14.02 – Principles of Macroeconomics
Spring 2023
Quiz 3– SOLUTIONS

Thursday, May 11, 2023
7:30-9:00 PM

NAME: _____ MIT ID: _____

QUIZ RULES – READ CAREFULLY

- Problem 1 contains 4 true/false questions. For Problem 1, you are not required to explain your answers. Explanations will not be taken into consideration when grading. Problems 2 and 3 contain short questions about Monetary Policy with Expectations and Discounting and Problem 4 is about the Mundell-Fleming model. This quiz will be graded on a 100-point scale, according to the breakdown specified below.
- Please write your answer below each question in the space provided, and do not remove the staple.
- Please **write your name on every page**.
- Read the questions carefully and keep your answers short and to the point. Points will be deducted for ambiguous or illegible answers, while partial credit will be assigned in case of a mistake carried over from earlier to later parts of the same problem.
- You will be asked to draw graphs. Please make sure to always label axes and curves, add arrows to denote shifts in the relevant curves, and mark the axes in the locations corresponding to equilibrium.
- By signing below, you agree to comply with the MIT honor code and one additional rule specific to this quiz: No discussing the quiz with anyone until 11:59 PM EST on May 12, 2023.
- There is **no Lecture on May 15.**
- Good luck!

DATE: _____ SIGNATURE: _____

1 True-False Questions (12 points)

1. [3 points] The yield to maturity on a bond is defined as the constant annual interest rate that makes the bond price today equal to the present discounted value of future payments of the bond. *Solution: true.*

2. [3 points] Bonds have two types of risk: Default risk and price risk. *Solution: True.*

3. [3 points] A bond that promises to pay \$100 each year for 20 years and \$1k final payment in 20 years, has a maturity of 21 years. *Solution: false.*

4. [3 points] If uncovered interest parity (UIP) holds, then the expected returns of foreign and domestic bonds must be equal. *Solution: true.*

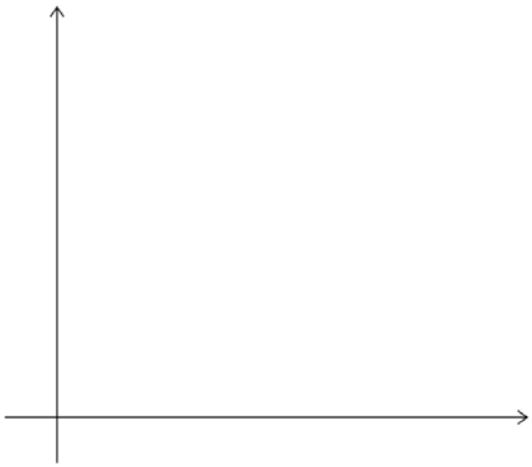
2 Monetary Policy with Expectations (8 points)

Consider the IS-LM model with expectations.

$$IS: \; Y = A(Y, T, r, Y'^{e}, T'^{e}, r'^{e}) + G$$

$$LM: \; r = \bar{r}$$

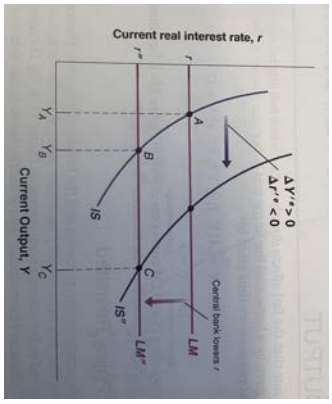
1. [4 points] Suppose the central bank decreases the current interest rate r and all other variables remain constant. Describe what happens to output Y using the IS-LM diagram.



(a) See shift of LM curve in figure below:

2. [4 points] Suppose now that after the decrease in r investors expect interest rates to also remain low in the future. Using the diagram you drew in the previous question, illustrate the effect of a decrease in $r^{'e}$. Does the change in expectations lead to a larger or smaller effect on output?

(a)



3 Interest Rate Sensitivity of Investment (10 points)

Consider two projects $j \in \{1,2\}$. Each project generates a different stream of earnings. At the **end of the first year** $t = 1$, project j generates $x_j > 0$ in earnings, and after that it perpetually generates j in earnings **at the end of each year** $t > 1$. All cash flows are expressed in constant dollars and the interest rate is assumed constant and equal to $r \in (0,1)$. The firm must pick one of the two projects.

