14.02 - Principles of Macroeconomics

Problem Set 7

SOLUTIONS

Spring 2023

Question 1: Monetary and Fiscal Policy with Flexible Exchange Rates [40 Points]

An open economy with flexible exchange rates is characterized by the following equations

$$\begin{aligned} C_t &= c_0 + c_1 (Y_t - T_t) \\ I_t &= b_0 + b_1 Y_t - b_2 i_t \\ X_t &= x_1 Y_t^* - x_2 \varepsilon_t \\ \frac{IM_t}{\varepsilon_t} &= x_0 Y_t \\ \varepsilon_t &= E_t \frac{P_t}{P_t^*} \end{aligned}$$

where C_t is consumption, I_t is investment, X_t is exports, T_t is taxes, IM_t is imports, Y_t is output, and Y_t^* is foreign output. Assume that Y_t^* and i_t^*, foreign output and the foreign interest rate, are both exogenous and can be taken as fixed. All the parameters, c_0, c_1, b_1, b_2, x_0, x_1, and x_2 are positive with $x_0 < c_1 + b_1 < 1$.

Equilibrium in the goods market is characterized by

$$Y_t = C_t + I_t - \frac{IM_t}{\varepsilon_t} + X_t + G_t$$

where G_t is government spending.

1. [7 Points] Using the uncovered interest parity (UIP) condition, express E_t in terms of i_t, i_t^*, and E_{t+1}^e. Recall that E_t is the value of one unit of home currency in units of foreign currency. Further, express the real exchange rate ε_t as a function of domestic and foreign interest rates, domestic and foreign prices, and the future expected exchange rate. If financial markets expect that the exchange rate in one year will be unity, $E_{t+1}^e = 1$, and if home prices are $P_t = 1$, and foreign prices are $P_t^* = 2$, derive an equation expressing the real exchange rate as a function of the current domestic and foreign interest rates. Do not use a linear approximation $\frac{1+z}{1+w} \approx 1 + z - w$ for this question!
   Solution:

   From the UIP

$$1 + i_t = \frac{E_t}{E_{t+1}^e} (1 + i_t^*) \implies E_t = E_{t+1}^e \frac{1 + i_t}{1 + i_t^*} = \frac{1 + i_t}{1 + i_t^*}$$

   The real exchange rate is

$$\varepsilon_t = E_t \frac{P_t}{P_t^*} = \frac{1}{2} \frac{1 + i_t}{1 + i_t^*}$$

2. [5 Points] Continuing with the assumptions in part 1, derive the IS curve: find Y_t as a function of i_t, T_t, and the exogenous variables and parameters.
Solution:

Equilibrium in the goods market implies

$$Y_t = c_0 + c_1 (Y_t - T_t) + b_0 + b_1 Y_t - b_2 i_t + x_1 Y_t^* - x_2 \varepsilon_t - x_0 Y_t + G_t$$

Rearranging terms

$$Y_t = \frac{1}{1 - c_1 - b_1 + x_0} (c_0 - c_1 T_t + b_0 - b_2 i_t + x_1 Y_t^* - x_2 \varepsilon_t + G_t)$$

Using our expression for the real exchange rate from 2.1 we have

$$Y_t = \frac{1}{1 - c_1 - b_1 + x_0} \left(c_0 - c_1 T_t + b_0 - b_2 i_t + x_1 Y_t^* - \frac{1}{2} x_2 \frac{1 + i_t}{1 + i_t^*} + G_t \right)$$

3. [5 Points] Now express net exports NX_t as a function of the domestic interest rate i_t and the set of exogenous variables and parameters. How does an increase in foreign output affect net exports (find $\frac{dNX_t}{dY_t^*}$). Show graphically how does an increase in foreign output shifts the IS curve.
Solution:

Net exports are given by

$$NX_t = X_t - \frac{IM_t}{\varepsilon_t} = x_1 Y_t^* - x_2 \varepsilon_t - x_0 Y_t = x_1 Y_t^* - \frac{1}{2} x_2 \frac{1 + i_t}{1 + i_t^*} - x_0 Y_t$$

