14.02 – Principles of Macroeconomics

Quiz 2

SOLUTIONS

Spring 2023

1 True or False [20 Points]

1. [2.5 Points] A change in the workers' bargaining power does not change the natural rate of unemployment.

   Solution: FALSE.

   If workers have more bargaining power then the natural rate of unemployment increases

2. [2.5 Points] If people assume that inflation will be the same as last year's inflation, the Phillips curve relation will be a relation between the change in the inflation rate and the unemployment gap.

   Solution: TRUE.

   The Phillips Curve is given by
   $$\pi_t = \pi^e + \alpha \left(u_t - u^n\right)$$
   If $\pi^e = \pi_{t-1}$ then
   $$\pi_t - \pi_{t-1} = \alpha \left(u_t - u^n\right)$$

3. [2.5 Points] If unemployment is constantly below its natural level and people expect that inflation will be the same as last year's inflation the inflation rate explodes over time (that is, it increases over time).

   Solution: TRUE.

   The Phillips curve is
   $$\pi_t = \pi_{t-1} + \alpha \left(u_t - u^n\right)$$
   If $u_t - u^n > 0$ for all t then $\pi_t > \pi_{t-1}$ and inflation explodes.

4. [2.5 Points] A negative financial shock in the IS-LM model shifts the IS curve to the left. **Solution:** TRUE.

   A negative financial shock in the IS-LM model reduces investment for a given interest rate. This represents a shift of the IS curve to the left.

Consider a modified version of the Solow Model with technological progress that includes human capital.

5. [2.5 Points] Adding human capital to the Solow model doesn't change steady state growth.

   Solution: TRUE.

   Steady state growth is given by the rate of technological progress, regardless of the presence of human capital.

6. [2.5 Points] Human capital doesn't influence the level of output per worker in steady state.

   Solution: FALSE.

   Even though human capital does not change steady state growth, it affects the level of output per worker in steady state.

7. [2.5 Points] If we assume no difference in productivity across countries, the model predicts higher inequality in output per worker than we observe in the data.

   Solution: FALSE.

   If we assume no difference in productivity across countries, the model predicts lower inequality than we observe in the data.

8. [2.5 Points] On average, poor countries grow faster than rich countries.

   Solution: TRUE.

   On average poor countries are further from their steady state than rich countries, so they tend to grow faster.

2 The IS-LM-PC Model [40 Points]

Consider an economy whose aggregate demand side is characterized by

$$Z_t = C\left(Y_t, T\right) + I\left(Y_t, r_t\right) + G$$

where

$$\begin{aligned} C\left(Y_t, T\right) =& c_0 + c_1\left(Y_t - T\right) \\ I\left(Y_t, r_t\right) =& b_0 + b_1 Y_t - r_t \end{aligned}$$

Taxes T and government spending G are exogenously given, and r_t is set by the central bank. The parameters c_0, c_1, b_0, and b_1 are all strictly positive with $c_1 < 1$, $b_1 < 1$, and $c_1 + b_1 < 1$. The supply side of the economy is characterized by the production function

$$Y_t = N_t$$

where the total employment N_t is given by $N_t = \left(1 - u_t\right) L_t$ and the labor force $L_t = \bar{L}$ is fixed. From the wage-setting relation and the price-setting relation, we can derive the Phillips curve

$$\pi_t = \pi_t^e - \alpha\left(u_t - u^n\right)$$

where the natural level of output is given by $Y^n = \left(1 - u^n\right)\bar{L}$ and the natural rate of unemployment is given by

$$u^n = \frac{m + z}{\alpha}$$

α is a parameter, z is an exogenous catch-all variable that stands for all other factors that can potentially affect the labor market equilibrium, and m is the exogenous markup. Assume that inflation expectations are anchored $\pi_t^e = \bar{\pi}$

1. [5 Points] Solve for equilibrium output as a function of r_t, G, and other exogenous variables/parameters.

   Solution: In equilibrium

$$\begin{aligned}Y_t =& Z_t \\ =& c_0 + c_1 \left(Y_t - T_t\right) + G + b_0 + b_1 Y_t - r_t \\ =& \frac{1}{1 - c_1 - b_1}\left(c_0 + b_0 + G - c_1 T - r_t\right)\end{aligned}$$

2. [5 Points] Rewrite the Phillips curve relationship in terms of $(Y_t - Y^n)$ instead of $(u_t - u^n)$, where Y^n is the natural output. Calculate the natural output Y^n in terms of the exogenous variables and fixed parameters.