1. [3 points] Find an expression for the net present value of each project in terms of j, x_j and r (Hint: since all positive cash flows are realized at the end of a year, all of them need to be discounted).

(a) $DPV = \frac{x_j}{1+r} + j\sum_{n=2}^{\infty}\left(\frac{1}{1+r}\right)^2 = \frac{x_j}{1+r} + j\frac{1}{1-1/(1+r)}(1+r)^{-2} = \frac{x_j}{1+r} + j\frac{1}{r}(1+r)^{-1}$

2. [3 points] Fixing r, for what values of x_1, x_2 is the firm indifferent between both projects?

(a) $\frac{x_1}{1+r} + \frac{1}{r}(1+r)^{-1} = \frac{x_2}{1+r} + \frac{2}{r}(1+r)^{-1} \iff x_1 = x_2 + \frac{1}{r}$

3. [4 points] Suppose r, x_1, x_2 are such that the firm is indifferent between both projects and suppose the interest rate increases. Which project will the firm implement? Justify your answer by computing the effect of the increase in the interest rate on the net present value of each project.

(a) using that $x_1 = x_2 + \frac{1}{r}$ and taking derivatives wrt r, $-\frac{x_1}{(1+r)^2} - \frac{1}{r^2}(1+r)^{-1} - \frac{1}{r}(1+r)^{-2} = -\frac{x_2}{(1+r)^2} - \frac{2}{r^2}(1+r)^{-1} - \frac{2}{r}(1+r)^{-2} \iff -\frac{1}{r}(1+r)^{-2} > -\frac{1}{r}(1+r)^{-2} - \frac{1}{r^2}(1+r)^{-1}$. So the decrease in the NPV of project 1 is lower than for project 2 and sothe firm should pick project 1.

4 Mundell-Fleming model [70 points]

Consider an open economy with demand for domestic goods:

$$Z = C + I + G + X - IM/\varepsilon$$

where government spending G, and taxes T are exogenously given, ε is the real exchange rate, and i denotes the domestic nominal interest rate. We assume

$$\begin{aligned} C &= c_0 + c_1(Y - T) \\ I &= b_0 - b_2 i \\ X &= e_0 \cdot Y^* / \varepsilon \\ IM &= d_0 \cdot Y \cdot \varepsilon \end{aligned}$$

where foreign output Y^* is exogenous, and all parameters are strictly positive with $d_0 \leq c_1 < 1$. The foreign nominal interest rate and the expected exchange rate are equal to i^* and E^e and assumed to be exogenous. Recall that E is the nominal exchange rate (the value of one unit of home currency in terms of foreign currency). Home prices are $P = 1$ and foreign prices are $P^* = 1$. Finally, assume that the UIP condition holds. Assume the country is under a flexible exchange rate regime.

1. [4 points] State and explain the uncovered interest parity condition (UIP).

   (a) The UIP condition
   $$E = \frac{(1+i)}{(1+i^*)} E^e$$
   It states that the expected return of a bond abroad must equal the expected return of a domestic bond.

2. [8 points] Derive an expression for Y as a function of domestic nominal interest rate i, exogenous variables, and parameters.

