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RICARDO CABALLERO: Today, we're going to talk about a very important topic in economics, which is expectations. We have barely mentioned expectations when we talk about the Phillips curve, we talked about expectations when we discussed the UIP and so on, but expectation is a much bigger issue in economics.

In fact, most decisions by firms, by consumers, governments involve considerations of the future. And it plays an even bigger role in finance in which essentially, everything is about the future. The price of an asset today is meaningless in itself. You have to compare it with what you expect to get out of that asset in the future. So it's all about expectations and so on.

So that's what we're going to do today we're going to talk about expectations, how to value things that you expect to receive in the future, and how to compare those things with things that you have in the present.

But before doing that, actually, let's talk a little bit about the news. Who knows who first Republic Bank is? Remember that a few weeks ago, I told you that the Silicon Valley Bank-- I mean, you read it. I just mentioned it, that the-- or discuss it that we had the second-largest bank by asset in US history-- it was-- Silicon Valley Bank was the second-largest asset-- bank in terms of assets to collapse in the US.

The first one was many years ago, and then we had this bank that had more than $200 billion in assets that essentially collapsed in a few days. It was a run on deposits. They had problems before, but what really did it, as is always the case with banks, is they have a run on deposits-- funding.

Well, it's no longer. The second-largest collapse in US bank history, now we have on the weekend the new second-largest bank to collapse, which is First Republic Bank that was essentially liquidated and sold to JPMorgan over-- today morning, very, very early in the morning. So you have an account in First Republic Bank, you soon are likely to have an account in JPMorgan.

But again, what made it collapse was something very similar to what made Silicon Valley Bank collapse, which is that they had invested on a series of things that were very vulnerable to the fast pace of hikes in interest rates in the US. And when they had those losses, depositors became worried about it, and eventually they decided not to wait, just run and see what happened. First Republic Bank lost about $100 billion in deposits just last week. The last few days of last week.

So it was obvious that it was not going to survive, and that's the reason something was arranged over the weekend to avoid the panics associated to collapse of the bank and so on. But anyways, by the way, this is all about expectations, is if people had expected the deposits to remain in the bank, then probably this bank would not have collapsed. It's all about people anticipating what other people will do and so on and so forth.

OK, but now let me get into the specifics of this lecture. So there, you have-- this is the most important index of equity-- equity index in the US, S&P 500. It's a very inclusive index that captures all the large-- most of the large companies in the US-- all of the large, I think-- companies in the US.

And that's an index, an average-- weighted average by this capitalization value of each of these shares, the weighted average of the major-- main shares in the US, equity shares in the US. And one thing you see is that it moves a lot around. Here, for example, when we became aware that COVID was going to be a serious issue, the US equity market collapsed by 35% or so. That's a very large collapse in a very short period of time.

And then, as a result of lots of policy support, actually we had a massive rally. So up to the end of 2021, the equity market had rallied by 114%, so big rally. Then we got inflation and the Fed began to worry about inflation, so they began to hike interest rates, and when they hike interest rates, that eventually led to a very large decline in asset prices of the order of 30% or so-- 25% or so, actually, from the peak to the bottom.

And then since the bottom, which was more or less October of last year, we have seen a recovery of about 16% or so of the equity market. And if you look at the NASDAQ, which is another one index that is very loaded towards technology companies, and you can see swings are even larger than that.

Now why do these prices move so much? Well, a lot of it has to do with expectations. Are things going to get worse in the future? Will the Fed cause a recession? How much higher will be the interest rates? And things like that matter a great deal. Another thing that matters a great deal is how much people want to take risk at any moment in time.

If you're very scared about environment, you're unlikely to want to have something that-- to invest on something that can move so much. And so risk is well-known, so it's called risk-off, when people don't want to take risk, these asset prices tend to collapse. Of Risky assets-- equity is a very risky asset.