   Solution: Use

$$Y_t - Y^n = \left(u^n - u_t\right)\bar{L}$$

   Replace in the Phillips Curve

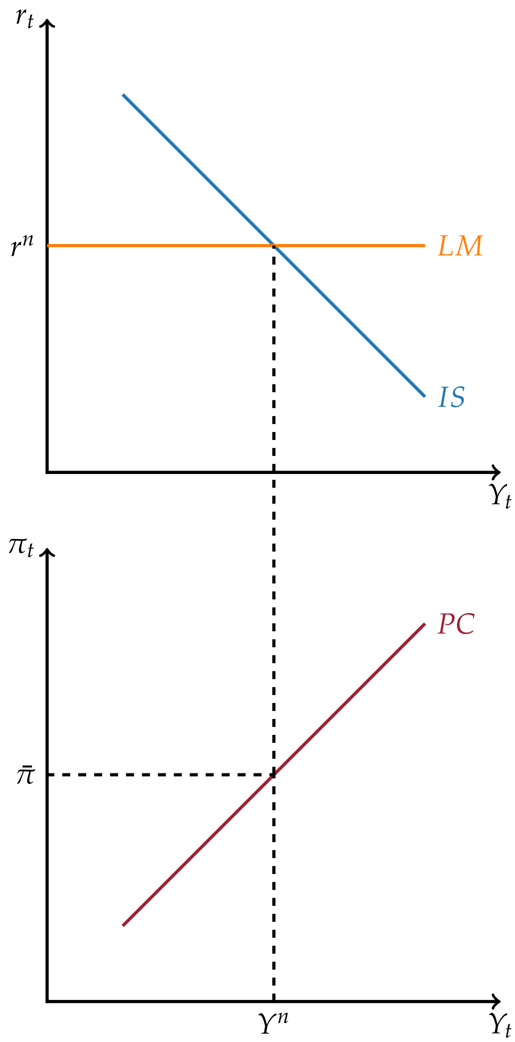
$$\pi_t = \pi_t^e + \frac{\alpha}{\bar{L}}\left(Y_t - Y^n\right)$$

   Finally, $Y^n = \left(1 - u^n\right)\bar{L}$. Replacing with $u^n = \frac{m+z}{\alpha}$

$$Y^n = \left(1 - \frac{m+z}{\alpha}\right)\bar{L}$$

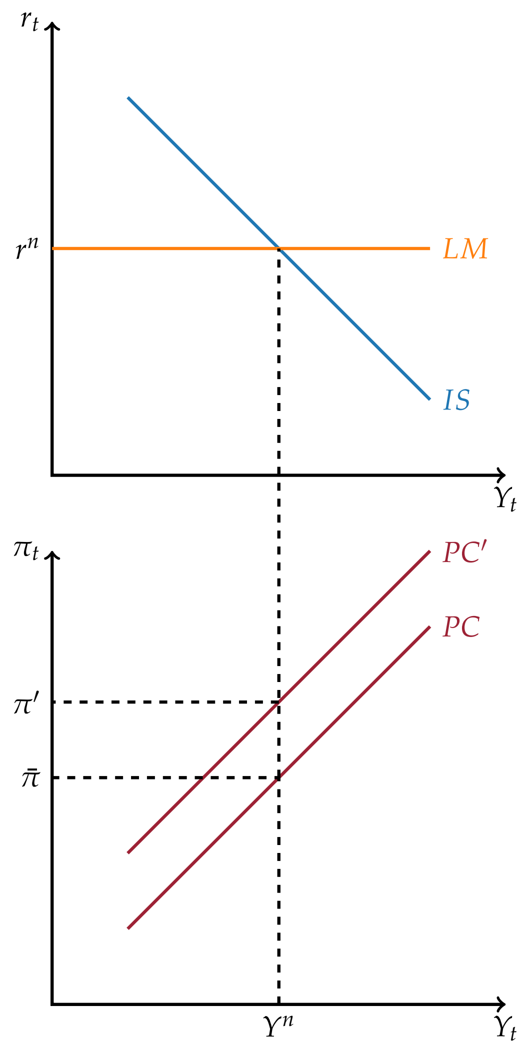
3. [5 Points] Draw the IS-LM and PC curves corresponding to a medium-run equilibrium in two diagrams, and label the natural level of output Y^n in each. Be sure to label your axes.