(a) From the UIP condition

$$E = \frac{(1+i)}{(1+i^*)} E^e$$

Then,

$$\varepsilon = \frac{EP}{P^*} = \frac{(1+i)}{(1+i^*)} E^e \tag{1}$$

To find an expression for Y we substitute out each term

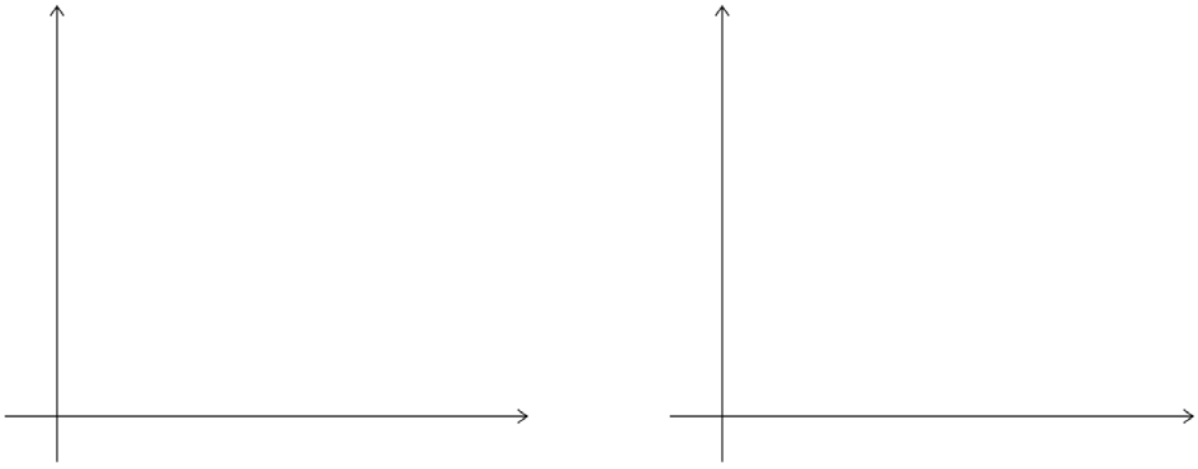
$$Y = c_0 + c_1(Y - T) + b_0 - b_2 i + G + e_0 \cdot Y^* / \varepsilon - d_0 \cdot Y$$

$$Y = \frac{1}{1 - c_1 + d_0} \left[b_0 + c_0 + G - c_1 T - b_2 i + \frac{e_0 \cdot Y^*}{\varepsilon} \right]$$

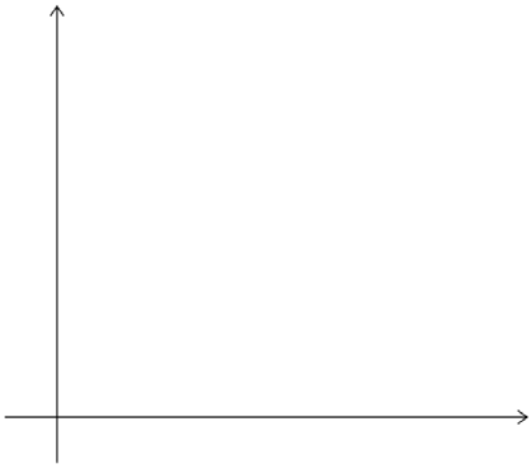
Finally, from (1)

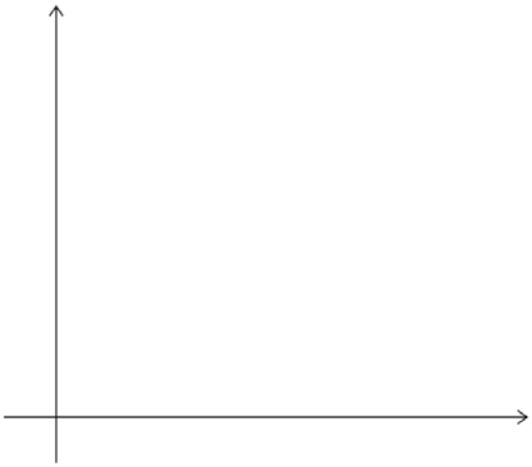
$$Y = \frac{1}{1 - c_1 + d_0} \left[b_0 + c_0 + G - c_1 T - b_2 i + \frac{e_0 \cdot Y^*}{E^e} \frac{(1+i^*)}{(1+i)} \right]$$

3. [10 points] Suppose that the foreign interest rate i^* increases. Derive the resulting change on equilibrium output $\frac{dY}{di^*}$. Show graphically what happens to output Y and the exchange rate E using the IS-LM-UIP diagram.



4. [8 points] Assume that markets now expect a higher future exchange rate, i.e., E^e increases. Describe what happens to output Y and the exchange rate E using the IS-LM-UIP diagram.

