14.02 - Principles of Macroeconomics

Problem Set 3

Spring 2023

NOTE: in the following questions you will be asked to draw some graphs. Please make sure to always label axes and curve, add arrows to denote shifts in the relevant curves, and mark the axes in the locations corresponding to equilibrium quantities and prices (otherwise points will be deducted!).

Question 1: Interest Rate Sensitivity of Investment [30 points]

Consider a start-up firm with ten possible projects of developing new apps: $j = 1, 2, ..., 10$. Each app j is projected to cost \$50 Million to develop, and generates a different stream of earnings. In the first year, project j generates \$2 Million in earnings annually, and after then it perpetually generates \$$j$ Million in earnings annually. All cash flows are expressed in constant dollars, i.e., prices are constant.

1. [6 Points] Calculate the present discounted value of the earnings stream associated with app j when the interest rate is i.

2. [6 points] If the interest rate is 5% $(i = .05)$, how many apps will the start-up firm develop?

3. [6 Points] What if the interest rate is 10%$(i = .1)$?

4. [6 Points] What is the highest value of i for which all the projects will be developed?

5. [6 Points] What does your answer suggest about the slope of the IS curve? Is it downward sloping or upward sloping?

Question 2: Basic IS-LM Model [45 Points]

Consider the following IS-LM model:

$$\begin{aligned} Y &= Z \\ Z &= C + I + G \\ C &= c_0 + c_1(Y - T) \\ I &= b_0 + b_1 Y - b_2 i \\ i &= \bar{i} \end{aligned}$$

where C is consumption, I is investment, G is government spending, T is taxes, i is the interest rate. We assume that the Central Bank sets the nominal interest rate $i = \bar{i}$, and that T and G are exogenous. Parameters (c_0, c_1, b_0, b_1, b_2) are positive and $b_1 + c_1 < 1$.

1. [5 points] Derive the mathematical expression for the IS curve in this model. i.e., i as a function of the equilibrium value of Y. What is the slope of the IS curve?

2. [5 points] Please describe how equilibrium output is determined in this model. Be specific about what role each of the curves plays.

3. [5 points] Please draw a graph depicting the IS and LM curves in this economy. Indicate equilibrium output. Place Y on the x axis and i on the y axis.

4. [10 points] Suppose that investors become worried about the future so that they invest less as a proportion of output than they had previously. i.e., b_1 declines to a new lower level b_1'. By how much will equilibrium output change? and why? Re-draw the graph from question 3 to represent this scenario.

5. [10 points] Monetary Policy response: Suppose that the central bank responds to the decline in b_1 described above by changing the interest rate i. By how much would the central bank need to change i to keep output at the same level? Discus the role played by the parameter b_2 in this expression. Re-draw the graph in question 4 to represent a scenario where the central bank changes i by exactly this amount.

6. [10 points] Fiscal Policy Response: Suppose instead that the government decides to maintain output constant by changing government spending. By how much would G need to change to achieve this objective? Re-draw the graph in question 4 to represent this scenario.

Question 3: Extended IS-LM Model [25 Points]

Consider the extended IS-LM model where investment depends on the real interest rate r, which is given by the nominal interest rate i minus inflation expectations π^e,

$$r = i - \pi^e.$$

This is known as the "Fisher equation". The extended IS-LM model is then pinned down by the following system of equations

$$
\begin{aligned}
Y &= Z \\
Z &= C + I + G \\
C &= c_0 + c_1(Y - T) \\
I &= b_0 + b_1 Y - b_2 r \\
i &= \bar{i}
\end{aligned}
$$

where as before, C is consumption, I is investment, G is government spending, T is taxes and r is the real interest rate. We assume that the Central Bank sets the nominal interest rate $i = \bar{i}$, and that T and G are exogenous. Parameters (c_0, c_1, b_0, b_1, b_2) are positive and $b_1 + c_1 < 1$.

1. [5 points] The University of Michigan runs a monthly survey of households, asking respondents for expected inflation over the next 12 months. Plot this (e.g. on FRED the series name is 'MICH'). What has happened to inflation expectations over the last two years?

2. [5 points] Solve for equilibrium output in the extended IS-LM model. In particular, please solve for Y as a function of the nominal interest rate i, rather than the real interest rate, and include π^e in the expression.

3. [5 points] Suppose inflation expectations rise 3 percentage points. By how much should the central bank raise the nominal interest rate to keep output Y constant?

4. [5 points] Suppose the central bank fixes the nominal interest rate at some level i, but announces that it plans to increase the money supply and generate inflation in the future. This announcement causes expected inflation π^e to rise. This is a policy known as "forward guidance".What happens to the real interest rate? What happens to output?

5. [5 points] Recall that nominal interest rates cannot go much below zero, because if a bond paid a negative nominal interest rate, holders of the bond would want to switch from holding bonds to holding cash, which always pays a 0% nominal interest rate. Suppose inflation expectations are currently 2%, $\pi^e = 2\%$. Assuming that $i \geq 0$, what is the lowest real interest rate that the central bank could achieve?

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