.

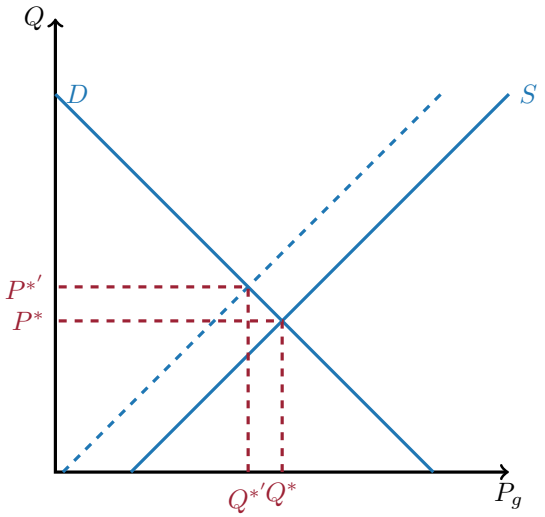
4. *(5 points)* Return to the original demand curve where $P_t = 1$. Suppose supply decreases to $Q_s = 5 + P_g$. Solve for the new equilibrium price $P^{*'}$ and quantity $Q^{*'}$. How do they compare with P^* and Q^*? Provide an economic intuition. In an xy graph, plot the old and new supply curves, and show the old and new equilibria.

Solution: Supply and demand are given by

$$Q_d = 11 - 2P_g$$
$$Q_s = 5 + P_g$$

In equilibrium

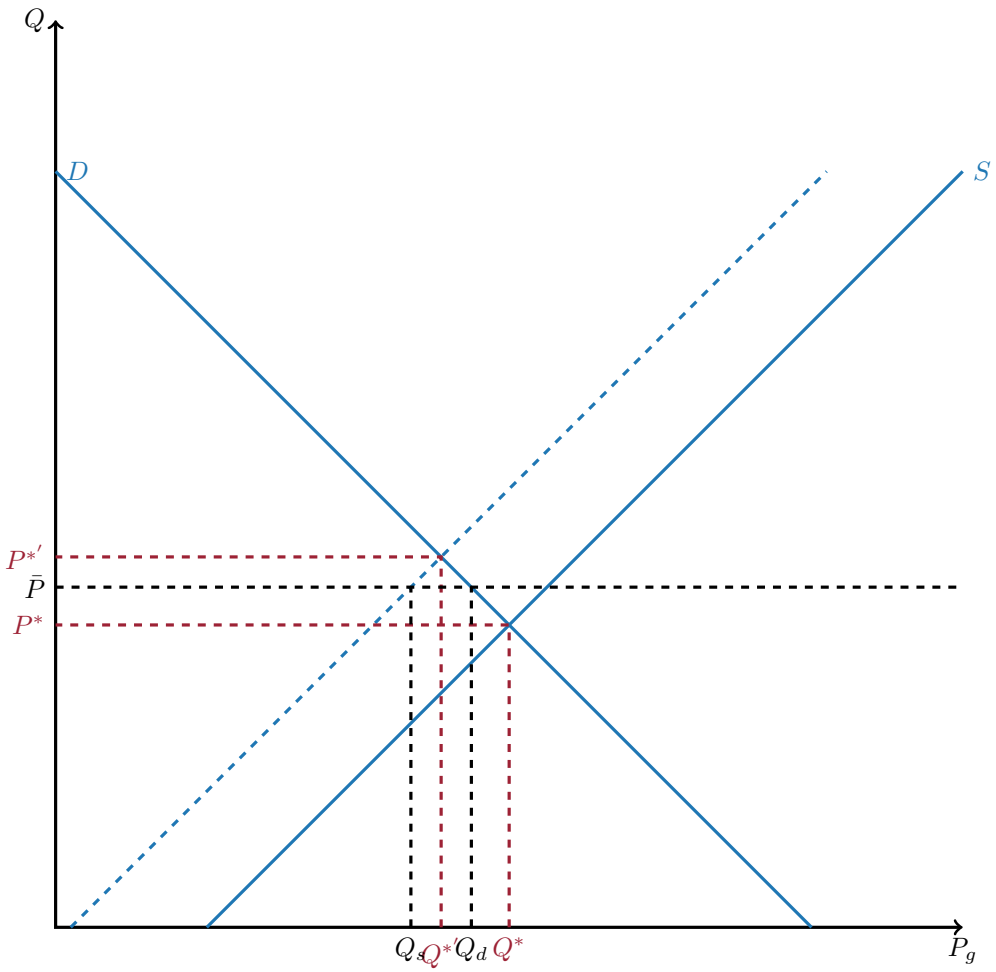
$$Q_d = Q_s \implies 11 - 2P_g = 5 + P_g \implies \boxed{P^{*'} = 2} \quad \boxed{Q^{*'} = 7}$$



A decrease in supply implies that at the old equilibrium price P^* there is excess demand. Therefore, the market adjusts by an in the price until demand matches supply.

5. *(5 points)* Suppose that in response to the decrease in supply the government puts a cap on gas prices. In particular, suppose the government places a price cap $\bar{P}$ lower than the equilibrium value from part 4. Under this policy, is the market for gas in equilibrium? If the answer is negative, show whether there is excess demand or excess supply. In an xy graph, plot what happens to the market for gas under this policy.

Solution: Under this policy the price is below the equilibrium price, so there is excess demand: since demand is increasing then at price $\bar{P}$ $Q_d > Q^{*'}$; and since supply is decreasing then at price $\bar{P}$ $Q_s < Q^{*'}$ and therefore $Q_d > Q_s$.