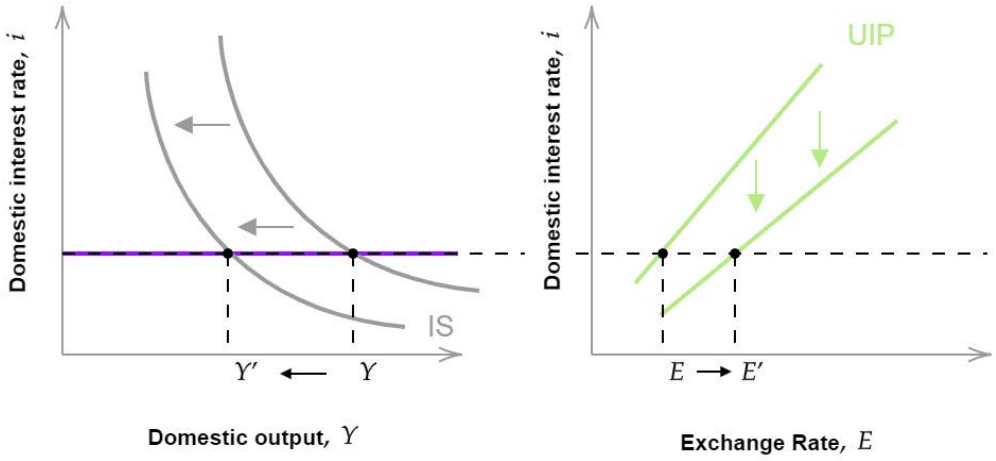




(a) Exchange rate appreciates: $E' > E$ and output decreases: $Y' < Y$

5. [8 points] Let NX be the net exports in this economy. Compute $\frac{dNX}{di^*}$. How will NX change when the foreign nominal interest rate i^* rises? Provide an economic intuition.

Figure 1:



(a) We have

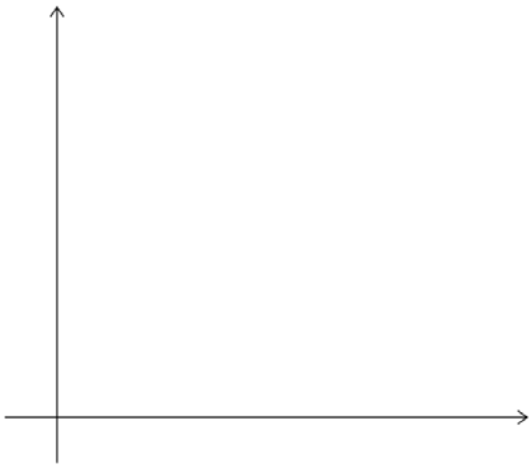
$$NX = \frac{e_0 Y^*}{\varepsilon} - d_0 Y = \frac{e_0 Y^*}{E^e}\frac{(1+i^*)}{(1+i)} - d_0 Y$$

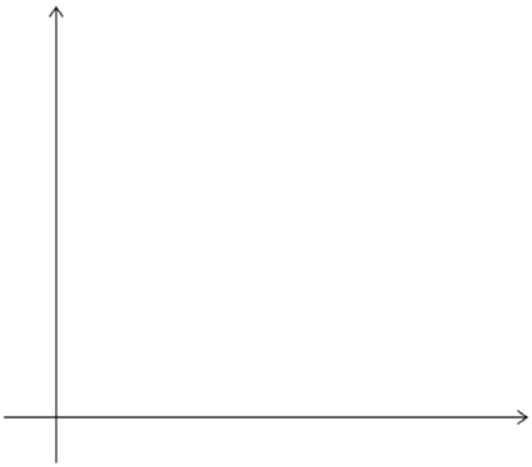
$$\frac{dNX}{di^*} = \frac{e_0 Y^*}{E^e}\frac{1}{(1+i)} - \frac{d_0}{1-c_1+d_0}\frac{e_0 \cdot Y^*}{E^e}\frac{1}{(1+i)}$$

$$= \frac{1-c_1}{1-c_1+d_0}\frac{e_0 \cdot Y^*}{E^e}\frac{1}{(1+i)} > 0$$

Net exports will increase when i^* increases. **Intuition:** A rise in foreign interest rate will depreciate the home currency, which will increase NX (since the assumption $d_0 \leq c_1 < 1$ ensures that the Marshall-Learner condition holds).

6. [10 Points] Now express net exports NX as a function of the domestic interest rate i and the set of exogenous variables and parameters. How does an increase in foreign output affect net exports (find $\frac{dNX}{dY^*}$). Show in a IS-LM-UIP diagram how an increase in foreign output shifts the IS curve.