But that's not the only thing that moves these assets around. It's not just the risk that the companies underlying companies may go bankrupt or anything like that. Here, you have, for example, the movement of a-- it's an ETF, but it doesn't matter. It's a portfolio of bonds-- of US Treasury bonds of very long duration, maturities beyond 20 years and so. So this is incredibly safe bonds because it's US treasuries. So there's no risk of default or anything like that.

Still, the price swings can be pretty large. I mean, over this period, you have seen an increase in value of 45%, then a decline in value of about 20%, another increase in 15%. Here, there was a huge decline, 40% since essentially-- what do you think happened here? Why is this big decline in bonds? You're going to be able to answer that very precisely later on, but I can tell you in advance, that was essentially the result of monetary policy tightening. Increasing interest rates caused the bonds to decline.

So even these instruments are very safe in the sense that if you hold them to maturity, you'll get your money back and all the promised coupons along the path, well still, the price can move a lot.

And it's obvious that that movement in price is something that you need to explain in terms of expectations, what people expect things to happen. In this case, it's not whether people expect to get paid or not because you will get paid, but it's expected-- but in this particular case, it's about expectations about future interest rate. You think the interest rate will be very high, then the price of bonds will tend to be very low and so on, but it's all about the future.

So a key concept that we're going to discuss today, and then you're going to use it to price specific assets, is a concept of expected present discounted value, and this is a loaded concept. There's lots of terms in there and we need to understand what each of these terms means. So the key issue that we're going to discuss is how do we decide-- for example, if you see the price of an asset out there that is 100, how do you decide whether that price is fair or not, it looks cheap or not?

And that question means you have to decide whether that price that you're paying today is consistent with the future cash flows that you're going to get from these assets. And that's the reason you buy an asset, is because you'll get something in return in the future. But how do we compare that? How do we compare the price today with those things that will happen in the future?

So answering that question, which is what we're going to do in this lecture, involves the following concepts. First, expectations. Big thing. That's it. This is expected present discounted value. The E part is for expectations. That comes there.

The expectations are very crucial because these are things that happen in the future. You need to expect-- even if it's a bond that promises you to pay $0.50 per dollar every six months, you still may have an expectation that if it is a bond issue by First Republic Bank, it may not pay. So you need to have an expectation about that. So a crucial term is expectation.

Then you need some method to compare payments received in the future with payments made today. I mean, if you buy an asset you pay today, but you're going to receive things returns for that asset in the future. So how do you compare that? Suppose I pay one today and I receive one one year from now. Does that seem like a good asset? Probably not. I mean, probably not.

And that's what the word "discounted" really means. When you say expected present discounted value, it says, somehow that things I receive in the future are valued less than things I have today. So if you're going to tell me that you're going to pay me $1 in the future and I have to pay you $1 today, most likely I won't take that deal. So in other words, I'm discounting the future. How do we discount the future? Well, something that we now have to figure out.

So let me first shut down this part, the expectations, and then we'll introduce it. So assume for now that the future. And I'm going to derive all the equations with assuming that you know the future. So there's no issue of trying to figure out what the future is, you know it. But still, you have to decide whether-- what is the right value for an asset.

OK, so let's start with the case where you know the future. Sorry. And let's do the comparison-- let's try to understand how do we move flows-- how do we value flows at different points in time? And the easiest thing is think first about comparing an asset that gives you $1 in the future, how much do you think is worth today?

Well, the easiest way to get to that value is to think on the alternatives. Suppose I have a dollar today, what can I do with it? Well-- in terms of investment? Well, suppose that you have available 1-year bonds, Treasury bonds, and that the interest rate is it. That's the interest rate on a 1-year bond.

So if you want-- if you have a dollar, you have the option to invest it in that asset, in that bond, which will give you 1 plus i dollars next year. Well, that means that I can get $1 next year by investing 1 over 1 plus i dollars today. Because if I invest 1 plus 1-- rather than $1, I invest 1 over 1 plus i today, then I multiply this by 1 plus i, and I get my dollar in the future.

