[SQUEAKING]

[RUSTLING]

[CLICKING]

RICARDEO CABALLERO: I couldn't connect. So the Fed just hiked by 25 basis points. And as people expected-- this is the way it works. When there's lots of uncertainty, essentially, the Fed starts communicating what it's going to do, and the communication was very clear that 25 basis points was to be expected.

And apparently-- I was reading this right now. It was released 3 minutes ago, 4 minutes ago-- they also said that further hikes are no longer guaranteed. So remember that we saw that expected hikes-- we saw several expected hikes for the next few months before the SVB mess. And right after it, we saw the whole thing declining.

And at least the minutes are consistent with that. So there we are. So not big uncertainty. I imagine the markets are rallying or something like that, at least for the next 10 minutes or so. But we shall see.

Anyway, but today, we're going to really start-- I'm going to show you the first model of economic growth. And before I do that, who knows who that person is? No, no clue?

Actually, he's Robert Solow. He's an emeritus professor at MIT. Together with Paul Samuelson, essentially, he's responsible for building the Economics Department at MIT. And he won the Nobel Prize in 1987-- I was a student then here-- primarily for his work on economic growth. And so what we're going to do in the next two or three lectures are essentially things that Bob Solow developed many, many years ago.

The basic mechanism-- you remember that we had this Keynesian cross before where we had this multiplier in the goods market and aggregate demand feeding into income and so on and so forth. That was the star mechanism in short-run macro. In long-run macro growth theory, this is the key mechanism, and you can think of it as the following.

At any point in time, an economy has factors of production-- primarily, labor and capital. That capital is stock. Labor is more or less fixed, so it depends on population, growth, things that are difficult to control, or they're not really that endogenous to economics, not, at least, in the current times.

Many centuries ago, yes, they were. We had this Malthusian theories, in which population growth determined growth because food scarcity and stuff like that. But that's no longer the case, fortunately, in most parts of the world.

But what can change over time and quite a bit, and it depends on economic decisions, is the capital stock. But at any point in time, there is certain capital stocks, which, combined with labor, give you some certain output. Output is income. Part of that income will be saved, as we have seen. And those savings will be used for investment.

But investment is nothing else than capital accumulation. So this income will lead to savings, which will fund investment, which will change the stock of capital, will feed into the capital stock, that will feed into income, and so on. All this is happening very slowly because the capital stock accumulates slowly.

But this is what is happening. And so all the models we're going to look at-- certainly, the model we're going to look at in this lecture is all about this mechanism.

So let's remember what we did in the previous lecture. And I'm going to assume that population is constant. I'm going to relax that at the very end. But assume that the population is constant and equal to N. And remember, we're not worried about unemployment and stuff like that here.

And so output per capita or per person is Y over N. And we remember we had a production function f of K and N. Then because of constant returns to scale, we could divide by N on both sides, everything. And we ended up with this relationship. So output per person is equal to-- is an increasing function of capital per person.

It's an increasing function of capital per person, but it's also a concave function of capital per person. Why is it concave? That is, why is it increasing at a smaller pace? Yeah?

AUDIENCE: It's decreasing marginal method.

RICARDEO CABALLERO: Decreasing marginal product of capital, exactly. For fixed amount of labor, the more capital you put into production, well, output keeps expanding, but by less and less because it has less and less labor to work with, each unit of capital. Perfect. That's very important.

Then we're going to work in closed economy. I haven't opened it. I'm going to do that after quiz 2. And I'm going to assume, also, no public deficits-- so g equal to T, capital T. And in that case, then, we know that private investment-- private savings equal to private investment. That's the way we derived our S curve. So that's not new.

I'm going to modify a little bit what we did in the short run, and I'm going to assume that savings is proportional to income. So savings-- little s times Y. Notice that this is different from what we did in the short run. In the short run, remember, we had a C0 floating around.

We had a constant in the consumption function. So savings, which was equal to income minus consumption, also had a constant floating around. Now, that constant was important in the short-run model because we were approximating for a bunch of things that are not related to short-term income-- wealth, the price of houses, and stuff like that. We put all that in that constant there.

When you think about the long run, though, most of those things that we excluded there-- asset prices, stuff like that-- tend to scale with output, as well. So these are inconsistent on the surface. But if you were to fully work out what is behind the C0 in the consumption function, then this is not a bad approximation.