Solution:



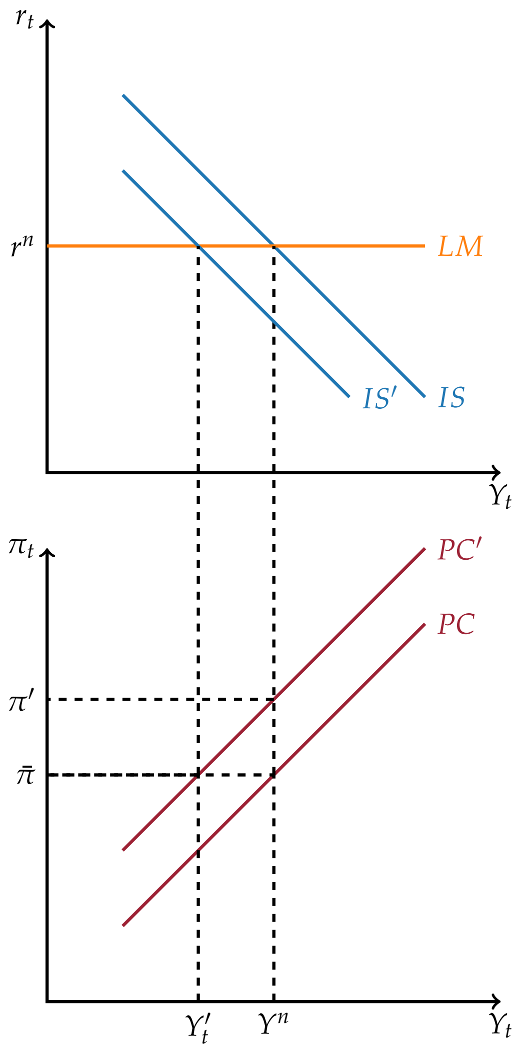
4. [5 Points] Suppose that this economy is initially at a medium-run equilibrium. During the pandemic, many people developed a strong preference for leisure and staying at home. This translated into a lower supply of work for a given wage, which can be viewed as an increase in the catch-all variable z to $z' = z + \Delta z$. Using the IS-LM-PC diagram, show graphically what happens to output Y and inflation π in the short run.

Solution:



5. [10 Points] Assume that after the increase in z, the fiscal authorities decide to adjust government spending to bring the economy to the new medium-run equilibrium. Calculate the required change in government spending. Using the IS-LM-PC diagram, show graphically how this policy change moves the economy to the new medium-run equilibrium. If, instead, policymakers decide to change taxes to move the economy to the new medium-run equilibrium, calculate the required change in taxes. Is the change in taxes needed to reach the new medium-run equilibrium smaller than, larger than or equal to (in absolute value) the required change in government spending that you just calculated? Why?

Solution:



The required change in output is

$$\Delta Y_t = -\Delta u^n \bar{L} = -\frac{\Delta z}{\alpha}\bar{L}$$

Using the IS equation:

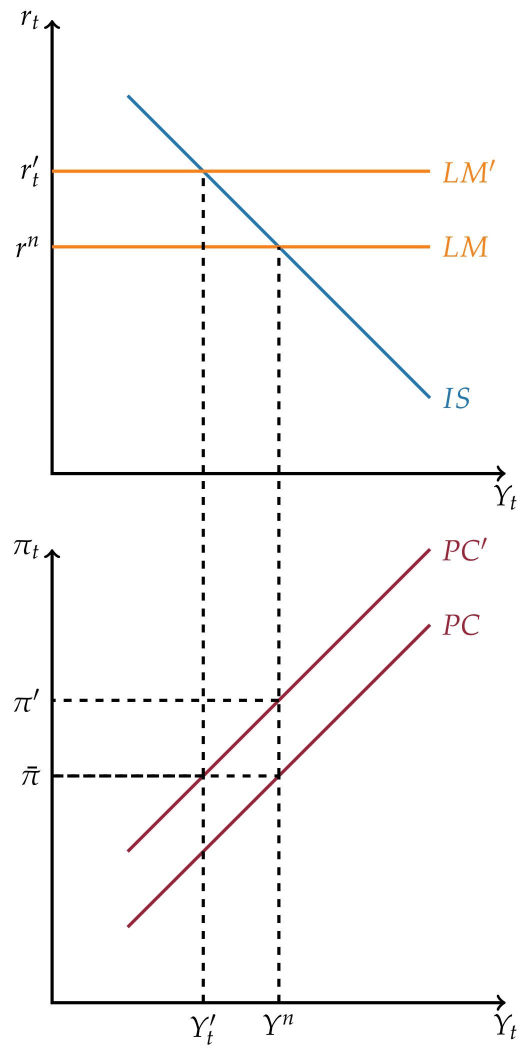
$$\Delta Y_t = \frac{1}{1 - c_1 - b_1}\left(-c_1 \Delta T + \Delta G\right)$$

