14.02 - Principles of Macroeconomics

Problem Set 5

Spring 2023

Question 1: Shocks in the IS-LM-PC Model [30 Points]

Consider a closed economy with aggregate demand given by

$$Y_t = c_0 + c_1(Y_t - T) + b_0 + b_1 Y_t - b_2(r_t + x_t) + G,$$

and the Phillps curve,

$$\pi_t - \pi_t^e = -\alpha\left(u_t - u_n\right)$$

where t is a time index, $\alpha, c_0, c_1, b_0, b_1, b_2 > 0$ and $c_1 + b_1 < 1$. The natural rate of unemployment is given by $u_n = \frac{m+z}{\alpha}$ where m is a markup and z is a catch-all variable that stands for all other variables that may affect the natural rate of unemployment. The labor force $L_t = L$ is constant and the production function is given by

$$Y_t = N_t.$$

Total employment N_t is given by $N_t = (1 - u_t) \cdot L_t$. The variables $T, G, m, z, L, x, \pi_t^e$ are all exogenous.

1. [10 Points] Rewrite the Phillips curve relationship in terms of $(Y_t - Y_n)$ instead of $(u_t - u_n)$, where Y_n is the natural rate of output or potential output. Calculate the natural rate of output Y_n in terms of the exogenous variables and fixed parameters. By how much does Y_n change if the labor force L increased by one unit? Provide an answer in terms of the exogenous variables and fixed parameters.

2. [10 Points] Solve for equilibrium output Y_t as a function of r_t, x_t and other exogenous variables/parameters (Hint: this is an IS-LM review).

3. [10 Points] Suppose now that Y_n is constant. Using the Phillips curve you derived in question 1 and the expression for Y_t derived in question 2, derive the change in inflation in response to a unit-increase in x. Express your answer in terms of exogenous variables and fixed parameters. Will inflation increase or decrease? Explain.

Question 2: Comparing Standards of Living Using PPP Numbers [20 Points]

Assume that the average consumer in the U.S. and India buy the quantities and pay the prices indicated in the following table:

	Food		Transportation Services	
	Price	Quantity	Price	Quantity
U.S.	$1	1,000	$2	2,000
India	50 rupees	500	150 rupees	300

1. [5 Points] For both countries, compute consumption per capita in its domestic currency.

2. [5 Points] Suppose that 1 dollar is worth 80 rupees. Compute India's consumption per capita in dollars.

3. [5 Points] Using the purchasing power parity method and U.S. prices, compute India's consumption per capita in dollars.

4. [5 Points] In questions 2 and 3 we asked you to compute consumption per capita using two different methods. Under each method, how much lower is the standard of living in India than in the United States? Why is there a difference between the two methods?

Question 3: The Aggregate Production Function [20 Points]

Consider the production function

$$Y = K^{0.4}N^{0.6}$$

where K denotes the amount of capital being used in production and N denotes the amount of labor.

1. [6 Points] Compute output when $K = 100$ and $N = 50$.

2. [8 Points] If both capital and labor double, what happens to output? Is this production function characterized by constant returns to scale? Explain.

3. [6 Points] Write this production function as a relation between output per worker and capital per worker.

Question 4: Solow Model [30 points]

Suppose that the economy's production function is given by

$$Y = K^{\alpha} N^{1-\alpha}$$

where Y is output, K is capital and N is labor. Assume that $\alpha = 1/3$, and that δ, the depreciation rate of capital, equals 0.10.

1. [5 Points] Is this production function characterized by constant returns to scale? Are there decreasing returns to capital? Are there decreasing returns to labor?

2. [5 Points] Transform the production function into a relationship between output per worker and capital per worker. For a given saving rate, s, and depreciation rate, δ, derive expressions for capital per worker and output per worker in the steady state.

3. [5 Points] Solve for the steady-state level of consumption per worker when $s = 0.1$ and $s = 0.2$. Explain the intuition behind the change as you increase s. Can you find the saving rate that maximizes the steady-state levels of consumption per worker?

4. [10 Points] Suppose the economy is in steady state with $s = 0.1$ in period $t - 1$. Assume that at the start of period t, the depreciation rate increases permanently from 0.1 to 0.2 while the savings rate remains $s = 0.1$.

   (a) What will be the new steady-state level of capital per worker and output per worker?

   (b) Compute the path of capital per worker and output per worker over the first five periods (t, t+1, t+2, t+3, t+4) after the change in the depreciation rate

5. [5 Points] Now assume that instead of δ rising in period t, δ remains fixed at 0.1 but s rises to 0.2 starting in period t. Illustrate this change graphically. Using capital per worker as the X-axis, plot output per worker, depreciation per worker, as well as investment per worker before and after the change. Indicate the previous and the new steady-state level of capital per worker. Indicate the dynamics of capital per worker starting from the initial steady-state. What is the new steady-state level of capital per worker and output per worker.