So that tells me that-- say the interest rate is 10%, then with $1 today, I can get $1.1 in the future. That means that investing $0.90 today, more or less, I can get a bucket in the future. That tells me that $1 in the future is equivalent to $0.90 today with that assumption.

So that's the reason when I told you the deal of, look, I have an asset that costs you $1, but it gives you $1 in the future, and that's not a good deal if the interest rate is positive. If the interest rate is 10%, then a fair comparison is $0.90 with $1, not $1 with $1. So that's a discounting of the future. The most obvious way of discounting the future is to discount it by the interest rate.

Which interest rate to pick? That's more subtle. That depends on risk, it depends on many other things, which we're going to discuss to some extent here. But for now, let's make it very simple. And in a world which you really know the future, really, the right interest to use is the safe interest-- the interest rate of Treasury bonds and things like that. So that's that.

What about the dollar that you receive-- what about if you are thinking about what is the value of $1 two years from now? Well, if I get a $1, can do the same logic. If I can use the same logic. If I get a $1 today, I can convert that into 1 plus it times 1 plus it plus $1. So say 10% and 10%, I get 1.1 next year, and then I get 1.1 times 1.1, 1.21 or something like that. That's my final result.

Well, then, how much is it worth to have $1-- $1 asset that gives you $1 two years from now? Well, it's going to be $1 divided by the product of this interest rate. Why is that? Well, because with this amount of dollars today-- at this point, $0.80, something like that, I can generate $1 two years from now. That means $1 two years from now is worth about $0.80 today.

We're going to use a lot this type of logic. And I know that it may not be that intuitive the first time you see it, but ask questions. You want me to repeat it? OK. The final goal is the following.

We're going to-- what comes next, we're going to see, which happens again with many decisions in life, but particularly for financial assets, we're going to try to value something that-- it was payoff happens at different times in the future. And the question is, how do I value an asset that pays me $5 one year from now, $25 three years from now, minus $10 10 years from now, plus $50 $100 from now? What is the value of that? Of having an asset like that?

And so I need some method to bring it to today's value because today, I have a meaning of what $1 is, and therefore, I can compare it with whatever price. People are asking me for that asset.

So what this is doing is it's doing that. It's telling you how to convert $1 at different parts in the future into $1 today. And by that logic, the recipe is, well, use the interest rate because you could always go the other way around. You could always-- you can ask a question, with $1 today, how many dollars can I get two years from now, say, that? Well, say x?

Well, then I need 1 over x, then $1 there is worth 1 over x dollars today. That's the logic. Because 1 over x times x is 1. So let's do first problem here. With $1 today-- oops. I can generally say $1.1 at t equal 2. Then the question I want to know is, how much is $1 worth-- how much is $1 received at time t equal 2 worth today? That's the question I'm trying to answer. Because an asset will be something that will pay you in the future. So I want to know, how much is $1 received in the future worth today?

And then the answer is, well, then it's-- I know the answer from this logic because I know that with 1, if I have 1 over $1.1 today, I can convert it into 1. How do I know that? Because 1 over 1.1 times 1.1 is equal to 1. If I invest this dollars today, I'm going to get this return on that. And the product of this thing gives me my dollar.

So if I tell you, do you prefer to have $1 two days from-- two years from now or today, you say, I prefer it-- obviously I prefer it today because I can get $1.1 two years from now. But then the more relevant question is, no, no, but then do you prefer to have $0.90 today versus $1 in the future? Then I need to do my multiplication because then I have to multiply the $0.90 by the 1.1 and see whether I get something comparable to $1 or not. But that's the logic behind that.

So the interest rate is what we discount the future by. And it's natural because if the interest rate is very high-- if the interest rate is 0, say, then $1 received two years from now and $1 received today is the same because I can-- if I invest $1 today and the interest rate is 0, I'm going to get my $1 two years from now.

If the interest rate is 50%, it makes a big difference receiving the dollar today versus receiving it two years from now. If you're in Argentina and the interest rate-- I don't know what it is, it's 700%. It makes a huge difference whether you receive it one year from now than today. And so that's the role of the interest rate.