Solution:

Net exports are given by

$$NX_t = X_t - \frac{IM_t}{\varepsilon_t} = \frac{e_0 Y^*}{\varepsilon} - d_0 Y = \frac{e_0 Y^*}{E^e}\frac{(1+i^*)}{(1+i)} - d_0 Y$$

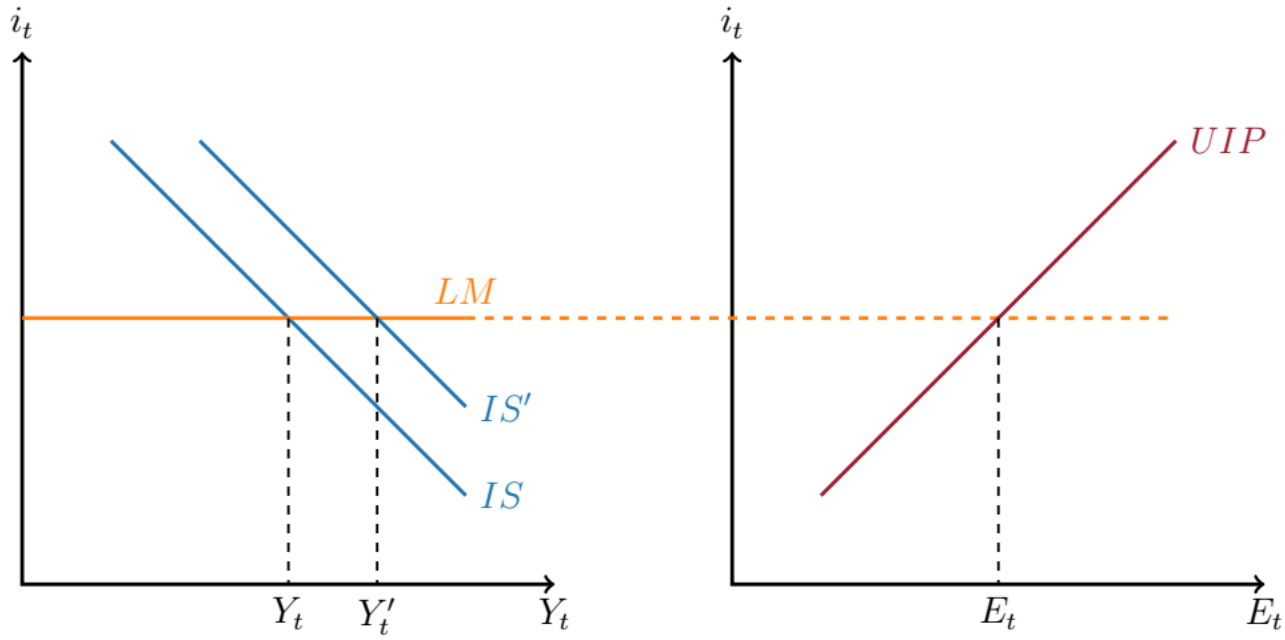
Replacing with our expression for Y_t

$$NX_t = \frac{e_0 Y^*}{E^e}\frac{(1+i^*)}{(1+i)} - d_0 \frac{1}{1-c_1+d_0}\left[b_0 + c_0 + G - c_1 T - b_2 i + \frac{e_0 \cdot Y^*}{E^e}\frac{(1+i^*)}{(1+i)}\right].$$

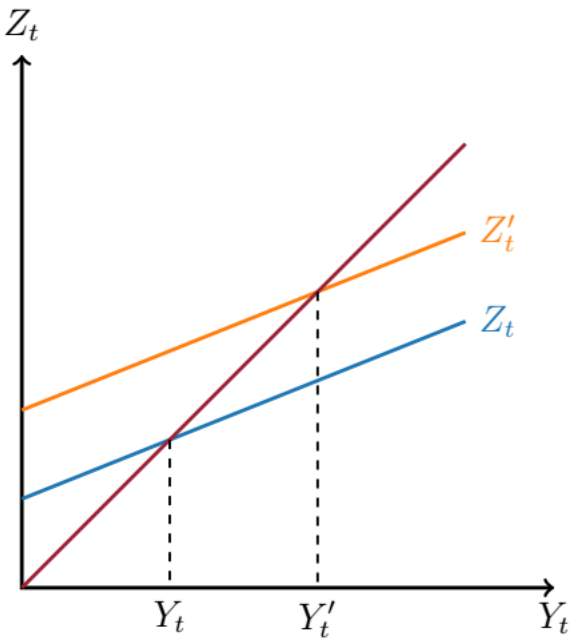
So we have

$$\frac{dNX_t}{dY_t^*} = \frac{e_0}{E^e}\frac{(1+i^*)}{(1+i)}(1 - d_0\frac{1}{1-c_1+d_0}) = \frac{e_0}{E^e}\frac{(1+i^*)}{(1+i)}(\frac{1-c_1}{1-c_1+d_0}) > 0.$$

Graphically, an increase in foreign output shifts the IS curve to the right:



Alternatively, (but not necessary) we can use the Keynesian Cross to see the same effect: An increase in foreign output shifts the aggregate demand curve upward, increasing the equilibrium output.



