14.01 Problem Set 3

Due at 5pm on October 13th, 2023
Late problem sets are **not** accepted.

1 Long-Run Supply Curve (20 Points)

Consider a firm with the following production function

$$F(K, L) = L^{\alpha} K^{\beta}$$

The firm faces a wage level w and rental rate of capital r.

1. *(5 Points)* Show that the long run supply curve for this production function is

$$p = \left(\frac{w}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{r}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} q^{\frac{1}{\alpha+\beta}-1}$$

Now suppose the firm has the following production function

$$F(K, L) = K + L$$

2. *(5 Points)* Find the long run supply as a function of r, w, and q.

3. *(5 Points)* True of False? Justify your answer: *An increase in wages always decreases supply in the long run.*

Let $L(w, r, q)$ denote the labor demand when wages are w, the rental rate of capital is r, and the quantity produced is q. Define the elasticity of labor demand as

$$\varepsilon_w^L = \frac{\partial L(w, r, q)}{\partial w} \frac{w}{L(w, r, q)}$$

5. *(5 Points)* True or False? Justify your answer: *The elasticity of labor demand is always larger (in absolute value) in the long run than in the short run.*

2 Supply with varying input costs (26 Points)

Consider a firm that has a production function $F(K, L) = K^{\frac{1}{2}} L^{\frac{1}{2}}$. It faces a wage level w and rental rate of capital r.

1. *(*5 Points) Find the long run supply curve for this production function. What is the long run supply curve for this firm?

Suppose demand is given by $Q_d = 10 - p$, and wages and the rental rate of capital are fixed at $w = 1$ and $r = 1$ respectively.

2. *(5 Points))* Find the equilibrium price and quantity.

Now suppose that the wage is not constant, but instead an increasing function of total production. In particular, suppose there are 6 firms and $w = \frac{q}{6}$.

3. *(5 Points)* Provide a justification of why wages may be an increasing function of the quantity produced.

4. *(5 Points)* What is the new long run supply curve? Graph and compare the old and new supply curves assuming wages are fixed at $w = 1$ for the old supply curve and rental rates are fixed at $r = 1$ for both curves. Provide an intuition. (Remember the firm still takes w and r as given when minimizing costs).

5. *(6 Points)* Suppose demand is still $Q_d = 10 - p$. How does the equilibrium price and quantity compare when $w = 1$ vs when $w = \frac{q}{6}$? You don't need to provide an algebraic expression, it is sufficient to show in a graph how they compare. Provide an intuition.

3 Aggregate Supply (22 Points)

In downtown Boston there is a vibrant farmers' market. In this market, there are three apple orchards in the area who all specialize in producing the same variety of apples. The total short-run cost functions for these producers are

$$c_1(q) = \frac{1}{4}q^2 + 4$$
$$c_2(q) = \frac{1}{3}q^2 + 3$$
$$c_3(q) = \frac{1}{2}q^2 + 2$$

1. *(5 Points)* Derive each firm's short-run supply curves. Do firms choose to produce at any price?

2. *(5 Points)* Let Q_s denote the aggregate supply of apples in the farmer's market. Derive the aggregate supply of apples in the farmers' market as a function of the price of apples p.

Suppose the demand for apples is given by $Q_d = 1 - p$.

3. *(5 Points)* What is the market equilibrium quantity Q^* and price p^*? How much does each firm produce in equilibrium?

4. *(7 Points)* In the long run, would you expect the number of apple orchards to increase or decrease over time? Justify your answer. Your answer needs to have an intuitive explanation, as well as a mathematical result to back your intuition.

4 Firm Entry (32 Points)

Consider a town that with population of size N. Each inhabitant has an individual demand for milk equal to

$$q_D(p) = 1 - \frac{1}{50}p$$

Each supplier of milk needs to pay a fixed cost of operation equal to 2 and their variable cost is equal to $\frac{1}{2}q^2$. Then, the supplier's total cost is given by

$$c(q) = \frac{1}{2}q^2 + 2$$

1. *(3 Points)* Derive the aggregate demand for milk, Q_d, as a function of price p and the size of the population N.

2. *(4 Points)* Derive the short-run individual supply of milk, q_s, as a function of the price p. For what range of prices does a supplier of milk choose to produce a positive quantity?

3. *(3 Points)* Suppose there are J suppliers in town. Derive the aggregate short-run supply of milk, Q_s, as a function of the price p and the number of suppliers J.

4. *(7 Points)* What is the market equilibrium quantity Q^* and price p^*? How do they depend on the size of the population, N, and the number of suppliers J? Provide an economic intuition.

5. *(7 Points)* How much does each firm produce in equilibrium as a function of J and N? Derive an expression for the firm's profits as a function of J and N.

6. *(8 Points)* Suppose now that there is free entry. Derive the number of suppliers, $\bar{J}$, that will produce in the long run as well as the aggregate quantity, Q^{LR} and price p^{LR} in the long run. How does it depend on the size of the town N? Provide an intuition.

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