## Lectures 19-20: Savings and Technology

- Review
- Cont.: Change in Saving rate
- Technological progress


## Solow's Growth Model

$$
\begin{aligned}
& A=1, N=1 \\
& Y=y=f(k) \\
& S=s Y \\
& I=S \\
& K(t+1)=(1-d) K(t)+I(t) \\
& \Rightarrow \\
& k(t+1)-k(t)=s f(k(t))-d k(t)
\end{aligned}
$$

## Steady State and the Saving Rate

In steady state: $\mathrm{k}(\mathrm{t}+1)=\mathrm{k}(\mathrm{t})=\mathrm{k}^{*}$
$\mathrm{k}(\mathrm{t}+1)-\mathrm{k}(\mathrm{t})=\mathrm{sf}(\mathrm{k}(\mathrm{t}))-\mathrm{d} \mathrm{k}(\mathrm{t})$
=>
$\operatorname{sf}\left(\mathrm{k}^{*}\right)=\mathrm{d} \mathrm{k}^{*}$
$g_{-} y^{*}=0 \quad\left(\right.$ if $\left.n>0, g_{-} y^{*}=0=>g_{-} Y=g_{-} K=n>0\right)$

In steady state, the saving rate does NOT matter for per-capita growth.
It does matter, however, for the level of per-capita output and transitional dynamics

Figures 11-3, 11-4

## Some numbers

- $\mathrm{Y}=(\mathrm{KN})^{0.5} \Rightarrow \mathrm{y}=(\mathrm{K} / \mathrm{N})^{0.5}=\mathrm{k}^{0.5}$
- $\mathrm{k}(\mathrm{t}+1)-\mathrm{k}(\mathrm{t})=\mathrm{sk}(\mathrm{t})^{0.5}-\mathrm{dk}(\mathrm{t})$
- St.St: $\mathrm{k}^{*}=(\mathrm{s} / \mathrm{d})^{\wedge} 2 ; \mathrm{y}^{*}=(\mathrm{s} / \mathrm{d})$
- $\mathrm{s} 0=\mathrm{d}=0.1 ; \mathrm{s} 1=0.2$ =>
- $\mathrm{k}^{*}$ goes from 1 to 4 and $\mathrm{y}^{*}$ from 1 to 2 .
- Higher saving=> need to maintain more capital

$$
c^{*}=y^{*}-\mathrm{dk}^{*}
$$

- The Golden Rule: Table 11-1


## Dynamics

- Dynamics: $k(1)=1+0.2-0.1=1.1>1$
- ... and so on
- Figure 11-7


## Technological Progress

- Table 12-2
- $\mathrm{Y}=\mathrm{F}(\mathrm{K}, \mathrm{N}, \mathrm{A}) \ldots . . \mathrm{Y}=\mathrm{F}(\mathrm{K}, \mathrm{NA})$
- $y=Y / N A=F(K / N A, 1)=f(K / N A)=f(k)$
- I/AN = s Y/AN
- In order to maintain a given k , we need to invest at least:

$$
\left(d+g_{-} A+g_{-} N\right) K
$$

## Technological Progress

$$
\begin{aligned}
& \mathrm{I} / \mathrm{AN}>\left(\mathrm{d}+\mathrm{g}_{-} \mathrm{A}+\mathrm{g}_{-} \mathrm{N}\right)(\mathrm{K} / \mathrm{AN}) \\
& =\mathrm{k} \text { grows }
\end{aligned}
$$

Figure 12-2
Table 12-1
Figure 12-3 / 12-4

## A Decline in g_A

- Table 12-2
- Table 12-1
- (use) Figure 12-2
- Why? (we don't know...)
- Measurement error?
- The rise of the Service Sector?
- Figure 12-5
- Decreased R\&D Expenditure?
- Table 12-3


## The New Economy and Productivity Growth

| Private Non-Farm <br> Business | $1948-$ <br> 1973 | $1973-$ <br> 1979 | $1979-$ <br> 1990 | $1990-$ <br> 1995 | $\mathbf{1 9 9 5}-$ <br> $\mathbf{2 0 0 0}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Labor productivity | 2.9 | 1.2 | 1.4 | 1.6 | $\mathbf{2 . 5}$ |
| Multifactor productivity | 1.9 | 0.4 | 0.3 | 0.6 | $\mathbf{1 . 1}$ |
| Manufacturing | 1.5 | -0.6 | 1.1 | 1.3 | $\mathbf{2 . 1}$ |
| Industrial Mach. | 0.7 | 0.2 | 3.2 | 3.1 | $\mathbf{5 . 8}$ |
| Electronic Mach. | 2.1 | 1.0 | 3.0 | 6.0 | $\mathbf{7 . 4}$ |

Source: BLS.

## Investment Has Increased



Figure by MIT OCW. After source: BEA; Datastream; St. Louis Federal Reserve.

## The Price of New Capital



Figure by MIT OCW. After source: BEA; Datastream; St. Louis Federal Reserve.