The higher the interest rate, the less is $1 received in the future worth relative to $1 received today because you can get a much higher return from the dollar you have today if the interest rate is high. The interest rate is low, you don't get that much. Much difference. OK, good. So this is a big principle. And, I mean, everything that I'll say next builds on this logic.

So let me give you a general formula. So let's ask, what is the value of an asset that gives payouts of zt dollars this year, zt plus 1 one year from now, zt plus 2 two years from now, and so on and so forth for n periods more? Well, I just need to do several of these operations.

I know that $1 received this year is worth $1. That's zt. $1 received one year from now is not the same as $1 received today. It's the same as 1 over 1 plus it dollar received today. So that cash flow I'm going to receive from this asset is worth this amount. For something that I received two years from now, then it's not-- certainly it's much less than receiving $1 today. It's going to be 1 over 1 plus it 1 plus it plus 1. And that I have to multiply by the number of dollars I will receive two years from now. And I keep going.

So that's the present value. Present discounted value-- present because I'm bringing all these future cash flows to the present. That's what each of these terms is doing. The 1 over that is bringing it to the present. Discounted because the interest rate is discounting things, making them smaller. And value because I'm trying to reduce them to the current value. That's a general formula, so it's a formula you need to understand.

It's just-- so that was an asset that gives you z dollars today, zt plus 1 one year from now, so you use this formula. Zt plus 2 two years from now, so you use this formula, and then you keep going.

What if we don't know the future? I had to remove the expected part. Well, if we don't know the future, then the best we can do-- in fact, we do fancier things, but that's what we want-- all that we'll do in this course. All that you can do is just replace the known quantities we have here for the expectations. So that's the closest. So I know zt, that's the cash flow I get now, but I don't know zt plus 1, so I can replace it by expectation.

I do know the interest rate on a 1-year bond from today to one year, so that's the reason I don't need an expectation here. But I don't know what the 1-year rate will be one year from now, so that's the reason I need an expectation there. And so on. And I don't know what the cash flow will be two years from now. I have an expectation about what the cash flow will be, but I don't know it, so I have an expectation there.

So all that I've done here is say, OK, acknowledge that this guy knew a little bit too much. He knew exactly what the cash flows were going to be in the future, and he knew what the 1-year rates were going to be in the future. This guy here knows less. He knows the cash flow today, he knows the interest rate today, but he doesn't know the cash flows-- really has a hunch, but he doesn't know the cash flows one year, two years, three years, and so on for the future, and it doesn't know the 1-year interest rate in the future.

So all these expectations, here is important the concept of time. These are expectations as of time t. At time t, you have some information and you make forecasts about the future. Use whatever you want. Machine learning, whatever. But you have information at time t, and then you have a forecast for the future at t plus 1. You have more information, so you make another forecast, and so on and so forth.

But in this-- we're valuing an asset at time t, then all these expectations are taken as of time t. That means given the information you have available at time t. That's the reason these guys don't have expectations in front of them because you know this at time t.

How do we take in the value at t minus 1, we would have not known that and we had to-- an expectation because there would have been expectation of t minus 1. OK, so that's your big formula there.

So there are some examples that are well-known, and so let me show you. They have nicer expressions. So that's an example of valuation of the same asset, but when the interest rate is constant. Then obviously I don't need all these products in the denominator. I have a constant interest rate, then I just get powers of that interest rate.

That's one in which you have constant payments. So the interest rate may be different, but the payments are the same over time. So that's that. So those are two easy formulas. That's one in which you have both constants, the interest rate and the payment, then you get a nice expression that's just that.

You recognize that-- if you have a constant interest rate here, you see that the value is declining-- is a geometric series. The value of two years from now is a square of 1 over 1 plus some-- is a square of a number less than 1. 1 over 1 plus i is some number less than 1. This is the square of that, then the cube, and so on.

So it's a geometric series that is declining in the rate 1 plus i-- 1 over 1 plus i. Or declining at the rate 1 plus i. So that's your geometric series. That's the value of that.