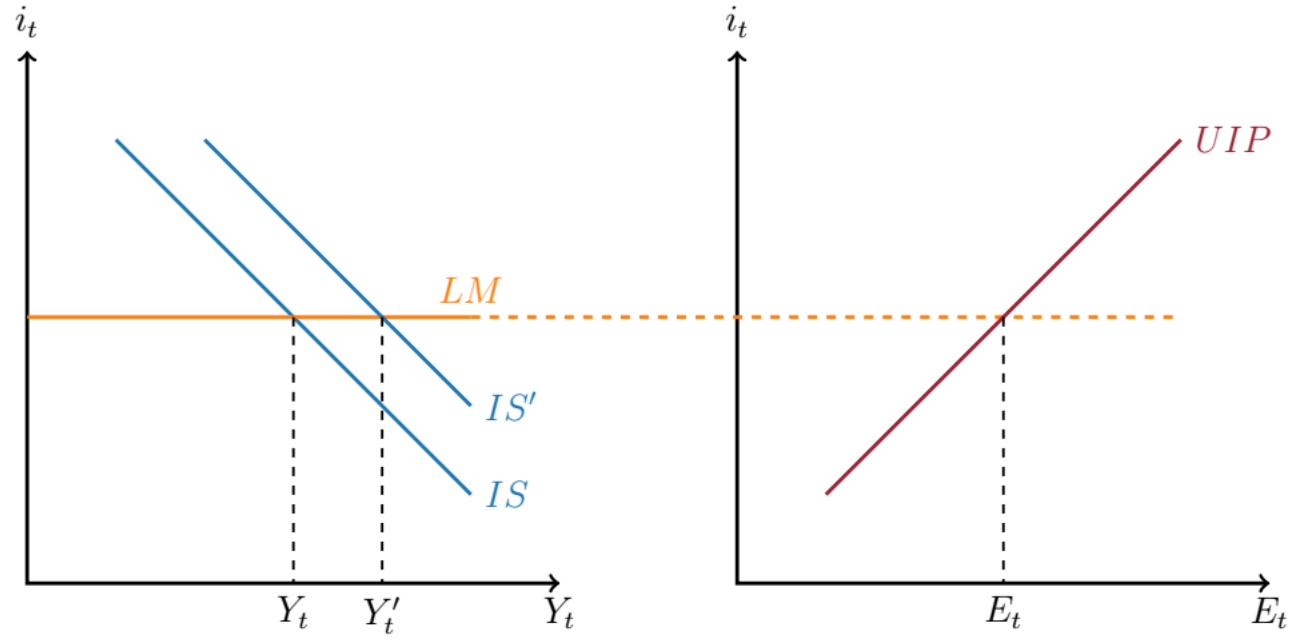
Replacing with our expression for Y_t

$$NX_t = \frac{1 - c_1 - b_1}{1 - c_1 - b_1 + x_0} \left(x_1 Y_t^* - x_2 \frac{1}{2} \frac{1 + i_t}{1 + i_t^*} \right) - \frac{x_0}{1 - c_1 - b_1 + x_0} (c_0 - c_1 T_t + b_0 - b_2 i_t + G_t).$$

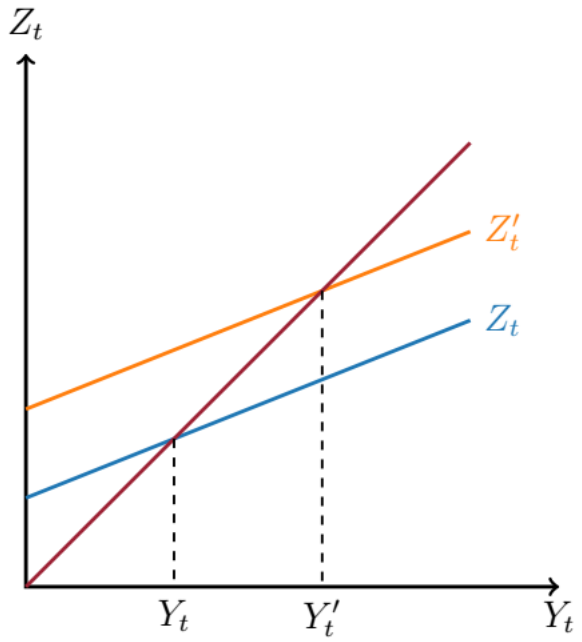
So we have

$$\frac{dNX_t}{dY_t^*} = \frac{1 - c_1 - b_1}{1 - c_1 - b_1 + x_0} x_1 > 0.$$

Graphically, an increase in foreign output shifts the IS curve to the right:



Alternatively, we can use the Keynesian Cross to see the same effect: An increase in foreign output shifts the aggregate demand curve upward, increasing the equilibrium output.