They are not that inconsistent because you endogenize things that, over the long run, scale with income. I mean, wealth tends to rise with income, and all these things tend to move together-- not at high frequency. You can have all sorts of fluctuations. But over the long run, they tend to scale up together.

So that's going to be our saving function. So that means that we know, in equilibrium-- this is not investment function. We know that, in equilibrium, investment will be equal to-- it will be proportional to income.

So remember where we were going through the box. At the top of the box, we had capital. That led to output. We're doing everything per capita. That led to savings, and that funded investment. So that's what we have.

So this growth model is really about these three functional forms and then a dynamic equation for the stock of capital. So the evolution of the stock of capital-- capital will increase because of investment. That's what investment is. It's an increase in the stock of capital.

But it will also decrease as a result of depreciation. I mean, things do break up once in a while. And different types of capital have different depreciation rates. Equipment depreciates much faster than structures, and buildings, and so on. But we're not going to make those distinctions here.

But you see this tells you, the capital stock at t plus 1 is equal to the capital stock we had before minus what is depreciated of that stock of capital plus any new investment we do today. In per worker terms-- and remember that, for now, I'm keeping population growth constant-- equal to 0-- not population growth constant-- yeah, constant, but equal to 0. So population is constant.

I can divide these both sides by N, and I get that capital per worker. Per worker or per person is equal to this expression here. I did two things here. I divided by N, and I replaced this I function, this investment, for savings because I know, in equilibrium, they have to be equal. So I have that.

I can rewrite this. Just subtract Kt over N on both sides, and then you get-- the change in capital per person is an increasing function of savings and decreasing of depreciation. So the last step that is important in this model is to-- so here, I have, essentially, a difference equation for capital, but we have an output per capita on the right-hand side.

But it turns out that I know that output per capita, per person-- I said per capita, per person. They're the same thing. Per worker-- it's the same thing in this part of the course. So this is equal to-- is a function, is an increasing and concave function of capital per person.

So this is-- I would say it's a fundamental equation of the Solow growth model. It says, the change in the stock of capital increases with investment, of course, and decreases with depreciation. And both of these expressions here are increasing functions of the stock of capital per person.

So let's try to understand what is in here. So why-- so this is linear, obviously, because the depreciation is linear. Say you lose 5% of your stock of capital every year because it breaks down. Obviously, the more capital per person you have, the more units of capital you're going to lose. This is units of capital per person. You have a larger stock of capital, you're going to lose-- 5% of a larger number is a larger number. And this is proportional. It's linear.

Now, this one-- remember, this comes here from the saving function. And this term here is equal to income per person. Now, suppose that you start in a situation where the capital stock is relatively low, and this is positive. What does it mean that this is positive? I mean, the implication of this being positive is that the stock of capital per person will be growing. But what does it mean that it's positive in words?

I mean, you have a stock of capital. There are things that reduce the stock of capital, and there are things that increase the stock of capital. This is the thing that increases the stock of capital. That's the thing that reduces the stock of capital. So if this is greater than that, what does that mean? That means it is positive. But in words, what is happening?

And we simplify, but remember, this is just investment per person. Well, this just says that in this economy, there is more investment than destruction of capital due to depreciation. That's what this means.

This is investment. And this is positive means that the investment, which is a function of saving, the saving rate, and stuff like that-- it's a function of the funding available for investment-- it's equal to the funding available for investment. If this is positive, well, this is greater than the stock of capital.

Another way of saying it-- you need a minimum level of investment in an economy to maintain the stock of capital. The minimum level of investment you need to maintain the stock of capital is equal to the depreciation. So 10 machine breaks, you need to invest at least 10 machines in order to maintain the stock of capital constant.

Now if this is positive, it means that you're investing more than the machines that are breaking down. Now, suppose you start in a situation where that's the case. So that means the stock of capital is growing. Suppose I ask you, the next period, do you think the gap will be larger or smaller than it used to be?

Actually, that's not a great question because I'm not doing it in the right units for that. Let me ask you a variation of that question. Suppose we keep going. After a while, do you think that number will get larger or smaller? Let it run for quite a while. Do you think that number will-- so remember what I'm saying.