Constant rate and payment forever. Suppose you have an asset that lives forever. There are some bonds like that. It's called perpetuities. The US has an issue-- the UK has and so on. So that's an asset, for example, that pays you a fixed amount forever. And it's the interest rate is constant, that's a trickier thing, then the value of that asset you can see, that this is going to 0, so the value of that asset is that.

And actually, a formula that you may see that is very often used as a first approximation is this one. This is the same asset, but it's called x dividend or x coupon. It's after the coupon of this year has been paid. So it's an asset that starts paying at t plus 1, is zt plus 1, plus 2, and so on. Well, that is the same as this minus the first coupon, so it's equal to that.

That's an interesting thing, huh? Look, what happened to this asset as the interest rate goes to 0? So this is an asset that lasts for a very long time. And look, we got to a valuation formula. What is happening as the interest rate goes to 0 to the line?

AUDIENCE: The value becomes very large.

RICARDO CABALLERO: Very large. It goes to infinity. And a lot of what has happened in global financial markets in the last few years has to do with that. Interest rates were very, very, very low. And so most assets that had long-duration had very high values. And it has a lot to do with that-- monetary policy has a lot to do-- whether it was a right monetary policy or not, that's something to be discussed. I think, on average, it was the right monetary policy, but one of the things it did, it increased the value of many assets.

In fact, that's one of the mechanisms through which monetary policy works in practice. It's not something we have discussed, but you can begin to see here because if the value of all assets go up a lot, people feel wealthier and that will tend to consume more, and so on. Well, this is one of the channels monetary policy does.

By the way this effect happens also to the assets that are finite end. It's just that this goes-- it's maximized when this asset lasts forever. This asset literally goes to infinity if the interest rate goes to 0. Well, if an acid lasts for 10 periods, it doesn't go to infinity, it goes to n times z. For some. If the interest rate is 0, just some things. You see that?

If an asset lasts for n periods, and it gives me a payment of z in every single period, then when the interest is 0, that asset is worth n times z because I will receive z coupons. And don't discount the future because the interest rate is 0.

What happens is, when the asset lasts forever, then n times z is a very large number. And that's what this expression captures here. OK. So let's talk about bonds now. We're going to start pricing bonds. So bonds differ along many dimensions, but one of them that's very important for bones is maturity the n that I had there in the previous expression.

So maturity means, essentially, how long the bond lasts. When does it pay you back the principal? The bonds typically pay coupons, and then there is a final payment, which we call face value of the bond or something like that. And when that final payment takes place, that's a maturity of a bond.

So a bond that promises to make a $1,000 final payment in six months has a maturity of six months. A bond that promises to pay $100 for 20 years and then $1,000-- a final payment in 20 years has a maturity of 20 years. Maturity is different from duration. I don't think I'm going to talk about duration here, but that's maturity, just when it's the final payment of a loan-- of a bond?

Bonds of different maturities. Each have a price and an associated interest rate. We're going to look at those things. And the associated interest rate is called the yield to maturity, or simply the yield of a bond. This is terminology, but we are going to calculate these things later on.

The relationship between maturity and yield is called the yield curve. Very important concept. Big fuss about the yield curve these days. I'll talk a little bit more about that. Or sometimes it's called the term structure of interest rate. Term, in the language of bonds, is really maturity. So term structure of interest rate really tells you what is the yield in a 1-year bond, 2-year bond, 3-year bond, 4-, 5-, 6-, 7-. You plot them, and that gives you a curve.

So for example, look at those-- these are two different yield curves. This is November 2000 and this is June 2001. So this tells you what the yield is on a 3-month bond. So a bond that matures in three months. On a 6-month bond, so on and so forth, up to 30-year bonds. What is the big difference between these-- what do you think happened here in between?

Notice that these two curves are more or less the same long-term interest rate, but they have very different-- these curves-- this is a very steep curve and this is a very flat or even inverted curve. What do you think may have happened there between November 2000 and June 2001?