4. [10 Points] Suppose now that the foreign interest rate i_t^* declines to $i_t^{*'} = i_t^* - \Delta i^*$. Assume that the domestic interest rate is unchanged. Assuming that the domestic and foreign price levels remain unchanged, how do the nominal and real exchange rates change? What is the change of net exports $NX_t' - NX_t$? Show graphically how the decline in the foreign interest rate i_t^* changes the IS-LM-UIP diagram, illustrating what happens to output as a result of this interest rate change.
Solution:

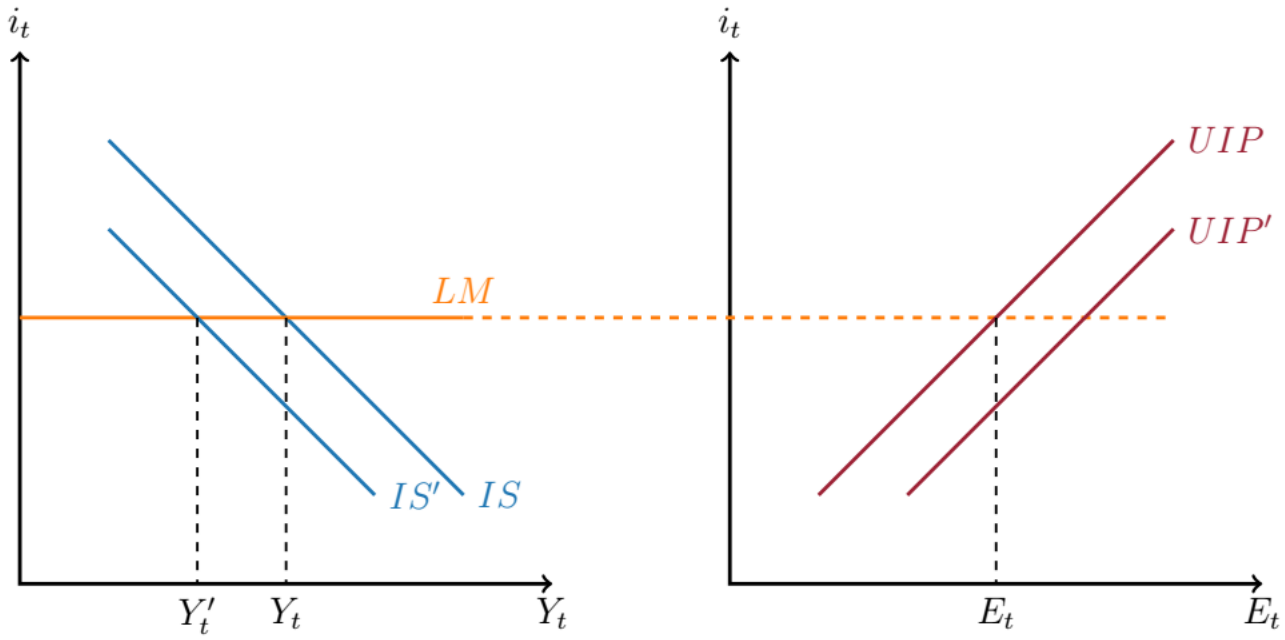
   By the UIP condition, the decrease in the foreign interest rate results in increases in the nominal and real exchange rates. The change in net exports is given by

$$\begin{aligned} NX_t' - NX_t &= -x_2 \tfrac{1-c_1-b_1}{1-c_1-b_1+x_0} \tfrac{1}{2} (1+i_t) \left(\tfrac{1}{1+i_t^*-\Delta i^*} - \tfrac{1}{1+i_t^*} \right) \\ &= -x_2 \tfrac{1-c_1-b_1}{1-c_1-b_1+x_0} \tfrac{1}{2} (1+i_t) \tfrac{\Delta i^*}{(1+i_t^*-\Delta i^*)(1+i_t^*)} < 0 \end{aligned}$$

   or (both answers are fine)

$$NX_t' - NX_t \approx \frac{\partial NX_t}{\partial i_t^*}(-\Delta i^*) = -x_2 \frac{1-c_1-b_1}{1-c_1-b_1+x_0} \frac{1}{2} \frac{1+i_t}{(1+i_t^*)^2} \Delta i^* < 0.$$

