14.02 - Principles of Macroeconomics

Problem Set 4
SOLUTIONS

Spring 2023

Question 1 - The WS-PS Model [20 Points]

Suppose the wage-setting equation is given by:

$$W = P\left(1-u\right)z$$

where z represents unemployment insurance, u represents the unemployment rate and W represents the nominal wage . Suppose the production function is given by $Y = N$, where N is employment, and the price-setting equation is given by:

$$P = W\left(1+m\right)$$

where m is the markup.

1. [6 Points] Explain how the wage-setting equation depends on the price level, the unemployment rate, and the unemployment insurance. Provide the intuition.
   Solution: *The wage-setting equation depends positively on the price level, because with a higher price workers will seek an increase in their wages that compensate this increase given that what they care about is their real wage, not their nominal wage. At the same time if firms sell their goods at a higher price they will be willing to accept the wage increase. The wage-setting equation depends negatively on the unemployment rate because with a higher unemployment rate the probability of finding a job elsewhere decreases and so workers are willing to accept a lower wage. The wage-setting equation depends positively on unemployment insurance because that raises the value of workers' outside option, and thus a higher wage is required for them to stay at their jobs.*

2. [6 Points] Explain how the price-setting equation depends on the wage and the markup m. Provide the intuition.

   Solution: *The price-setting equation depends positively on the wage because the unit cost of an extra unit of output is given by W, so firms will charge W at a minimum for their products. The price-setting equation depends positively on the markup because when firms have, for example, higher market power they are able to charge more for their products.*

3. [8 Points] Express both the wage-setting equation and the price-setting equation in terms of the real wage W/P. Find algebraically the natural rate of unemployment, u_n, by equating the real wage implied by the wage-setting equation with the one implied by the price-setting equation. Does the natural rate of unemployment increase or decrease in the mark-up? Provide intution.

   Solution: *Solving for u_n we have:*

   From the wage-setting equation:

$$\frac{W}{P} = (1-u)z$$

   From the price-setting equation:

$$\frac{W}{P} = \frac{1}{(1+m)}$$

   Setting the two equal,

$$\begin{aligned}(1+m)^{-1} &= (1-u_n)\,z \\ u_n &= 1 - \frac{1}{z\,(1+m)}.\end{aligned}$$

   u_n increases in the mark-up. A higher mark-up means firms can set higher prices leading real wages to decrease. In equilibrium the natural rate of unemployment must adjust.

Question 2 - The Phillips Curve (30 points)

Consider the WS and PS relations in a dynamic setting where the subscript t represents the year:

$$\begin{aligned}W_t &= P_t^e F\,(u_t, z) \\ P_t &= (1+m)\,W_t\end{aligned}$$

We shall assume for this question that:

$$F\,(u_t, z) = 1 - \alpha u_t + z$$

where α captures the strength of the effect of the unemployment rate on the equilibrium wage, and z represents the catch-all variable that captures the effect on wages of the policy environment (e.g., minimum wage, employment protection legislation, etc).

1. [8 points] Go to the FRED website (link here). Download the data series for median usual weekly earnings for those with less than a high school degree and those with a bachelor's degree and higher (series LEU0252916700Q and series LEU0252918500Q). Also download the data series for the Consumer Price Index for All Urban Consumers: All Items in US City Average (series CPI-AUCSL) since 2007. Calculate the percent change from a year ago at a quarterly frequency for each series and plot them together in one chart. Calculate the average for each of these three series from 2007 to 2021. Using these averages, what can you say about wage inflation and price inflation in the last 15 years? Is there a difference across educational groups? What has happened to the real wage for each of these groups since 2007?
Solution:

Wage inflation and price inflation appear to track each other quite closely. However, there are some notable periods where they diverge. In particular, wage inflation was higher than price inflation towards the end of the 2008 recession, and for most of 2013 onwards. Only recently has price inflation surpassed wage inflation. This means that real wages have been increasing for most of the period since 2007. However, they have begun to fall since the recovery from the recession. The changes in wages are more extreme for less-educated workers, also their real wage has suffered more in the recovery from the pandemic vis a vis more-educated workers.

Courtesy of Federal Reserve Bank of St. Louis. https://fred.stlouisfed.org.

2. [8 Points] Use the WS and PS relations shown above, together with the specification of the function F, to derive a formula for P_t in terms of P_t^e, u_t and parameters α, m, z.
***Solution**: Plug the first in the second:*

$$P_t = (1+m)\, P_t^e\, (1 - \alpha u_t + z)$$

3. [8 Points] Consider the equation you have derived in the previous part, and use it to derive the Phillips Curve

$$\pi_t = \pi_t^e - \alpha\,(u_t - u^n)\,,$$

where we have defined the *natural unemployment rate* as $u^n = {}^{(m+z)}/_{\alpha}$.
Hint: to get started, divide both sides of your equation by P_{t-1}. Then, use the following facts:

$$\frac{P_t}{P_{t-1}} = 1 + \pi_t \qquad \frac{P_t^e}{P_{t-1}} = 1 + \pi_t^e \qquad \frac{1+\pi_t}{(1+\pi_t^e)\,(1+m)} \approx 1 + \pi_t - \pi_t^e - m.$$

***Solution**: Follow the hint:*

$$\begin{aligned}
\frac{P_t}{P_{t-1}} &= \frac{P_t^e}{P_{t-1}}(1+m)(1-\alpha u_t+z) \\
1+\pi_t &= (1+\pi_t^e)(1+m)(1-\alpha u_t+z) \\
\frac{1+\pi_t}{(1+\pi_t^e)(1+m)} &= 1-\alpha u_t+z \\
1+\pi_t-\pi_t^e-m &= 1-\alpha u_t+z \\
\pi_t &= \pi_t^e-\alpha(u_t-u^n)
\end{aligned}$$

to recover the Phillips Curve.

4. [6 points] Go to the FRED website (link here). Download the data series for the Consumer Price Index for All Urban Consumers: All Items in US City Average (series CPIAUCSL) and the rate of unemployment (UNRATE) of the years 2021 and 2022. Compute monthly inflation in both years and plot the evolution of monthly inflation and the rate of unemployment. What do you observe? Assuming inflation expectations were constant, was the rate of unemployment below or above its natural rate?***Solution**:*

Question 3 - The IS-LM-PC Model [50 Points]

Consider an economy with aggregate demand,

$$Z_t = c_0 + c_1(Y_t - T) + b_0 + b_1 Y_t - b_2 r_t + G,$$

where t is a time index, Y_t is output, T taxes, r_t the real interest rate, G government spending and $c_0, c_1 b_0, b_1, b_2$ are positive constants so that $c_1 + b_1 < 1$. Next, consider the Phillps curve,

$$\pi_t - \pi_t^e = -\alpha(u_t - u_n),$$

where $\alpha > 0$ is a positive parameter, u_t is the unemployment rate. The natural rate of unemployment is given by $u_n = \frac{m+z}{\alpha}$ where m is a markup and z is a catch-all variable that stands for all other variables that may affect the natural rate of unemployment.

The labor force $L_t = L$ is constant and the production function is given by

$$Y_t = N_t$$

where the total employment N_t is given by $N_t = (1-u_t)\cdot L_t$. The variables T, G, m, z, L are all exogenous.

Part (a) Equilibrium of the IS-LM-PC Model [30 Points]

1. [10 Points] Rewrite the Phillips curve relationship in terms of $(Y_t - Y_n)$ instead of $(u_t - u_n)$. Calculate the potential output Y_n in terms of exogenous variables/parameters. (Note: $u_n$is NOT an exogenous variable)

   Solution: Note that if $Y_t = Y_n$, then $u_t = u_n$. Using the production function, express $Y_t - Y_n$ as:

$$\begin{aligned}
Y_t - Y_n &= N_t - N_n \\
&= (1-u_t)L - (1-u_n)L \\
&= -(u_t - u_n)L
\end{aligned}$$

hence

$$u_t - u_n = -\frac{1}{L}(Y_t - Y_n)$$

Then, substituting inside (PC) we get:

$$\pi_t - \pi_t^e = \frac{\alpha}{L}(Y_t - Y_n)$$

The potential output is

$$Y_n = L \cdot (1 - u_n) = L \cdot (1 - \frac{m+z}{\alpha}).$$

2. [10 Points] Solve for the equilibrium output Y_t as a function of r_t and other exogenous variables/parameters.