7. [8 points] Derive the government spending multiplier (i.e., $\frac{dY}{dG}$). If we were to close the economy—so exports X and imports IM were identically equal to zero—what would the multiplier be? Why would the multiplier be different in a closed economy? Provide the economic intuition. Solu-

tion:

(a) We have

$$\frac{\partial Y}{\partial G} = \frac{1}{1 - c_1 + d_0}$$

in an open economy, while

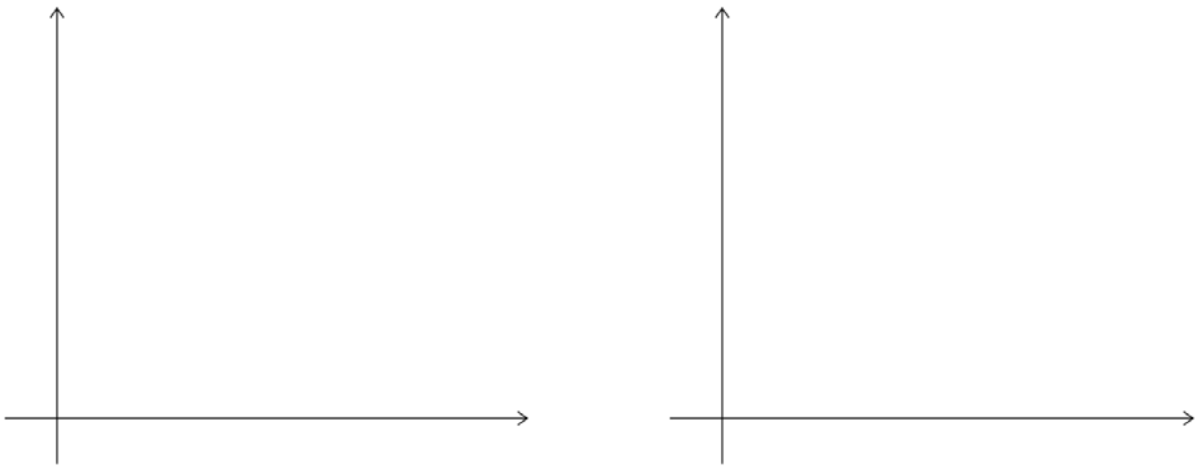
$$\frac{\partial Y}{\partial G} = \frac{1}{1 - c_1}$$

in a closed economy. Thus,

$$\left.\frac{\partial Y}{\partial G}\right|_{\text{open}} < \left.\frac{\partial Y}{\partial G}\right|_{\text{closed}}.$$