   Graphically, a decrease in the foreign interest rate shifts the IS curve to the left and shifts the UIP curve downward. So we have lower output $Y_t' < Y_t$ and the higher nominal exchange rate $E_t' > E_t$.



5. [5 Points] Continuing with the analysis in part 4, find the change in domestic output $Y_t' - Y_t$ associated with a foreign interest rate decline of Δi^*.
Solution:

   The change in domestic output is

$$\begin{aligned} Y_t' - Y_t &= -x_2 \tfrac{1}{1-c_1-b_1+x_0} \tfrac{1}{2} (1+i_t) \left(\tfrac{1}{1+i_t^*-\Delta i^*} - \tfrac{1}{1+i_t^*} \right) \\ &= -x_2 \tfrac{1}{1-c_1-b_1+x_0} \tfrac{1}{2} (1+i_t) \tfrac{\Delta i^*}{(1+i_t^*-\Delta i^*)(1+i_t^*)} < 0 \end{aligned}$$

   or (both answers are fine)

$$Y_t' - Y_t \approx \frac{\partial Y_t}{\partial i_t^*}(-\Delta i^*) = -x_2 \frac{1}{1-c_1-b_1+x_0} \frac{1}{2} \frac{1+i_t}{(1+i_t^*)^2} \Delta i^* < 0.$$

6. [8 Points] Now suppose that the domestic central bank reduces the domestic interest rate by the same amount as the drop in the foreign interest rate. How does this change the uncovered interest parity condition? Re-draw the IS-LM-UIP diagram to include a drop in the domestic interest rate as well as the foreign interest rate.
Solution:

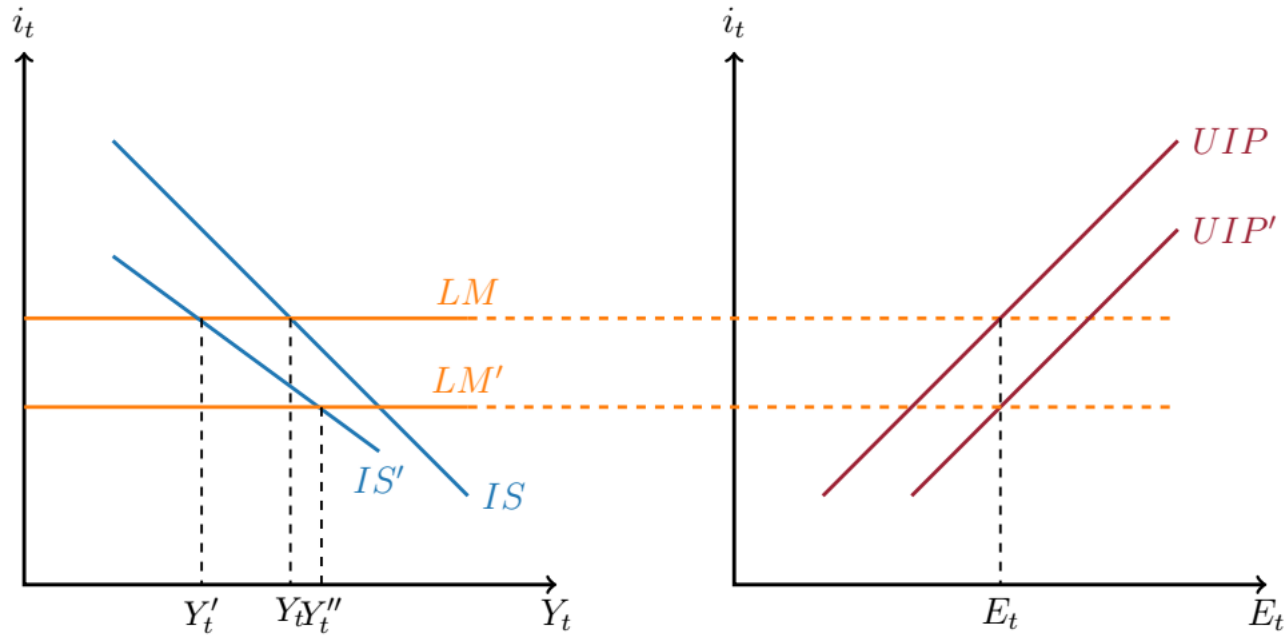
   If the domestic central bank reduces the domestic interest rate by the same amount as the drop in the foreign interest rate, the UIP condition implies that E_t approximately remains the same. (Technically, the new nominal exchange rate is given by