Then the required change in government spending/taxes is

$$\Delta G = -\left(1 - c_1 - b_1\right)\bar{L}\frac{\Delta z}{\alpha} \qquad \Delta T = \frac{1 - c_1 - b_1}{c_1}\bar{L}\frac{\Delta z}{\alpha}$$

6. [10 Points] Now assume that fiscal policy does not respond to the increase in z, but that monetary policy does. Calculate the required change in the real interest rate that would bring the economy to the new medium-run equilibrium. Using the IS-LM-PC diagram, show graphically how this policy change moves the economy to the new medium-run equilibrium.

Solution:



The required change in output is

$$\Delta Y_t = -\Delta u^n \bar{L} = -\frac{\Delta z}{\alpha}\bar{L}$$

Using the IS equation:

$$\Delta Y_t = -\frac{1}{1 - c_1 - b_1}\Delta r_t$$

Then the required change in government spending is

$$\Delta r_t = (1 - c_1 - b_1)\,\bar{L}\frac{\Delta z}{\alpha}$$

3 The Solow Growth Model [40 Points]

Consider the Solow growth model:

$$Y_t = K_t^{\alpha} (A_t N_t)^{1-\alpha}$$

where the subscript $t = 0, 1, 2, \ldots$ is the time index; Y_t denotes output, K_t denotes the amount of capital being used in production, N_t denotes the amount of labor and A_t is a measure of technology. Assume that capital depreciates at rate $\delta \in (0, 1)$. Savings rate is constant and satisfies $s \in (0, 1)$. Finally, assume that N_t and A_t grow at a constant rate n and g, respectively. That is,

$$N_{t+1} = (1+n) N_t$$

$$A_{t+1} = (1+g) A_t$$

The relationship between investment and capital accumulation is given by

$$K_{t+1} = I_t + (1-\delta) K_t$$

where $I_t = sY_t$ denotes investment. Assume that $g = 0$ and at $t = 0$ $A_t = 1$

1. [5 Points] Show that this production function has constant returns to scale and decreasing marginal returns to capital and labor.

   Solution: First we show the function has constant returns to scale

   $$F(\lambda K_t, \lambda A_t N_t) = (\lambda K_t)^{\alpha} (\lambda A_t N_t)^{1-\alpha} = \lambda K_t^{\alpha} (A_t N_t)^{1-\alpha} = \lambda F(K_t, A_t N_t)$$

   Now we show that the function has decreasing marginal returns

   $$F_K = \alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha} \implies F_{KK} = \alpha (1-\alpha) K_t^{\alpha-2} (A_t N_t)^{1-\alpha} < 0$$

   $$F_N = (1-\alpha)\alpha K_t^{\alpha} A_t^{1-\alpha} N_t^{\alpha} \implies F_{NN} = -\alpha (1-\alpha) K_t^{\alpha} A^{1-\alpha} N_t)^{-\alpha-1} < 0$$

2. [5 Points] Let k_{t+1} be the capital per worker. Derive the law of motion for k_t. That is, express k_{t+1} as a function of k_t, s, n, δ, and α. (Hint: divide by N_t on both sides, and use $k_{t+1}n \approx k_t n$)

   Solution:

   $$\begin{aligned} K_{t+1} =& sY_t + (1-\delta) K_t \\ \frac{K_{t+1}}{N_t} =& sy_t + (1-\delta) k_t \\ \frac{K_{t+1}}{N_{t+1}}(1+n) =& sy_t + (1-\delta) k_t \\ k_{t+1}(1+n) =& sy_t + (1-\delta) k_t \\ k_{t+1} \approx& sy_t + (1-\delta-n) k_t \\ k_{t+1} \approx& sk_t^{\alpha} + (1-\delta-n) k_t \end{aligned}$$

3. [5 Points] Find the steady state level of k_t, denote it k^*. How does it depend on δ? Provide an economic intuition.

