Question 1: The Goods Market continued [40 Points]

Consider the following economy. The demand for goods, Z, is given by

\[ Z \equiv C + I + G \]  

(1)

where \( C \) is consumption, \( I \) is investment, and \( G \) is government spending. The consumption function is given by

\[ C = c_0 + c_1 Y_D \]  

(2)

where \( c_0 \geq 0, 0 < c_1 < 1 \), \( Y_D \equiv Y - T \) is disposable income, and \( T \) is taxes.

When we say “exogenous variables”, this refers to variables determined outside of the model such as, for example, \( c_0, c_1 \). In contrast, \( C, Y \) are endogenous variables. \( G \) and \( I \) will always be treated as exogenous.

In Problem Set 1, you found that when \( T \) is exogenously given by \( T = t_0 \), equilibrium in the goods market implies:

\[ Y = \frac{1}{1 - c_1} [c_0 - c_1 t_0 + G + I] \]

In this case, the government spending multiplier is given by:

\[ \frac{dY}{dG} = \frac{1}{1 - c_1} \]

(a) Endogenous \( T \):

Now, consider the case where \( T \) is endogenously given. Specifically, it depends on income in the following way: \( T = t_0 + t_1 Y \).

1. [5 Points] Solve for equilibrium output \( Y \) assuming that \( t_1 \in (0,1) \).
2. [5 Points] What is the government spending multiplier now? Does the economy respond more to government spending in this case or in the previous one?

3. [5 Points] How much does public saving, $T - G$, decrease in response to an increase in $G$? Is it less than or greater than one?

(b) Balanced Budget

Now suppose that the government decided to choose $G$ such that budget is balanced:

$$G = T,$$

that is, now we treat $G$ as another variable we need to solve for. The rest of the equations remain the same as in Part (a), i.e., taxes depend on consumption.

1. [5 Points] Among the variables $Y$, $C$, $G$, $I$ and $T$, identify which variable is exogenous and which is endogenous in this case.

2. [5 Points] Solve for equilibrium output $Y$ as a function of exogenous variables/parameters.

3. [5 Points] Now, suppose $G$ is exogenous but that taxes are set to match government spending (i.e., so the budget is still balanced but taxes no longer depend on consumption as in part (a)). Solve for equilibrium output again. How much does equilibrium output change in response to an increase in government spending $G$? Compare this new multiplier to the case of exogenous taxes (in the prompt) and endogenous taxes (Part (a)).

4. [5 Points] Interpret the result in b.3.

5. [5 Points] Express $S$ in terms of exogenous variables when the government follows a balanced budget. Remember $S = Y - T - C$. How does $S$ respond to a change in $G$?

**Question 2: Financial markets [45 Points]**

(a) Demand for money and bonds

Suppose there are only two assets in the economy: money and bonds. Suppose that money demand in an economy is given by:

$$M^d = SY(a - i)$$

where $SY$ is nominal GDP, $a > 0$ is a parameter, and $i \geq 0$ is the interest rate.
1. [5 points] Describe how money demand depends on the interest rate and income (that is, say whether it increases, decreases, or is non-monotonic in each variable). Explain the intuition.

2. [5 points] A bond will pay $120 in one year. Express the price of the bond as a function of the interest rate. How does the price of the bond depend on the interest rate? Explain how this is consistent with the idea that increasing the interest rate increases the demand for bonds.

(b) Determining the interest rate

For the rest of Question 2 you can assume $a = .6$.

1. [5 Points] Suppose nominal GDP is $20T$. When the central bank supplies $10T$ of money, what is the equilibrium interest rate?

2. [5 Points] Now suppose the central bank wants to control the interest rate. Assuming again that nominal GDP is $20T$, how much money does the central bank need to supply in order to set the interest rate equal to $i = .05$?

(c) Maintaining the interest rate

1. [5 Points] Now suppose that the economy grows and nominal GDP increases to $25T$. If the central bank keeps the same money supply as in b.2, what is the equilibrium interest rate?

2. [5 Points] If nominal GDP increases to $25T$, how much money should the central bank supply in order to maintain the targeted interest rate of $i = .05$?

(d) Changing the interest rate

1. [5 points] Consider again the case where nominal GDP is $20T$ and the Fed wants to decrease the targeted interest rate from $i = .05$ to $i = 0$. How much money should it supply to implement this interest rate?

2. [5 points] Comparing your solutions to b.2 and d.1, describe how the central bank needs to change the money supply to decrease the interest rate. Describe the process by which the central bank can implement this change in the money supply.

3. [5 Points] If the central bank supplies more money than the amount derived in d.1, how will the interest rate change? Explain the reasoning behind your answer.

Question 3: Core IS-LM [15 Points]

Consider the following version of the IS-LM model. Consumption is given by

$$C = c_0 + c_1(Y - T),$$

(3)
where \( c_0 > 0, \) \( 0 < c_1 < 1, \) \( Y \) is income, and \( T \) is taxes. Investment is given by

\[
I = b_0 + b_1 Y - b_2 i,
\]

(4)

where \( b_0 > 0, b_1 > 0, b_2 > 0, \) and \( i \) is the nominal interest rate. Also assume \( c_1 + b_1 < 1. \) The demand for goods is given by

\[
Z = C + I + G.
\]

Assume \( G \) and \( T \) are exogenous.

(a) Equilibrium in Goods Market

1. [5 points] Explain intuitively why it is reasonable to assume that investment negatively depends on the interest rate. Provide an explanation for the case of a firm that must borrow to invest, and also provide an explanation for the case of a firm that has enough of its own funds to finance investment projects without borrowing. (Hint: If the firm does not use its funds to finance investment projects, what can it do with its money?)

2. [5 points] Using equilibrium condition, \( Y = Z, \) derive the mathematical expression for equilibrium output for a given interest rate, \( i. \) Explain also graphically how the equilibrium output is determined. (Put \( Z \) on the y-axis and \( Y \) on the x-axis).

3. [5 points] What is the government spending multiplier in this economy for a given interest rate, \( i? \) (That is, how much does the equilibrium output change in response to an increase in \( G? \).) Is the government spending multiplier larger than the case where investment is exogenous, \( I = \bar{I}? \) Why or why not?