$$E_t' = \frac{1+i_t-\Delta i^*}{1+i_t^*-\Delta i^*}$$

   which can be lower or higher than the original nominal exchange rate, $E_t = \frac{1+i_t}{1+i_t^*}$, depending on whether $i_t \geq i_t^*$ or $i_t < i_t^*$. In the first case case $E_t' \geq E_t$whereas in the second case $E_t' < E_t$. If i_t, i_t^*, and Δi^* are small, however, we approximately have $E_t' \approx E_t$ because

$$E_t' = \frac{1+i_t-\Delta i^*}{1+i_t^*-\Delta i^*} \approx 1+i_t-\Delta i^*-(i_t^*-\Delta i^*) = 1+i_t-i_t^* \approx \frac{1+i_t}{1+i_t^*} = E_t$$

where the approximations use $\frac{1+x}{1+y} \approx 1 + x - y$ if x and y are small) In the diagram below we assume that $E'_t = E_t$.



The decrease in the foreign interest rate shifts the UIP curve to the right and the IS curve to the left as in the previous question. The change in the domestic interest rate is captured by the downward shift of the LM curve. Note that output Y''_t should be higher than the initial level, Y_t, because the exchange rate is fixed, and the decrease in the domestic interest rate increases investment.

Question 2 - Fiscal Policy and Fixed Exchange Rates [30 Points]

Consider an open economy similar to the one in problem 2, with one key difference: it maintains fixed exchange rates with all major trading partners. The economy is again described by these equations:

$$\begin{aligned} C_t &= c_0 + c_1 (Y_t - T_t) \\ I_t &= b_0 + b_1 Y_t - b_2 i_t \\ X_t &= x_1 Y^*_t - x_2 \varepsilon_t \\ \frac{IM_t}{\varepsilon_t} &= x_0 Y_t \\ \varepsilon_t &= E_t \frac{P_t}{P^*_t} \end{aligned}$$

For this economy, the nominal exchange rate is fixed at $E_t = E^*$. Equilibrium in the goods market is characterized by

$$Y_t = C_t + I_t - \frac{IM_t}{\varepsilon_t} + X_t + G_t$$

1. [10 Points] Rewrite the equations for IM_t and X_t so that they depend on E^*. Assume that at the exchange rate E^*, the domestic economy is in short-run equilibrium.
   Solution:

The real exchange rate is

$$\varepsilon_t = E^* \frac{P_t}{P_t^*}$$

Then exports and imports are

$$X_t = x_1 Y_t^* - x_2 \varepsilon_t = x_1 Y_t^* - x_2 E^* \frac{P_t}{P_t^*}$$

$$IM_t = x_0 Y_t \varepsilon_t = x_0 Y_t E^* \frac{P_t}{P_t^*}$$

2. [10 Points] Assume that there is only one other country, and that its central bank cuts its interest rate by an amount Δi^*. If the domestic central bank would like to restore the economy to equilibrium, how much would it need to adjust its interest rates? Draw the IS-LM-UIP diagram and explain which, if any, of the IS, LM, and UIP curves shift when the foreign central bank cuts its interest rate.
Solution:

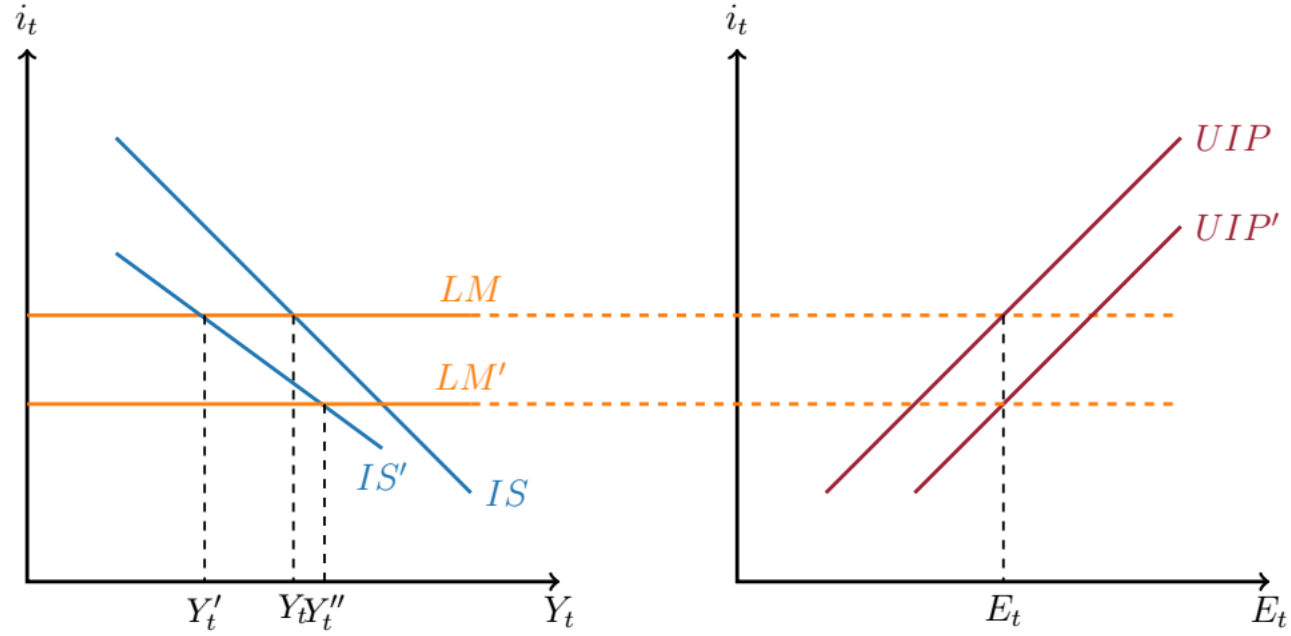
From the UIP, we need to have

$$\frac{1+i_t}{1+i_t^*} = \frac{1+i_t-\Delta i_t}{1+i_t^*-\Delta i_t^*}$$

which gives (when i_t and i_t^* are small)

$$\Delta i_t = 1 + i_t - (1 + i_t^* - \Delta i_t^*)\frac{1+i_t}{1+i_t^*} = \frac{1+i_t}{1+i_t^*}\Delta i_t^* \approx \Delta i_t^*.$$

The decrease in the foreign interest rate shifts the UIP curve to the right and the IS curve to the left. The change in the domestic interest rate is captured by the downward shift of the LM curve. Note that output Y_t'' should be higher than the initial level, Y_t, because the exchange rate is fixed, and the decrease in the domestic interest rate increases investment.



3. [10 Points] If an economic shock, such as a disruption in energy supplies, leads to a reduction in output in the foreign country to $Y_t^{*'} = Y^* - \Delta Y_t^*$, what is the impact on domestic output Y_t? Why does foreign output matter for domestic output? If political leaders decided to change government spending by an amount that would just offset the impact of the drop in foreign output, how much would they need to change G_t, and in what direction?
Solution:

   The reduction in output in the foreign country leads to a reduction in exports and therefore a reduction in output. The change in output is

$$Y_t' - Y_t = -\frac{1}{1 - c_1 - b_1 + x_0} x_1 \Delta Y_t^*$$

   The effect of a chagne in government spending ΔG is

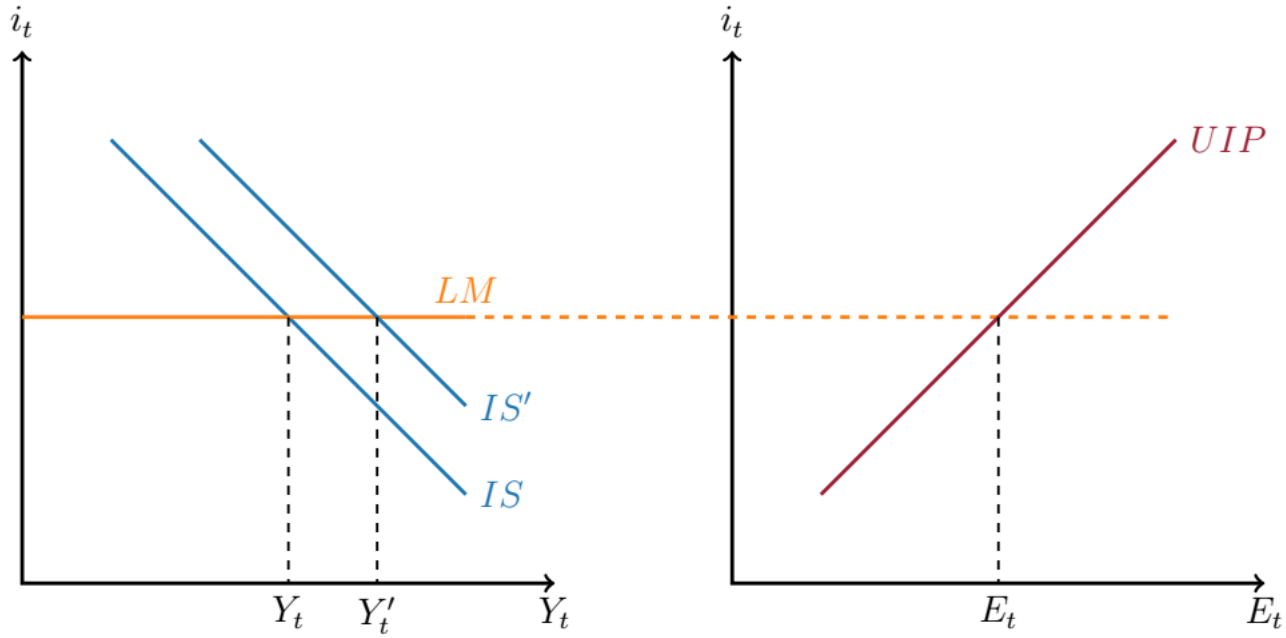
$$Y_t' - Y_t = \frac{1}{1 - c_1 - b_1 + x_0} \Delta G$$

   So the needed change in government spending to offset the decrease in output due to the reduction in foreign output is $\Delta G = x_1 \Delta Y_t^* > 0$.

Question 3 - The Exchange Rate as a Policy Tool [30 Points]

A flexible exchange rate combined with a willingness to change the domestic interest rate can increase the effectiveness of monetary policy in an open economy. Consider an economy that suffers from in business confidence (which tends to reduce investment).

1. [10 Points] In an IS-LM-UIP diagram, show the short-run effect of the fall in business confidence on output and the exchange rate when the central bank leaves the interest rate unchanged. How does the composition of output change?
Solution:



   In response to a decrease in business confidence investment decreases. The second round effects are a further decrease in investment, consumption and imports.

2. [10 Points] The central bank is willing to cut the interest rate to restore the level of output to its original value. How does this change the composition of output?
   Solution:

   From the UIP a decrease in the interest rate decreases the exchange rate. From the decrease in the interest rate, investment increasers. From the decerase in the exchange rate exports increase. The increase in outuput has positive second round effects on consumption, investment and importas.

3. [5 Points] If the exchange rate was fixed and the central bank could not change the interest rate what policy options are left for the central bank?
   Solution:

   The central bank cannot do anything to restore the level of output to its original value.

4. [5 Points] Central banks generally favor flexible exchange rates. Explain why.
   Solution:

   Central banks generally favor flexible exchange rates because it allows them to use the interest rate to react to shocks.

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