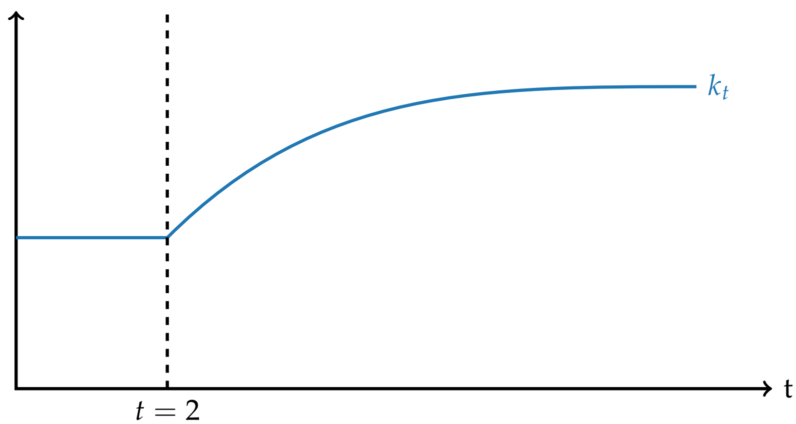
Solution: In a steady state

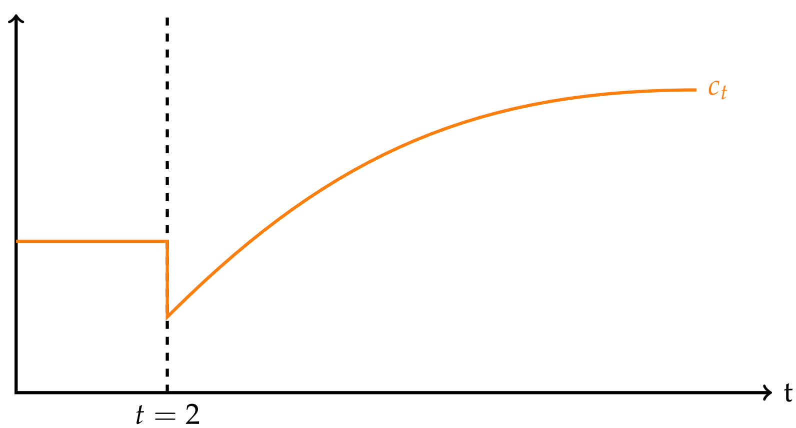
$$k^* = s\,(k^*)^{\alpha} + (1-\delta-n)\,k^* \implies k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

It depends negatively on δ: if capital depreciates at a faster rate then the level at which it will be constant is lower.

4. [10 Points] Assume that at $t = 0$ the economy is in the steady state, i.e., $k_0 = k^*$ and that $s < \alpha$. However, at $t = 2$ the savings rate changes permanently to $s = \alpha$. Describe the transition path for capital per worker and consumption per worker. You don't need to do any math here, you can show the result graphically by sketching the time evolution of each variable.

Solution:





Now consider the case of $g > 0$.

5. [5 Points] Find the steady state level of the capital stock per effective worker and output per worker.
Solution: The law of motion for capital per effective worker is

$$\hat{k}_{t+1} = s\left(\hat{k}\right)^{\alpha} + (1 - \delta - n - g)\,\hat{k}$$

Then in a steady state $\hat{k}^{*} = \left(\frac{s}{\delta+n+g}\right)^{\frac{1}{1-\alpha}}$. Output per worker is given by $y_t = A_t \hat{y}$. Then in a steady state $\hat{y} = \left(\hat{k}^{*}\right)^{\alpha}$. If $A_t = 1$ then $y_t = (1+g)^{t}\left(\hat{k}^{*}\right)^{\alpha}$

6. [10 Points] Assume that at $t = 0$ the economy is at the steady state with saving rate $s < \alpha$. However, at $t = 2$ the savings rate decreases permanently. Plot the evolution of output per effective worker, consumption per effective worker, total output and consumption per worker.

Solution: