Question 1: The Equity Premium and The Value of Stocks [30 Points]

In the case of stocks, the risk premium is called the equity premium. Equilibrium requires that the expected rate of return from holding stocks for one year be the same as the rate of return of one-year bonds plus the equity premium.

\[
\frac{D_{t+1}^e + Q_{t+1}^e}{Q_t} = 1 + i_{t+1} + x
\]

where \( x \) denotes the equity premium, \( D \) are dividends, and \( Q \) is the price of the stock. Rewrite this equation as

\[
Q_t = \frac{D_{t+1}^e}{1 + i_{t+1} + x} + \frac{Q_{t+1}}{1 + i_{t+1} + x}
\]

That is, arbitrage implies that the price of the stock today must be equal to the present value of the expected dividend plus the present value of the expected stock price next year.

1. [5 Points] Explain why in this equation it is important that the stock is ex-dividend. That is, it has just paid its dividend and expects to pay its next dividend in one year.

2. [5 Points] Using this equation, explain the contribution of each component to today’s stock price.

3. [5 Points] If the risk premium is larger, all else equal, what happens to the price of the stock today?

4. [5 Points] If the one-period interest rate increases, what happens to the price of the stock today? If the expected value of the stock at the beginning of period \( t + 1 \) increases, what happens to the value of the stock today?

5. [10 Points] Assume all policy rates, current and expected into the future had been 2%. Suppose the Fed decides to tighten monetary policy and increase the short-term policy rate \( i_{t+1} \) from 2% to 3%. What happens to the stock prices if the change in \( i_{t+1} \) is expected to be temporary, that is, last for only one period? Assume expected dividends do not change. What happens to stock prices today if the change in \( i_{t+1} \) is expected to be permanent? Assume also that expected dividends do not change.
Question 2: Interest Rate Sensitivity of Investment [30 points]

Consider a start-up firm with ten possible investments projects, each of which will develop a new app: \( j = 1, 2, \ldots, 10 \). Each app \( j \) is projected to cost $50 million to develop, and generates a different stream of earnings. At the end of the first year, project \( j \) generates $2 million in earnings annually, and after then it perpetually generates \( j \) million in earnings annually. All cash flows are expressed in constant dollars.

1. [6 Points] Calculate the present discounted value of the earnings stream associated with app \( j \) when the real interest rate is \( r \).

2. [6 points] If the real interest rate is 5% (\( r = .05 \)), how many apps will the start-up firm develop?

3. [6 Points] What if the real interest rate is 10% (\( r = .1 \))? 

4. [6 Points] What is the highest value of \( r \) for which all the projects will be developed? What will the firm’s investment spending be in this case?

5. [6 Points] What does your answer suggest about the slope of the IS curve? Is it downward sloping or upward sloping?

Question 3- Individual Saving and Aggregate Capital Accumulation [40 Points]

1. Suppose that every consumer is born with zero financial wealth and lives for three periods: youth, middle age and old age. Consumers work in the first two periods and retire in the last one. Their income is $5 in the first period, $25 in in the second, and $0 in the last one. Inflation and expected inflation are equal to zero, and so is the real interest rate.

   a. [5 Points] What is the present discounted value of labor income at the beginning of the period of life? What is the highest sustainable level of consumption such that consumption is equal in all three periods?

   b. [5 Points] For each age group, what is the amount of saving that allows consumers to maintain the constant level of consumption you found in part 1?

   c. [5 Points] Suppose there are \( n \) people born each period. What is total saving in this economy? Explain.

   d. [5 Points] What is total financial wealth in the economy?

2. Suppose now that borrowing restrictions do not allow young consumers to borrow. If we call the sum of income and total financial wealth “cash on hand” then the borrowing restriction means that consumers cannot consume more than their cash on hand. In each age group, consumers compute their total wealth and then determine their desired level of consumption as the highest level that allows their consumption to be equal in all three periods. However, if at any time, desired consumption exceeds cash on hand, then consumers are constrained to consume exactly their cash on hand.

   a. [5 Points] Calculate consumption in each period of life. Compare this answer to your answer to part 1.a and explain any differences.
(b) [5 Points] Calculate total saving for the economy. Compare this answer to your answer in part 1.c and explain any differences.

(c) [5 Points] Derive total financial wealth for the economy. Compare this answer to your answer in part 1.d and explain any differences.

(d) [5 Points] Consider the following statement: “Financial liberalization may be good for individual consumers, but it is bad for overall capital accumulation”. Discuss