AUDIENCE: People changed their expectations about interest rates.

RICARDO CABALLERO: Yeah. That's true. That's for sure true about that. But look also that-- but that these three months, there is very little uncertainty about three months. It was a lot lower than that. So yes, people change their expectations, but why do you think they change their expectations?

AUDIENCE: [INAUDIBLE] rising [INAUDIBLE]

RICARDO CABALLERO: Rising inflation from here to here. These are nominal interest rates. Up to now, we've been talking about nominal interest rate. What happens here is there was a mini-recession, so the Fed cut interest rates. When you are in recessions, the curve tends to look like this because the central bank is cutting interest rates in the short-run to deal with the current recession.

What happened 30 years from now has nothing to do with the business cycle today, so that interest rate doesn't need to move a lot. But the Fed is bringing interest rates down a lot in the front end. So that's a typical shape of a curve in a recession.

That's a typical shape of a curve in the opposite situation where the inflation is too high and so on. Because what happens? The Fed is trying to-- the Fed really controls the very front end of the curve. That's what the Fed really controls-- the central bank in general, but the Fed, they control the very front end of the curve because they're setting the very short-term interest rate.

So this is a situation where they're tighten-- the monetary policy is very tight because they have a situation of overheating in the economy. And in fact, they got too carried away. That's the reason that we ended up in a recession here. OK. How do you think it looks today? Do you think it looks more like this or more like that? Is inflation low or high today? High. I mean, that's a problem. The Fed is trying to hike interest rates.

Now recently, because of the mess in the banking sector, then expectations of interest rates have begun to decline a little, but the situation was very inverted. Here you are. The green line is today. OK, so it's very inverted. A year ago, it looked like that. So you see, the longer hasn't changed much, but a year ago, there was no sense that inflation was getting so much out of line. It happened a little later than that. There was some concern that interest rates would rise, but now it's very clear that the economy is overheating.

And this-- I should have plot you something for a month ago, it would have been even steeper. OK. Anyways. But that's because the Fed is trying to slow down the economy. It's hiking interest rates. That's the reason the curve is very, very inverted today.

So let me calculate these rates. How do we go about it? So the first thing we're going to do is we're going to use the expected present discounted value formula to calculate the price of a bond. And then we want to start doing it for different bonds, and we're going to construct the yield curve. So suppose you have a bond that pays $100, nothing in between, $100 one year from now. So this is a bond with maturity, 1-year maturity. I'm going to call that bond with 1-year maturity-- the price of a bond with a 1-year maturity at time t P1t.

Well, that's easy to calculate. It's expected present discounted value for you. You have the interest rate, whatever it-- say 1-year interest rate, then I know that the price of the bond is 100 divided by 1 plus the interest rate, the 1-year interest rate today. That's the price. That's the expected discounted value.

So I tell you-- what I'm showing you is the relationship between interest rates and prices. Our price of a bond. The price of that bond is just 100 divided by 1 plus the-- 1 plus-- the 1-year interest rate today. So important observation is that the price of a 1-year bond varies inversely with the current 1-year nominal interest rate. This is all nominal. Why is that inverse relationship? Why is it the price of a 1-year bond is inversely related to the 1-year interest rate?

In other words, I'm asking, what do you think happens to the price as the nominal interest rate rises? And why do you think that's what happens to the price? Well, the first question doesn't have a-- I mean, it's very easy, you know the answer to the first question. What happens if i goes up? Well, it's obvious that this price comes down, but why? And use the concepts we have developed here. Remember, we spent like 20 minutes in one slide. Use that slide for that answer.

Hint-- this $100 you are not receiving today, you are receiving a year from now. What happens with $1 received a year from now? What is the value of a dollar received one year from now when the interest is high? It's low because you'd much rather have the dollar today, invest it, and get this big return on the dollar.

That means, naturally, a bond that is paying you $100 tomorrow is going to be worth less when the interest rate is very high. It's going to be worth less today when the interest is very high. You'd rather have the money today, invest it in the interest rate and get the interest rate. And I need to invest 1 over 1 plus i1t dollars to get $100. That's another way of saying it.

What about the bond that pays $100 in two years? Well, I need to discount that by this, which is-- it's a product of the two interest rate. And since I don't know what the 1-year rate will be one year from now, I have to use expectation here rather than the actual rate. But look at the notation. I'm calling P2t-- $P2t, the price of a 2-year bond, a bond with maturity of two years, as of time t. And this is a bond that has no coupons. Yes, pays you $100 at the end of the two years.

Now note that this price is inversely related to both the 1-year rate today and the expectation of the 1-year rate one year from now. If either one of these goes up, the bond is worthless today. You discount more $1 received two years from now. I don't care which one-- either of them that goes up is bad news for the price of a bond. It's clear?

So there is an alternative. So this is the way you price bonds using just expected discounted value approach. Now it turns out that in practice, a lot of the asset pricing is done by arbitrage, meaning you compare different assets, and that have similar risks, they should give you more or less the same return. That's what you-- so let me do this arbitrage thing.

Suppose you're considering investing $1 for one year. So that's your decision. I'm going to invest one-- I have a dollar, which I want to invest for one year. But I have two options to do that. I can invest $1 in a 1-year bond. I know exactly what I'm going to get in that bond. Or I can invest in a 2-year bond and sell it at the end of the first year. Those are two ways of investing for one year.

Arbitrage has to be compared over the same period of time and everything. It's not the return of a bond that you hold for 10 years versus one that you hold for one year. It has to be something a similar investment. Suppose I need to invest for one year. OK. Then if I have these two bonds, the option is not buy one or the other and then hold to maturity because that would be comparing an investment of one year with an investment of two years. I need to compare the strategies of getting my return in one year.

In the 1-year bond that's trivial because I get my return at the end at the maturity of the bond. In the 2-year bond, it means I need to sell it in between after one year. So those are the two strategies I want to compare. And since I'm not-- considering risk here as a central element, those two strategies are going to have to give me the same expected return. That's arbitrage. That's what we call arbitrage. The two strategies have to give me the same expected return.

So what do we get from this strategies? Well, if I go through the 1-year bond, I know I'm going to get my dollar times 1 plus i1t. That's what I get for one year-- out of investing $1 in a 1-year bond.

If I go through the 2-year bond strategy, buy it and sell it at the end of the year, then I'm going to get-- I invest $1 today, I'm going to pay P2t-- that's what I pay today for a 2-year bond, that's what I pay here for a 2-year bond. And I expect to get the price of a 1-year bond one year from now.

I mean, the 2-year bond will be a 1-year bond after a year has passed. It's a 2-year bond today, but after one year, it's going to have only one year to mature. So that's the reason the price I need to forecast is the price of a 1-year bond one year from now. That's what this is here.

And that's my return on this strategy because I'm going to pay this today, these dollars, and I expect to get that one year from now. OK so arbitrage means I need to set these to equal.

So that means I have to get the same return with two strategies. That means I'm investing the same. So you only need to compare the returns. This needs to be equal to that. That's what I have here. Which tells you that you are solving from here that the price of a 2-year bond at time t is equal to the expected price of a 1-year bond at t plus 1 discounted by 1 plus the 1-year interest rate.

This was like my cash flow-- my cash flow now is not the cash flow-- it's just the price I'm going to get a price for that asset. That's like the z's I had in my formula. And for a 1-year strategy, I only needed to worry about the z plus 1. There was no dividend at day 0. And that's exactly that formula.

But notice that at t plus 1, that will hold. So at t plus 1-- I'm at t plus 1, I don't need expectations. I know that P1t plus 1 will be equal to 100 divided by 1 plus i1, the 1-year rate, at t plus 1. Therefore, the expected is something like this, approximately. The expected price is something like that. I expect-- I mean, this will be-- without the e, will be the price of this 1-year bond at t plus 1. I don't know exactly what the interest rate will be next year, so I have-- the best I can do is have an expectation-- that's my expectation, approximately.