5 Income and Substitution Effects [20 points]

Ben consumes only apples (x) and t-shirts (y). His preferences can be represented by the following utility function: $U(x,y) = x^2y^3$ The price of apples is p_x, the price of t-shirts is p_y, and Ben has an income of m dollars.

1. *(5 points)* Find the demand for apples and t-shirts as a function of p_x , p_y and m.

   Solution: In an optimum

   $$MRS = \frac{p_x}{p_y} \implies \frac{2}{3}\frac{y}{x} = \frac{p_x}{p_y} \implies y = \frac{3}{2}\frac{p_x}{p_y}x$$

   In the budget constraint

   $$p_x x + p_y \frac{3}{2}\frac{p_x}{p_y}x = m \implies \boxed{x = \frac{2}{5}\frac{m}{p_x}} \quad \boxed{y = \frac{3}{5}\frac{m}{p_y}}$$

2. *(5 points)* What is the price elasticity of the demand for apples? What is the cross-price elasticity of the demand for apples with respect to the price of t-shirts?

   Solution: The price elasticity of the demand for apples is

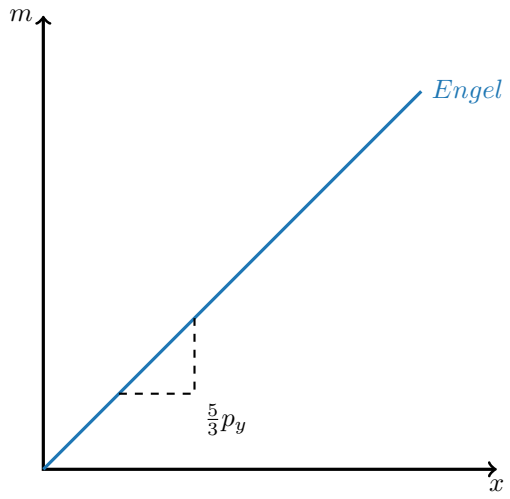
   $$\varepsilon^x_{p_x} = \frac{\partial x}{\partial p_x}\frac{p_x}{x} = -\frac{2}{5}\frac{m}{p_x^2}\frac{p_x}{\frac{2}{5}\frac{m}{p_x}} = -1$$

   The cross-price elasticity of the demand for apples with respect to the price of t-shirts is

   $$\varepsilon^x_{p_y} = \frac{\partial x}{\partial p_y}\frac{p_y}{x} = 0$$

3. *(5 points)* Draw the Engel curve for t-shirts. Are t-shirts an inferior or a normal good? What is the income elasticity of the demand for t-shirts?

   Solution: The Engel curve for t-shirts is Since the demand for t-shirts is increasing in income, then they are

   a normal good. The income elasticity of the demand for t-shirts is

   $$\varepsilon^y_m = \frac{\partial y}{\partial m}\frac{m}{y} = \frac{3}{5}\frac{1}{p_y}\frac{p_y}{\frac{3}{5}\frac{m}{p_y}} = 1$$

4. *(5 points)* True or False? Provide an explanation: *Since Ben's demand of t-shirts does not depend on the price of apples then a change in the price of apples has no substitution nor income effect on the demand for t-shirts.*

   Solution: False: From the substitution effect the demand for t-shirts increase in response to an increase in the price of apples, whereas from the income effect the demand decreases. With these preferences, they offset each other so the total effect is zero.

6 Income and Substitution Effect Concepts [25 points]

1. *(7 points)* In a world with only two goods, can it be that both of these goods are inferior? In a world with two goods, can both of them be normal? Explain.

   Solution: It cannot be the case that in a world with two goods, both of these goods are inferior. When one goes up, you have to spend more money on something (assuming non-satiation holds). If both were inferior goods, you'd end up spending less on both. That can't be optimal if non-satiation is satisfied. (In fact, even if non-satiation isn't satisfied, you still can't have both goods be inferior. Your original bundle is still affordable when your income rises, so you can't 'switch' to something else that was also originally affordable when the income goes up. That would imply a violation of transitivity.) In a world with two goods, both can be normal: consider $U(x, y) = xy$.

2. *(8 points)* Suppose there is an increase in prices and consumption decreases. Given this information, the good must be a normal good. True or false? Explain.

   Solution: False. The good can be inferior, but the substitution effect is larger than the income effect.

Suppose a consumer desires only two goods, x and y, and her prefereces are represented by the utility function $U(x, y) = x + y$. In addition, suppose that $p_y = 3$ and her income is $m = 6$.

3. *(2 points)* Suppose $p_x = 4$, how much does the consumer demand of each good?

   Solution: If $p_x > p_y$ then the consumer only consumes of good y, so $x = 0$ and $y = 2$.

4. *(2 points)* Suppose p_x decreases from $p_x = 4$ to $p'_x = 2$. How much does the consuemr demand of each good?

   Solution: Now $p'_x < p'_y$, so the consumer only consumes good x and therefore $x' = 3$ and $y' = 0$

5. *(2 points)* Suppose $p'_x = 2$ to $p''_x = 1$. How much does the consumer demand of each good?

   Solution: We still have that the price of x is cheaper, so the consumer only consumes good x and therefore $x'' = 6$ and $y'' = 0$

6. *(4 points)* True or False? Provide an explanation: when p_x changes from $p_x = 4$ to $p'_x = 2$ there are both income and substitution effects, whereas when p_x changes from $p'_x = 2$ to $p''_x = 1$ there is no substitution effect.

   Solution: True: when p_x changes from 4 to 2 the consumer substitutes from good y to x because now x became cheaper, and also income effects because now prices are lower and their purchasing power is higher. When p_x further decreases to 1 there is only income effect since the consumer was already consuming all of tood x and all the increase in consumption of x is due to an increase in his purchasing power.

14.01 Principles of Microeconomics
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