   Solution: by direct computation

$$Y_t = c_0 + c_1(Y_t - T) + b_0 + b_1 Y_t - b_2 r_t + G$$
$$Y_t = \frac{1}{1 - c_1 - b_1}(c_0 - c_1 T + b_0 - b_2 r_t + G).$$

   This gives the equilibrium output when the central bank chooses r_t.

3. [10 Points] Using the Phillips curve, solve for the natural rate of interest r_n associated with the medium-run equilibrium, i.e., the interest rate that achieves $Y_t - Y_n = \pi_t - \pi_t^e = 0$. The answer should be in terms of exogenous variables/parameters.

   Solution: by direct computation

$$Y_n = \frac{1}{[1 - c_1 - b_1]}(c_0 - c_1 T + b_0 - b_2 r_n + G)$$
$$r_n = -Y_n\left(\frac{1 - c_1 - b_1}{b_2}\right) + \left(\frac{c_0 - c_1 T + b_0 + G}{b_2}\right).$$
$$= -L \cdot (1 - \frac{m+z}{\alpha})\left(\frac{1 - c_1 - b_1}{b_2}\right) + \left(\frac{c_0 - c_1 T + b_0 + G}{b_2}\right).$$

Part (b) The Effects of an Increase in the Price of Oil [20 Points]

Remember that the medium run equilibrium is characterized by four conditions:

- Output Y equals its potential level Y_n
- Unemployment rate u equals the natural rate of unemployment u_n
- The real interest rate is equal to the natural rate of interest r_n
- The expected rate of inflation π^e is equal to the actual rate of inflation π.

Using the results in Part (a), answer the following two questions. (provide either a qualitative answer based on economic intuition OR an algebraic derivation using the results in Part (a))

1. [10 Points] Assume $\pi_t^e = \overline{\pi}$, i.e., that inflation expectations are *anchored.* Suppose the economy is initially at the medium-run equilibrium and, suddenly, OPEC succesfully increases oil prices, which can be equivalently seen as an increase in the markup: m increases to m' ($m' > m$). What happens to output and inflation in the new short-run equilibrium? What will the central bank do with the real interest rate to reach the new medium-run equilibrium? What will happen with inflation and output along the transition?

   Solution: From the result of 2.(a).2, we see that, after the change in m, Y_t stays constant. From the formula of u_n, we see that u_n increases. Thus, the potential output $Y_n = (1 - u_n) L$ decreases. The Phillps curve then implies $\pi_t - \overline{\pi} = \frac{\alpha}{L}(Y_t - Y_n) > 0$, so inflation exceeds $\overline{\pi}$. So in the new short-run equilibrium we have the same level of output and a higher level of inflation.

   From the result of 2.(a).3, we see that the natural rate of interest increases, i.e., the central bank will need to increase the real interest rate to restore the medium-run equilibrium.

   From the result of 2.(a).2, we see that this increase in the interest rate will decrease Y_t along the transition. From the PC this implies that inflation will decrease to $\overline{\pi}$ again.

2. [10 Points] Suppose we had $\pi_t^e = \pi_{t-1}$ instead. How does this affect your answers to the previous subpoint? What happens with the level of inflation in the new medium-run equilibrium? Explain the intuition.

   Solution: Output is still constant in the new short-run equilibrium after the change in m, but in this case we have $\pi_t - \pi_{t-1} = \frac{\alpha}{L}(Y_t - Y_n) > 0$, so inflation keeps increasing in the new short-run equilibrium.

   From the result of 2.(a).3, we see that the central bank needs to increase the interest rate to restore the medium-run equilibrium.

   From the result of 2.(a).2,, we see that this increase in the interest rate will decrease Y_t along the transition. From the PC this implies that the "increments" in inflation will shrink to zero along the transition.

   In this case, inflation will keep on increasing during the transition and in the new medium-run equilibrium we will end up with a higher level of inflation. When expectations are anchored as in the previous subpoint, however, inflation decreases along the transition so the new medium-run equilibrium has the same level of inflation.

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