

14.03/14.003 Exam 2

Fall 2014

SOLUTIONS

Professor David Autor

Wednesday, 11/12/2014

Instructions

- Please do not open this exam until directed
- There are 80 points on this exam and you have 80 minutes to complete it.
- You may not use a calculator (there is no need for it).
- No other reference material is allowed.
- There are 3 parts to the exam: please use a separate exam book for each part.
- Correct answers without explanation do not receive credit.
- Part I is SHORT ANSWER. Answer 4 problems of 6. (5 points each, 20 points total.) You must explain your answers with one or two sentences or graphs. Correct answers without explanation do not receive credit. *If you answer more than 4 questions, we will only grade the first 4.*
- Part II has 1 long question that you must answer. (30 points)
- Part III has 1 long question that you must answer. (30 points)

Part I

Short Questions: Answer 4 of 6 questions [5 points each, 20 points total]

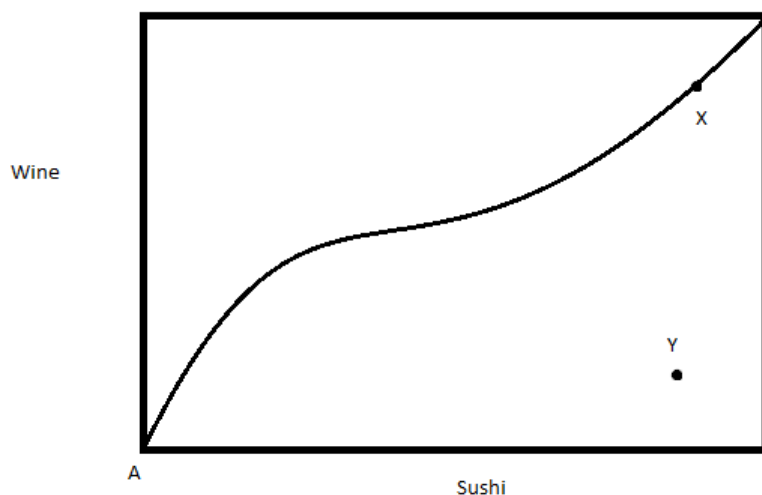
1. [Trade-Feyrer] Feyrer (2009) uses an instrumental variables (IV) strategy to estimate the causal effect of trade on income. The fact that not *only* goods are transported by airplane but *also* people (for example, diplomats, CEOs, and tourists) is a potential problem for his IV strategy. *True, False or Uncertain and why?*

Solution: *True. The important thing here is that we require two assumptions for instrumental variables to be an appropriate strategy: first stage (that the instrument has a measurable effect on the endogenous variable, which is trade in this case) and the exclusion restriction – air vs. sea distance only affects income through trade. Since Feyrer instruments for trade in goods, the fact that air transport is used for other things (e.g. trade in ideas through CEOs traveling to other countries to share knowledge) suggests a possible failure of the exclusion restriction (i.e. that the instrument affects income through mechanisms other than trade in goods). That said, one counter-argument is that the trade in goods measure is merely a proxy for a broader notion of ‘openness’, in which case we get a causal estimate of the effect of ‘openness’ on income (though this notion of openness is less well-defined).*

2. [Coase theorem] The First Welfare Theorem says that under certain conditions, a free market in equilibrium will reach a Pareto efficient outcome. Does the Coase Theorem *increase*, *decrease* or *leave unchanged* the number of necessary conditions required for the First Welfare Theorem to apply? *Explain.*

Solution: *You could answer that it either **leaves unchanged** or **decreases** the number of necessary conditions. In the discussion of the FWT, we noted that Pareto efficiency requires (1) **no externalities**, (2) **no transaction costs**, (3) **no market power**, and (4) **complete information**. The Coase Theorem says that externalities are **not** a problem **if** property rights are complete **and** transactions cost are low. The Coase Theorem therefore allows us to drop the requirement of **no externalities** and replace it with the requirement of **complete property rights**. (And no transaction costs implies low transactions costs, so low transaction costs is not an additional condition.) You could therefore conclude that the Coase Theorem leaves **unchanged** the number of necessary conditions: (1) **complete property rights**, (2) **no transaction costs**, (3) **no market power**, and (4) **complete information**. We will also accept the answer that the Coase Theorem **decreases** the number of conditions since we normally assume that all goods in the Edgeworth box are owned by someone. The logic behind this question is also evident in our in-class example of Ed and Fiona negotiating over beans and smoke. When property rights over air quality are assigned to either Ed or Fiona, Pareto efficiency is achieved, even though there are externalities.*

3. **[Gains from trade]** The figure below depicts an Edgeworth Box exchange economy with two consumers, two commodities (Sushi and Wine), and one contract curve as drawn. Under autarky, the two consumers will trade with each other from any initial endowment point until they reach a competitive equilibrium on the contract curve. *True, False or Uncertain and why:*
- (a) If the initial endowment is at point X , introducing international trade at some fixed price ratio p will make both consumers weakly better off;
- (b) if the initial endowment is at point Y , introducing international trade at some fixed price ratio p will make both consumers weakly better off.