But now I can stick this expression in here. I have this I'm going out and I can stick that in there and I get this expression. So that's the price for the 2-year bond. Do you recognize this? You saw it before. That's the same expression that we got when we used the expected present discounted value formula.

We said we're going to go-- say $100 years-- 100 years-- two years from now, I know that discount factor for that is 1 over 1 plus i1t times 1 plus i1t plus 1 expected. Well, that's what I got. Get some arbitrage. OK. From an arbitrage logic. This is used a lot in finance. I'm going to say something complicated, but just ignore it if it's-- it's not relevant for the quiz or anything.

But there is a big debate in the US today about-- not big debate, a big concern about the US Treasury debt because there is a debt ceiling, meaning there is a maximum amount that the government can-- of debt they can issue, and that ceiling has been moved over time, but every time we get close to a deadline, when this needs to be agreed again, there is a concern and there's negotiations and so on, and-- well, I mean, everyone at this moment at least thinks that, as in every instance in the past, they're going to reach some sort of agreement the day before of the deadline or not.

But if they don't, and there is a mess, this is huge for finance. It's huge for finance because US Treasury bonds, especially short-term bonds, are used for pricing everything through arbitrage and so on. So you get a mess there. That's a mess in every single financial market. You wouldn't know how to price many financial assets, actually. So it would be a disaster.

But the reason I mention this here is because, again, lots of prices are priced in reference-- in finance are priced in reference, especially derivatives, options, and stuff like that, you price them relative to something using this type of logic. So if the thing you use as a base, as a reference becomes highly unstable and uncertain and risky, then obviously everything becomes very complicated, very risky, and financial markets do not like risk, that's for sure.

Anyways, ignore that, that's irrelevant for your quiz, but that's the reason this-- the whole discussion can over the summer can get to be very, very tricky for finance.

So the yield to maturity-- remember I mentioned this concept before-- of an n-year bond is also-- whenever you hear the 3-year rate, it's that. It's the yield to maturity. A which is different from-- OK, let me tell you-- show you a formula that's easy to explain, then. And it's defined-- it's important-- as the constant annual interest rate that makes the bond price today equal to the present discounted value or expected discounted value.

So notice the highlight there, it's defined as the constant annual interest rate that makes the bond price today equal to the present discounted value of future payments of the bond. So, for example, in our 2-year bond, that's the price. This is the price of the asset. That we know the price. We already got the price from the previous slides of the bond, which was based on the short-term interest rate, 1-year interest rate, and our forecast of the short-term-- the 1-year interest rate one year from now.

I know that price, take that as a number. So then the yield the yield to maturity is calculated as that constant interest rate-- constant-- how do I see this constant? Because, well, I'm using the same interest rate for the first period and the second period, and now I'm calling it i2t. It's a 2-year interest rate, but it's constant.

Constant doesn't mean that it doesn't move over time. It means I'm discounting all the cash flows as a constant interest rate. It means I'm using this equation. OK. So the yield to maturity is find an interest rate that allows me to use this constant thing-- constant-- assume-- use this formula and get back the same price as I got by using the expected discounted value or the arbitrage or something like that. So that's the definition.

You this price. Now you look for that interest rate that allows you to match that price. And that's called the yield. That's the thing-- remember, I plotted some curves. Well, those interest rates and those curves were computed this way.

Now notice that we know what this price is. This price is, by the expected discounted value or the arbitrage approach, is equal to 100 divided by this. So I know that these two things are-- this is equal to that, which means that this denominator is equal to that. And that implies, for a small interest rate, that these two-year interest rate is approximately equal to the average of the expected interest rate-- 1-year rates.

So this is called actually the expectation or hypothesis, by the way. Is that the 2-year rate is approximately equal to the average of the 1-year rate this year plus the expected 1-year rate one year from now. So that's an important concept. I'm going to start from here again in the next lecture.