14.02 - Principles of Macroeconomics

Problem Set 8
SOLUTIONS

Spring 2023

Question 1: The Equity Premium and The Value of Stocks [30 Points]

In the case of stocks, the risk premium is called the equity premium. Equilibrium requires that the expected rate of return from holding stocks for one year be the same as the rate of return of one-year bonds plus the equity premium

$$\frac{D^e_{t+1} + Q^e_{t+1}}{Q_t} = 1 + i_{1t} + x$$

where x denotes the equity premium, D are dividends, and Q is the price of the stock. Rewrite this equation as

$$Q_t = \frac{D^e_{t+1}}{1 + i_{1t} + x} + \frac{Q_{t+1}}{1 + i_{1t} + x}$$

That is, arbitrage implies that the price of the stock today must be equal to the present value of the expected dividend plus the present value of the expected stock price next year.

1. [5 Points] Explain why in this equation it is important that the stock is ex-dividend. That is, it has just paid its dividend and expects to pay its next dividend in one year.
   Solution

   It is important because we are comparing the future payoff of investing a dollar in stockk or in bond. The cost today is given by the price of the stock. If the stock was not ex-dividend then the cost should be the price minus the dividend at that period.

2. [5 Points] Using this equation, explain the contribution of each component to today's stock price.
   Solution

   By no arbitrage, the price of holding a bond is equal to the discounted payoffs. The payoff has two components. First, the dividend we expect to receive. Second, we can sell the stock in t+1.

3. [5 Points] If the risk premium is larger, all else equal, what happens to the price of the stock today?
   Solution

   The price of the stock decreases.

4. **[5 Points]** If the one-period interest rate increases, what happens to the price of the stock today? If the expected value of the stock at the beggining of period $t + 1$ increases, what happens to the value of the stock today?

   Solution

   If the one-period interest rate increases the price of the stock today decreases. If the expected value of the stock in the next period increases then the current price increases. .

5. **[10 Points]** Assume all policy rates, current and expected into the future had been 2%. Suppose the Fed decides to tighten monetary policy and increase the short-term policy rate i_{1t} from 2% to 3%. What happens to the stock prices if the change in i_{1i} is expected to be temporary, that is, last for only one period? Assume expected dividends do not change. What happens to stock prices today if the change in i_{1t} is expected to be permanent? Assume also that expected dividends do not change. *Solution*

   If the change is permanent prices will change more because the expected future price will also decrease.

Question 2: Interest Rate Sensitivity of Investment [30 points]

Consider a start-up firm with ten possible investments projects, each of which will develop a new app: $j = 1, 2, ..., 10$. Each app j is projected to cost $50 million to develop, and generates a different stream of earnings. At the end of the first year, project j generates $2 million in earnings annually, and after then it perpetually generates $$j$ million in earnings annually. All cash flows are expressed in constant dollars.

1. **[6 Points]** Calculate the present discounted value of the earnings stream associated with app j when the real interest rate is r.

   Solution:

   The present discounted value of the earnings stream is given by:

   $$PDV = \frac{\$2M}{1+r} + \frac{\$jM}{(1+r)^2} + \frac{\$jM}{(1+r)^3} + \frac{\$jM}{(1+r)^4}...$$
   $$= \frac{2}{1+r} + \frac{j}{r(1+r)} \text{ million dollars}$$

   note: M stands for million

2. **[6 points]** If the real interest rate is 5% ($r = .05$), how many apps will the start-up firm develop?

   Solution:

   If $r = 0.05$, the present discounted value is given by:

   $$\frac{2}{1.05} + \frac{j}{0.05(1.05)} \approx 1.90 + 19.05j$$

   which exceeds 50 when $j \geq 3$. Thus, the start-up firm will develop 8 apps.

3. **[6 Points]** What if the real interest rate is $10\%(r = .1)$?

Solution:

If $r = 0.1$, the present discounted value is given by:

$$\frac{2}{1.1} + \frac{j}{0.1(1.1)} \approx 1.82 + 9.09j$$

This exceeds 50 when $j \geq 6$. Thus, the start-up firm will develop 5 apps.

4. **[6 Points]** What is the highest value of r for which all the projects will be developed? What will the firm's investment spending be in this case?

Solution:

App $j = 1$ is the least profitable. Thus, we need to have:

$$\frac{2}{1 + r} + \frac{1}{r(1 + r)} \geq 50,$$

which requires:

$$50i^2 + 48r - 1 \leq 0$$

or

$$r \leq 2.04\%$$

With 10 projects all costing $50M, the total investment spending is $500M

5. **[6 Points]** What does your answer suggest about the slope of the IS curve? Is it downward sloping or upward sloping?

Solution:

As the interest rate rises, the stream of earnings becomes less valuable in present value terms. So, fewer investments would become worth their upfront costs. This finding suggests that the IS curve is downward sloping. As the interest rate increases, investment decreases, lowering output both directly and indirectly (by the Keynesian cross logic).

Question 3- Individual Saving and Aggregate Capital Accumulation [40 Points]

1. Suppose that every consumer is born with zero financial wealth and lives for three periods: youth, middle age and old age. Consumers work in the first two periods and retire in the last one. Their income is $5 in the first period, $25 in in the second, and $0 in the last one. Inflation and expected inflation are equal to zero, and so is the real interest rate.

(a) [5 Points] What is the present discounted value of labor income at the beggining of the period of life? What is the highest sustainable level of consumption such that consumption is equal in all three periods?

Solution:

The present discounted value of labor income at the beggining of the period of life is 30. The highest sustainable level of consumption such that consumption is equal in all three periods is 10.

(b) [5 Points] For each age group, what is the amount of saving that allows consumers to maintain the constant level of consumption you found in part 1.?

Solution:

The youth save -5, the middle age save 10 and the old save zero.

(c) [5 Points] Suppose there are n people born each period. What is total saving in this economy? Explain.

Solution:

Total saving in this economy is $(-5 + 10 + 0) * n = 5n$

(d) [5 Points] What is total financial wealth in the economy?

Solution:

Total financial wealth is $(0 - 5 + 10) * n = 5n$

2. Suppose now that borrowing restrictions do not allow young consumers to borrow. If we call the sum of income and total financial wealth "cash on hand" then the borrowing restriction means that consumers cannot consume more than their cash on hand. In each age group, consumers compute their total wealth and then determine their desired level of consumption as the highest level that allows their consumption to be equal in all three periods. However, if at any time, desired consumption exceeds cash on hand, then consumers are constrained to consume exactly their cash on hand.

(a) [5 Points] Calculate consumption in each period of life. Compare this answer to your answer to part 1.a and explain any differences.

Solution:

The youth consume 5, middle aged consume 12.5 and the old consume 12.5. The youth would like to save more but cannot, so they consume their cash on hand. In turn, the middle aged and the old consume more because they have higher cash on hand.

(b) [5 Points] Calculate total saving for the economy. Compare this answer to your answer in part 1.c and explain any differences.

Solution:

Total saving in this economy is $(0 + 12.5 + 0) * n = 12.5$ Saving is larger because the young cannot incur in debt.

(c) [5 Points] Derive total financial wealth for the economy. Compare this answer to your answer in part 1.d and explain any differences. *Solution:*

Total financial wealth is $(0 + 0 + 12.5) * n = 12/5$

(d) [5 Points] Consider the following statement: "Financial liberalization may be good for individual consumers, but it is bad for overall capital accumulation". Discuss

Solution:

Financial liberalization can allow consumers to incur in debt if they anticipate higher income in the future, but at the cost of reducing saving and therefore investment.

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