Solution: *True for X , Uncertain for Y . X is both the endowment point and the competitive equilibrium (since it is on the contract curve). If we introduce trade here, each consumer always has the possibility of just consuming their endowment (not trading) which means they will always be at least as well off as in the initial competitive equilibrium. The same is not true for Y since the endowment Y is not on the contract curve, so the competitive equilibrium reached from Y will be different. It is then ambiguous whether trade will make both consumers weakly better off than the initial equilibrium. In particular, if the relative price of Sushi is very high in international markets, it is likely that A (the consumer with a large endowment of Sushi) will be better off after trade, and B will be worse off. (See the Figure in solutions to part 4 of the first Long Question for graphical illustration of this).*

4. **[Partial equilibrium]** A fully competitive labor market (with no monopsony and no minimum wage) is in equilibrium at hourly wage W^* and employment level L^* . The government grants employers a tax subsidy of \$2 per worker per hour. *True, false, uncertain and why:* The equilibrium wage W^* will rise by no more than \$1 per hour.

Solution: *You could answer either false or uncertain. The increase in the wage (that is, the incidence of the tax subsidy to wages) depends on both the elasticity of supply and demand for*

labor. The wage could rise by anywhere from \$0 if labor supply is perfectly elastic to \$2 if labor supply is perfectly inelastic. We can be sure that \$1 is not an upper bound on the increase.

5. **[Jensen-fishermen]** In the Indian state of Kerala prior to the introduction of cell phones, the fish market was clearly Pareto inefficient. On any given day, fish were thrown away on some beaches while on other beaches, no fish were available despite many buyers. *What was the primary source of the market failure? Explain.*

Solution: Incomplete information. Neither buyers nor sellers knew where the other was located, so potentially Pareto-improving trades did not take place. You could also answer **transaction costs**. The high transaction cost of locating customers by motoring from beach to beach prevented Pareto-improving trades from occurring.

6. **[Externalities]** On hot days in Los Angeles, most businesses run their air conditioning at maximum capacity. This creates a congestion externality that can overload the electrical grid, leading to a brownout or power failure. *True, false, uncertain and why:* An efficient way for a regulator to correct this externality is to restrict the energy provided to each business to Q/N kilowatt hours, where Q is the total kilowatt hours of power generation available and N is the number of businesses.

Solution: This is false. Restricting the energy supply to a uniform quantity for each business **cannot** be efficient since the demand for energy will differ across businesses. Some businesses won't use their full allotments. Others will be willing to pay a high price for additional energy. It would clearly be more efficient for a regulator to either allow firms to trade their energy quotas in a market (tradable permits), or to set a market price for electricity that rose when demand increased (for example, on hot days).

Part II

The Pokémon Principle [30 points]

Holly and Ash are Pokémon trainers. Holly has an endowment of 10 water-type pokémon w and 0 fire-type pokémon f . Ash has an endowment of 10 fire-type pokémon f and 0 water-type pokémon w . We write their initial endowments as $E(w, f)$:

$$E_H = (10, 0), E_A = (0, 10).$$

Holly and Ash have identical utility functions:

$$\begin{aligned} U_H(w_H, f_H) &= w_H f_H + w_H + f_H, \\ U_A(w_A, f_A) &= w_A f_A + w_A + f_A. \end{aligned}$$

Both Holly and Ash want a mix of fire and water types for two reasons: the desire to catch them all, and the desire to win battles (which is easier if your pokémon stock is more diverse). *You can ignore discreteness throughout this question.*

1. Holly and Ash decide to trade with each other. Show that trade is Pareto-improving over autarky. You can take a short-cut by using that the market-clearing relative price is equal to one (which comes from the symmetry of their utility functions and endowments). Be sure to solve for Holly's and Ash's demands for w and f in terms of the prices p_w and p_f . You will also need these demand functions below. *Please use the normalization $p_f = 1$.* **(9 points)**