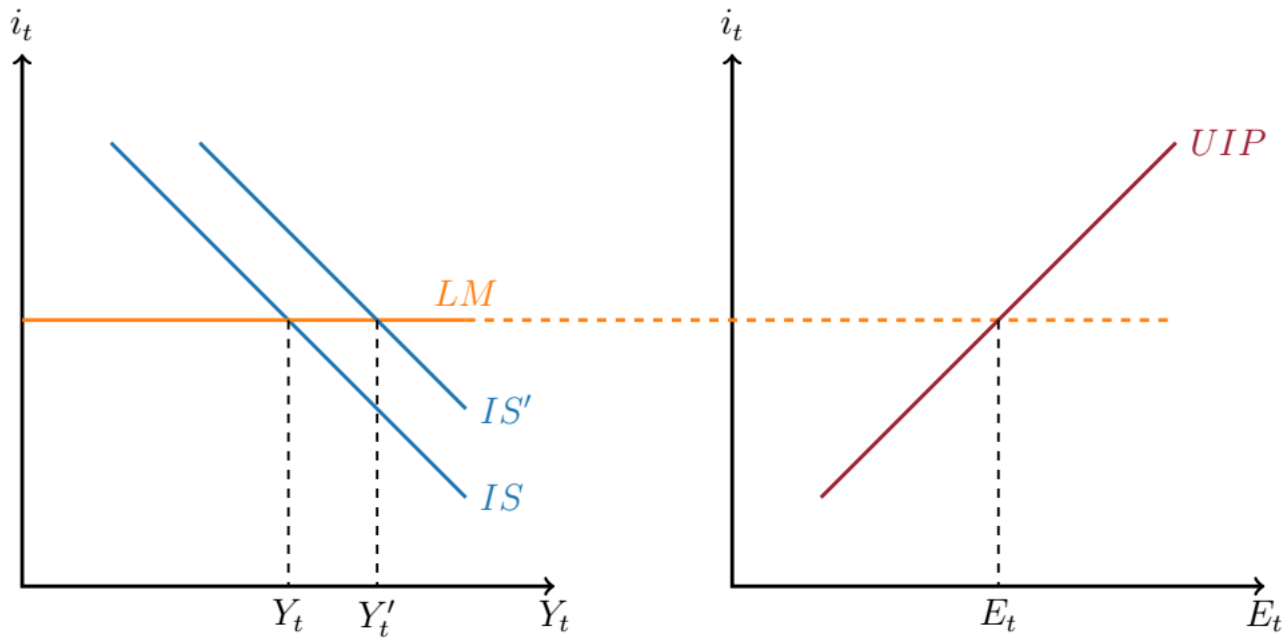
The intuition for this is that government spending will raise output. Agents will use their extra income not only to buy domestic goods (raising output further by a multiplier effect), but also to buy foreign goods. In this model, the income used to buy foreign goods leaves the domestic economy and does not recirculate. Therefore, it dampens the multiplier effect.

8. [7 points] Suppose that people become more pessimistic about the future, which decreases the value of c_0. Using the IS-LM-UIP diagram, graphically show what happens to output Y and the nominal exchange rate E when the domestic central bank leaves the policy interest rate i unchanged. For this question, you do not need to do any algebra.

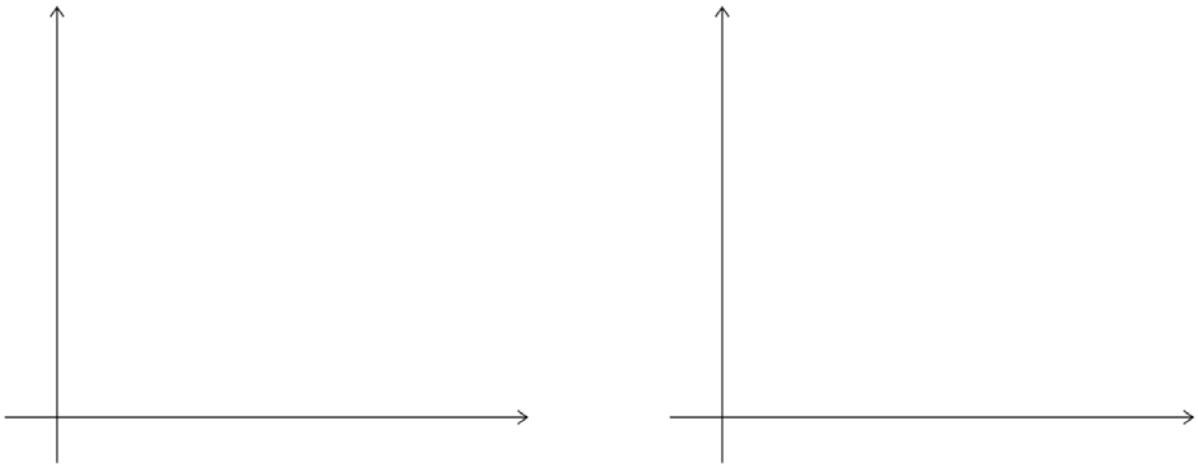


Soluti

(a) See Figure below. The IS curve shifts downward. As a result, Y decreases, while the nominal interest rate i and the nominal exchange rate remain the same.



9. [7 points] Suppose that foreign output Y^* increases, and the domestic central bank raises the interest rate to *partially* offset the change in output induced by the change in foreign output. Using the IS-LM-UIP diagram, graphically show what happens to output Y and the exchange rate E. Again, you do not need to do any algebra.



Soluti

(a) See Figure below. This shock increases domestic demand, but is counteracted by an increase in the interest rate. The exchange rate increases because of the UIP condition. Output increases by assumption (partially offset).

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