We start from some stock of capital. This is positive. If this is positive, it means that the capital stock is growing. That means this guy is growing, and that guy is growing, and they're growing equally. But after a while, do you think this number will get smaller or bigger-- after a long while, just to make sure that my approximation is not tidy.

AUDIENCE: Smaller because of the decrease in [INAUDIBLE].

RICARDEO CABALLERO: Exactly. It's going to get smaller because this guy keeps growing linearly [INAUDIBLE] and this one is not. It's concave. At some point, this income-- you need to put a lot of capital for income to keep rising, and therefore, for saving to keep rising, and therefore, for investment to keep rising.

And at some point, yes, it won't be able to really grow. I mean, you're going to be using all your investment, really, to maintain the stock of capital. That's the logic of the Solow model.

And it's all in this diagram. So this is the diagram you should really, really understand well and control it, and play with it, and all that. It's the equivalent to your IS-LM model in the first part of the course. So look at what we have here.

So I'm going to plot output per worker-- per worker, per person-- against capital per worker here. And so this red line here is just the depreciation, this term here, and thus, is a linear function of the capital per worker. That's what it is.

The blue line here is output per worker, which, as we said, is a concave function of K over N. Remember, I showed you that production function in the previous lecture. There you are. What is the green line? It's investment per worker, which is equal to saving per worker.

And saving per worker is little s, the saving rate, times a output. So it's little s, which is a number like 0.1, if we're talking about the US, and 0.4 if we're talking about Singapore. It varies a lot across countries. So this green line here is nothing else than this blue line multiplied by a number that is less than 1. That's the reason it's lower.

OK, good. So the point I was describing before was a point like this. Remember? The point I was describing-- suppose that the economy starts in a point like this one, K0 over N. And I want to understand the dynamics of this economy. How will it grow over time?

So what you see here is that, at this level of capital per worker, investment is greater than depreciation. So that's exactly a situation where this is positive That distance here is that.

And the reason I said, ah, I'm not going to do any local analysis because we could have started with a K over 0 over here, and then that number is growing. But it's growing-- if you were to normalize by the stock of capital, it's declining. But I didn't want to do that then.

But now, that's what I-- so let's look at this case. You're in a situation where this is positive. If this is positive, it means the capital stock per worker is growing. So you're moving to the right. In the next period, you're going to be here. That means the capital-- so it keeps growing but by smaller steps.

Eventually, the investment is entirely used for recovering from the depreciation of capital, so covering the depreciation of capital. And at that point, the capital stock stops growing. We call that a steady state, stationary state. We stop there.

So that's the steady state of this model. That means this economy, regardless of where I-- I'll do the analysis from the other side. Suppose that you start from a situation like this. You start with a lot of capital. Well, if you start with a lot of capital in this economy, what happens here?

Well, what happens here is that the investment you're putting into the ground in this economy is less than what you need to maintain the stock of capital, which is the depreciation. And that means the stock of capital will be shrinking over time. You're moving that way.

So regardless of where you start in this economy, if I ask you the question A hundred years from now, where are you? you tell me, I don't need to know where you start from. I know that we're going to end up around there. You can-- either you start from here, you can go there. From here, you go there and so on. That's the reason we call this a steady state. This is where you converge in the long run.

Now, this is already interesting because it tells you, at this point here, the economy was growing. The capital stock was growing, and output was growing. You see the capital-- if you start from here, the capital stock is growing. Well, output is also growing. You're moving up. So you had growth.

That kind of growth we call transitional growth. It goes from one point to another point. It's not a permanent growth. It's a transitional growth. It's the fact that you were away from your steady state, and then you're going convergent towards your steady state.

A lot of the growth we observed and the difference of growth we observed across countries-- remember, I showed you those downward-sloping curves and all that-- is as a result of that. Poorer economies tend to have lower capital-- capital labor, capital employment ratios, capital population ratios. And therefore, they tend to grow faster because they're catching up with the steady state.

Very advanced economies that have been more or less in the same place for a long time are moving around there, so there's less catching up growth. And that's the main responsible for the downward-sloping curve I showed you within OECD countries and even broader than that. Africa was a little bit of a problem there.

OK, so this is an important model for you, important diagram. Let's play a little with it. So suppose that, at the time-- this is a very simple model. But at the time, the view was that, well, what really supports growth is savings. So economies that save a lot grow a lot.

And this sort of makes sense here because investment, which is what leads to capital accumulation, is entirely funded by savings. It makes sense. You have more savings, you should grow more.

OK, so this is something-- we can do an experiment. Suppose you start at a steady state, if you will. And now we increase the savings rate. What moves? Which curve? This is the kind of thing you should know when you work with this model.

If I change the saving rate, which curve moves in this model? Let me go one by one. Does the red line move? No. It has nothing to do with savings. It has to do with depreciation.

If I move depreciation rate, that curve will move, but not-- will the production function move? No. So the blue line cannot move. All that will move is the green line because the green line is the saving rate times the blue line. So if I increase the saving rate, I'm going to move the green line up. That's what we have here.

So you see what happens is you start-- this was a steady state for this saving rate in this economy. Now, all of a sudden, this economy starts saving more. What happens? This tells you very much the story of Asia. The Asia miracle of the '60s, '70s, and so on is very much something like that-- a little more complicated.

A big part of what explains the fast growth of Asia during that period is that something like that happened. Now, why the savings rate increase and so on-- that's all very interesting and so on. But it's not what I want to discuss today.

So what happened here, then? So what happens-- this economy was in a steady state. So there was no growth. It was growing at 0 in a steady state because this says, in a steady state, output per worker remains constant. And since we have no population growth, that means output is not growing, either. The only way you can have that ratio constant with the denominator not moving is that the numerator is not moving, either. OK, good.

So now, boom, all of a sudden, we get a higher saving rate. So what happens now? What reacts? So the saving rates go up. It's a closed economy. It means the investment rate will go up. What happens now?

What does that gap tell you? Now you have a positive gap there, which means you're investing more than what you need in order to maintain the stock of capital at the previous steady state. So that means that the stock of capital is going to start growing to the right. It's going to start growing.

And as the stock of capital grows, then output per capita also grows. And this will keep happening until you reach the new steady state. So a higher saving rate-- so important conclusion there.

This, as simple as it is, proves something-- that the conventional wisdom that the higher saving rate would give you sustained growth, higher growth, isn't really true, certainly not in this model. Eventually, you'll go back to growth equal to 0. When you reach a new steady state, you're going to be also growing at 0.

What is true, though, is that you get, again, what is called transitional growth because here, you're going to start growing very fast, in fact. And then you're going to keep growing at the lowest-- lower, lower pace until you go back to 0. But you're going to get lots of growth in the transition as a result of that.

And it turns out, in the data, when you're looking at 20, 30 years of data, it's difficult to disentangle very permanent rates of growth versus transitional rate of growth. This is one of the things that has concerned China quite a bit. They grow very, very fast. They have been growing very, very fast for a long time.

But it's very clear that it's becoming harder and harder for them to grow at the type of rates of growth that they had 20 years ago. They had rates of growth 15% or so. They had very high-- they had a very low initial capital population ratio-- big population, little capital, and enormous savings rates. So they grew very, very fast.

They had the green line very close to the blue line, the capital stock very low, so they grew very, very fast but they have been growing very fast for a very long period of time. So now it's getting a lot harder because they're getting closer and closer to their steady state. That's the issue.

There are other sources of growth, and that's what we're going to talk about in the next lecture. But this is sometimes called the easy part of growth. It's sort of running out in China. And it has run out in all developed economies for quite a while.

Good. Is this clear? It's important. I mean, a question like that is guaranteed in your quiz for 81. What happens if the saving rate does something?

So this is a plot over time. So this is a case in which you were in a steady state. And at time t, the saving rate goes up as 1 greater than 0 jump. Then output cannot jump. So the saving rate goes up, but output cannot jump at day 0. Why? Why is it that the output doesn't jump immediately to a new steady state?

This is the-- I'm saying, this is what will happen to output. You're going to start growing very fast early on, and then keep growing, keep growing at a slower and lower pace because of decreasing returns to capital. And eventually, you'll convert to the new steady state with the rate of growth equal to 0, like the one you had before, this savings shock.

And the question I'm asking now is, why does output have to do this? Why doesn't it just jump? What is the only variable that could make it jump?

So you need to look at the production function. The production function is the function of K over N. N is fixed. The only thing that can make it jump is the capital stock jumps, but the capital stock is not jumping. That's a stock.

And in order to accumulate a larger stock of the new steady state, you're going to go through a lot of flows. That's investment. Every year, you're going to be adding a little more to the stock of capital. That's the way you grow. It's not that, all of a sudden, your stock of capital jumps.

That's very much because this is a closed economy. If you're in an open economy, the capital stock can move a lot faster in a transition because you can borrow from abroad. You don't need to fund it all with domestic sources. And in fact, that's what typically happens in emerging markets and so on is they typically borrow for a long time.

Problem is that they tend to consume it rather than invest it, and that's the reason you end up in financial crisis and so on. But in principle, things could go much faster if you have an open economy, and you have capital inflows into your country. But we'll talk more about that five or six lectures from now.

But anyways, but this is what happens with an increase in the savings rate. So yes, it affects the rate of growth of the economy during the transition but not in the long run. Now, this transition can be very long.

Now, what about consumption? So invariably-- and there's no way around that-- given a technology and so on, if the saving rate goes up, then output per worker will go up. The next question is, what happens to consumption per worker? Does consumption per worker go up or not?

You are inclined to say, well, I mean, it makes sense that it goes up because we have more income. If the saving rate is little less times y, then consumption is 1 minus little s times Y. So income goes up, consumption should go up.

And yes, that's a dominant source. But it's not all the story because remember what I told you. So consumption here is going to be equal to 1 minus little s times Y. So consumption per person will be that.

Remember that what is increasing Y over N there-- so what is making this guy go up, which will lead to an increase in consumption over N-- is that this guy went up. And that's a force in the opposite direction.

And in fact, that was one of the debates with the Southeast Asian miracle is that it was fueled by lots of savings. So people say, OK, that's wonderful. Your output growth is very fast, but consumption growth is not so fast. And at some point, it may be hurting you. I think that they were right, though, for other reasons. But that picture makes the point.

So if your saving rate to start with-- this is a general lesson. If the saving rate you start with is very, very low, then an increase in the saving rate will lead to a strong increase in consumption because this change is small relative to the big bang get in output because, if you have low saving rate, that also means that the capital stock is very low.

And if the capital stock is very low, f prime is very big. This is a concave function, and you're in the steep part of the function. Later on, if saving is very high, you're going to tend to have capital stock very high. And then, first of all, more capital won't increase output per worker a lot because of decreasing returns. And this is a big number, so it starts dominating.

And that's what you see here. This economy has increased the saving rate. Consumption per worker rises. But at some point, it reaches a maximum, and then it starts declining.

I mean, think of the limit. If you save 100% of your income, you don't consume anything. No matter how much is your output, if your saving rate is 100%, then you're not going to consume anything. If you have no income, no savings rate, no savings, no capital stock, no income, you're not going to consume anything either.

So at least you know these two points. And since you know there are some positive points in the middle. You know that the curve is going to tend to have that kind of change. It's going to be nonmonotonic, and that's the way it is.

So let me just play with a few numbers. This is-- yeah, let me play with a few numbers. It's not that crazy. Suppose you have a production function that gives equal weight to capital and workers-- so this production function. Does the production function have constant return to scale?

It better be because that's what we're doing. What do you think? Yes. The sum of the exponents is 1, so it's K to the 1/2, N to the 1/2. The sum of the exponents is 1, so you know that it's proportional to a scaling factor.

So we're going to use as a scaling-- as before, N. S we have this. This is a f of little f of K over N is the square root of K over N minus delta K over N. All that I'm doing is I'm plugging in that function.

So here, I'm replacing all these functions by a specific example, one in which this is a square root of K over N. That's a concave function-- square root. Good.

Now, do it as an exercise. If you solve for the steady state, how do you solve for the steady state? Well, set this equal to 0. That will give you the steady state. The steady state is when the capital is not growing anymore. It's when this is equal to 0.

When this is equal to 0, I can solve for the steady state level of K over N from here. This equal to 0, I can solve for K over N. And I'm going to call that the steady state, K star. We typically use stars for steady states in growth theory.

Well, the answer to this is K-- the steady state, stock of capital per person is the saving rate over delta squared. That's what it is. Output per person, which is the square root of K over N, is, therefore, the square root of s over delta squared, so it's over s delta.

So in this particular model, in the long run, output per worker doubles when the savings rate doubles. If I double the saving rate, then output per worker will double. Notice that the stock of capital is going to grow a lot more when you increase the savings rate. It's squared.

So in that economy, if you do increase the savings rate from 10% to 20%, this is the way it goes. So remember, 10% to 20%-- that means that the new steady state output per worker will be twice what it was in the previous steady state. So you go from 1 to 2. But it takes a long time. The numbers are [INAUDIBLE]. 50 years it takes you to go to the new state.

And so that's the time frame we're talking about. So it is true that the saving rate will not change the long-run rate of growth, absent other mechanisms. But you can grow faster than your average, your steady state level for quite some time. And again, a lot of the Asian miracle has been of that kind.

This is what I was telling you of China before, no? Well, yeah, you can grow very fast, especially if you have saving rate much higher than 20%-- I mean 50% or so. But the rate of growth will have a tendency to decline, absent some other miracle.

There are a lot of the reasons why we have all this fight about technology and so on. It has to do with-- because that's the main mechanism-- alternative mechanism to grow is technology. We're going to talk about that in the next lecture. But this force, which is, what I said before, is the easy part of growth-- it's very difficult to fight this pattern.

So here you have numbers for the steady states. So if the saving rate is 0, obviously, everything is 0-- no way around. If saving rate is 0.1, 10%, then, in this model, capital per worker is 1. Output per worker is 1. Consumption per worker then go from 0 to 1. Why? Because you were saving something. So it's 1 minus 0.1, which is a saving rate.

Suppose you double the saving rate. Well, we know that we're going to double output per worker in this economy. We said that. We're going to go from 1 to 2. The capital stock is going to have to grow a lot more to double the amount of output.

Why is that? Decreasing returns. To double output, you're going to have to much more than double capital because you're going to be fighting decreasing returns. What about consumption? Well, it won't double because you're doing this out of increasing the saving rate. So you get the 2 minus, now, 0.2, not 0.1-- minus 0.2 times 2, so you get 1.6 and so on.

And the higher you go with your saving rate, the harder it gets for capital to bring along output per capita, and the more the drag on consumption because you need to be saving a lot in order to maintain this high stock of capital that you're having. You have a very large stock of capital. That means you need to save a lot just for the sake of maintaining that stock of capital. And so little is left for extra output per capita.

And so you see that, here, for this particular model, when the saving rate exceeds 0.5, then output obviously keeps rising when you increase the saving rate, but output starts declining. So you're in the declining part. And if you get to 1, of course, there is no consumption. So that's the curve that we trace.

Is everything clear? Now I'm going to-- that's the basic Solow model. And that's a model that, again, you need to control completely.

All that I'm going to do now is very simple. I'm going to just modify a little bit this model to add population growth. So what happens-- by the way, for centuries, population growth has been one of the main-- in this model, we concluded that output per worker was not growing. What we're going to conclude in a second is that output per worker will not grow if population is growing.

But that means that output is growing. If population is growing, and output per worker is not growing, it's constant, that means output is also growing. And for a long time, growth of output, not of output per worker, was driven by large population growth. And sometimes, you get big migration flows into a country that leads to growth and so on.

Now big parts of the world have negative population growth. So now we're going through a cycle in which things are going the other way around in many large parts of the world. This is true in almost all of continental Europe, certainly in Japan. I said South Korea, China, and even some places in Latin America. So the drag, actually, is against that. We don't have the natural force for growth that we had for many, many years.

So let me introduce population growth. So assume now that population, rather than being constant, grows at the related gN, which could be positive or negative. I'm going to do the example for a positive population growth example.

So there's no equation that changes, in the sense that this is still true. It's still true that investment is equal to savings. It's still true that output is equal to output per worker, and output is equal to f of K and N and so on and so forth.

The thing that is a little trickier is that, in this model, if I don't normalize things for-- in this case here, where population was not growing, I could have just eliminated this N as a constant. And I would have done everything in capital, in a space of capital here, and output here would have been the same, just scaled by a number, a constant N.

When I have population growth, I'm not indifferent between doing one way or the other because, if I don't have-- if I do it in the space of K and Y, and population is growing, then all these curves are moving. This is a very unfriendly diagram because my curves are all moving. As N is moving, everything is moving.

So the trick in all these growth models, and it's going to be even more important in the next lecture, is to find the right scaling of capital so there is a steady state, so your curves are not moving around as population growth. It's very easy to find the scaling factor. It's population.

So that's what I'm going to do. But remember what is different here is-- so what I'm saying is I want to get all my variables scaled by population at some point in time. That's what I want to do because I know-- I practice enough with these things-- that's going to give me a steady state.

Now, what is tricky relative to what I showed you before is that, before, I just divided by N both sides, and I was home. Now I can't really do that. OK, let me divide by Nt plus 1 both sides. So that's nice. I get my capital per worker at t plus 1.

But there are certain things that are not as nice. What I have on the right-hand side is not what I really want. I don't want capital over population next period. My steady state is going to be in the space of capital overpopulation at the same time. That's my steady state.

So this is not so nice. So what I have to do is I want to convert this, the right-hand side, in assumption that these are the kind of things that I want to have. So what I'm going to do is divide and multiply each of these sides by Nt over Nt plus 1-- sorry-- I'm going to divide. I multiply each of these by Nt.

So multiply by Nt, divide by Nt, so multiplying by 1. And then I can rearrange the terms in this way so I get what I want, which is capital per person at time t, all at time t. But then I get this ratio here. And I can do the same for this expression here.

Now, what is that ratio? It's population today divided by population tomorrow. Well, it's 1 over 1 plus the rate of growth of population. Nt plus 1 is equal to Nt times 1 plus gN. That's the rate of growth of population.

So what I have here is 1 over 1 plus gN. Now, gN is not a big number. So 1 over 1 plus gN is approximately equal to minus gN. So 1 over 1 plus gN-- gN is very close to 0-- is approximately equal to minus gN.

So that's the reason this guy became that guy-- approximately that guy. I can do the same here, but it turns out that there's an extra term here which is equal to s times gN times Yt over Nt. Well, that's second-order. That's the reason I'm going to drop it.

It's a saving rate which is-- it's a small number times a rate of population growth, which is a number like 0.01 or something like that. So that's a small number, so I'm dropping it. That's a bigger approximation than that one, actually, but I'm going to do it. Everything becomes a lot simpler. So this is an approximation. I'm just dropping second-order terms.

And once I have that, I have the system I want because now I have a system for the evolution of the capital per worker or per person. And if you see, it looks exactly as we had before. Remember, this is exactly what we had before-- sf K over-- we used to have N subscript t. Now it's K over Nt.

But what is different is that now, rather than having only the depreciation rate here, we have the depreciation rate plus the rate of growth of population. Why do you think we have the rate of growth of population there? Remember the economics behind this expression before.

It was this is what adds to capital, to capital per worker. This is what you need to maintain, what takes away from capital. Now, it's what takes away from-- given we're doing everything in the space of capital per worker, that takes away from capital-- oh, that's a typo. There's a t there-- t.

So why do you think I have this gN here? Well, I have only one minute, so I don't have time to-- because if I want to maintain a stock of capital per worker, and workers are growing, then I need to be growing the capital. So even if I had no depreciation, if I want to maintain the capital per worker constant, and workers are growing, then I need to grow the stock of capital.

So in order to maintain the capital-- I still need to spend what I used to spend for depreciation of the capital stock. But if I want to maintain the capital per worker constant, then I'm going to need more investment, just to make up for that extra component.

So now, set gA equal to 0. Your diagram is exactly as before in this space. Set a equal to 1 and constant. But this line, the red line here will have delta plus gN. So it rotates up.

So you can play here and see what happens if there is a change in population growth and so on and so forth. It's going to be counterintuitive initially because you see, if I increase population growth, this curve will rotate up, and then it will appear as if that leads to negative growth. But you don't get negative growth.

In this diagram, you do get that Y over N will decline. But that doesn't mean that you get negative growth. It just means that output is not growing as fast as population. But both are growing, just the population is growing faster than output.

I'll start from that-- oh, I think it's after your break, so when you have forgotten everything by then. So I'll do a review of this, and then we-- OK, have a nice break.