Solution: Initially we have $u_H = u_A = 10$. To solve for the equilibrium price (I will show this to be one – alternatively can use this fact straight away and just find demands given $p = 1$), we first find demands. Holly faces the problem (normalizing the price of f to one)

$$\begin{aligned} &\max_{w, f} wf + w + f \text{ s.t. } pw + f = 10p \\ &\Rightarrow \max_w (10w - w^2)p + w + (10 - w)p \\ &\Rightarrow FOC : 10p - 2wp + 1 - p = 0 \\ &\Rightarrow w_H^* = \frac{9p + 1}{2p}, f_H^* = \frac{11p - 1}{2} \end{aligned}$$

Ash faces the problem

$$\begin{aligned} &\max_{w, f} wf + w + f \text{ s.t. } pw + f = 10 \\ &\Rightarrow \max_w (10w - pw^2) + w + (10 - pw) \\ &\Rightarrow FOC : 10 - 2wp + 1 - p = 0 \end{aligned}$$

$$\Rightarrow w_A^* = \frac{11-p}{2p}, f_A^* = \frac{9+p}{2}$$

Now we find the price that clears the market for w :

$$w_H^* + w_A^* = \frac{9p+1}{2p} + \frac{11-p}{2p} = 10$$

$$\Rightarrow p = 1$$

Follows that post-trade we have

$$u'_H = w_H f_H + w_H + f_H = 25 + 5 + 5 = 35 > 10 = u_H$$

$$u'_A = w_A f_A + w_A + f_A = 25 + 5 + 5 = 35 > 10 = u_A$$

Therefore trade is Pareto-improving.

2. Assume Holly and Ash are back at their initial endowments. Holly tells Ash that she wants to see the world, so they leave their home town in upstate New York and travel to Kyrgyzstan. Now they can trade with market vendors there, and there is an unlimited supply of w and f pokémon available at market prices. It is well known that fire-type pokémon are common in Kyrgyzstan, and as a result the relative price of water-type pokémon is high, i.e. $\frac{p_w}{p_f} = p_w = 5$ (normalizing $p_f = 1$). Assume that Holly and Ash's endowments are small enough that they cannot affect this relative price. Show that this 'international trade' is not Pareto-improving. **(6 points)**

Solution: now Holly and Ash trade at world prices. Holly has the same demands from before, but we now substitute in $p = 5$:

$$w_H^* = \frac{9p+1}{2p} = 4.6, f_H^* = \frac{11p-1}{2} = 27$$

$$\Rightarrow u_H^T = 27(4.6) + 4.6 + 27 = 155.8 > 35 = u'_H$$

Holly is better off than if they just traded amongst themselves. Ash has demands

$$w_A^* = \frac{11-p}{2p} = 0.6, f_A^* = \frac{9+p}{2} = 7$$

$$\Rightarrow u_A^T = 0.6(7) + 0.6 + 7 = 11.8 < 35 = u'_A$$

but Ash is worse off – international trade is not Pareto-improving.

3. Explain why one party is strictly worse off under trade. **(4 points)**

Solution: Despite being a very well-known pokémon trainer, Ash is made worse off by international trade. Ash is well-endowed with fire-type pokémon, but since these are common in Kyrgyzstan, his endowment is not very valuable at 'world prices'. The opposite is true for

Holly, whose water-type pokémon are in demand. Ash ends up doing worse than if he just trades with Holly, but still better off than autarky (where he just consumes his endowment and gets $u_A = 10$).

4. What can be done to make the outcome of trade with Kyrgyzstan a Pareto improvement over just trade with each other? Show the result graphically. What theorem are we invoking? (8 points)

Solution: See Figure 1 with Edgeworth Box: I show here that with ex ante endowment transfers we can have both Holly and Ash better off in the free trade equilibrium. We are using the Second Welfare Theorem – that any Pareto efficient allocation can be achieved as competitive equilibrium given the right lump sum transfers.

5. Given that trading with Kyrgyzstan did not affect either Ash’s or Holly’s endowment, explain why trading with Kyrgyzstan leads to a potential Pareto improvement (after transfers). [Hint: Could we have achieved a Pareto improvement simply by setting $p_w = 5$ without trading with Kyrgyzstan?] (3 points)

Solution: Trade with Kyrgyzstan eliminates the constraint that consumption of each type of pokémon cannot exceed 10. Notice that when trading with Kyrgyzstan, Ash and Holly consume more fire-type pokémon than would be feasible under autarky (34 total!).

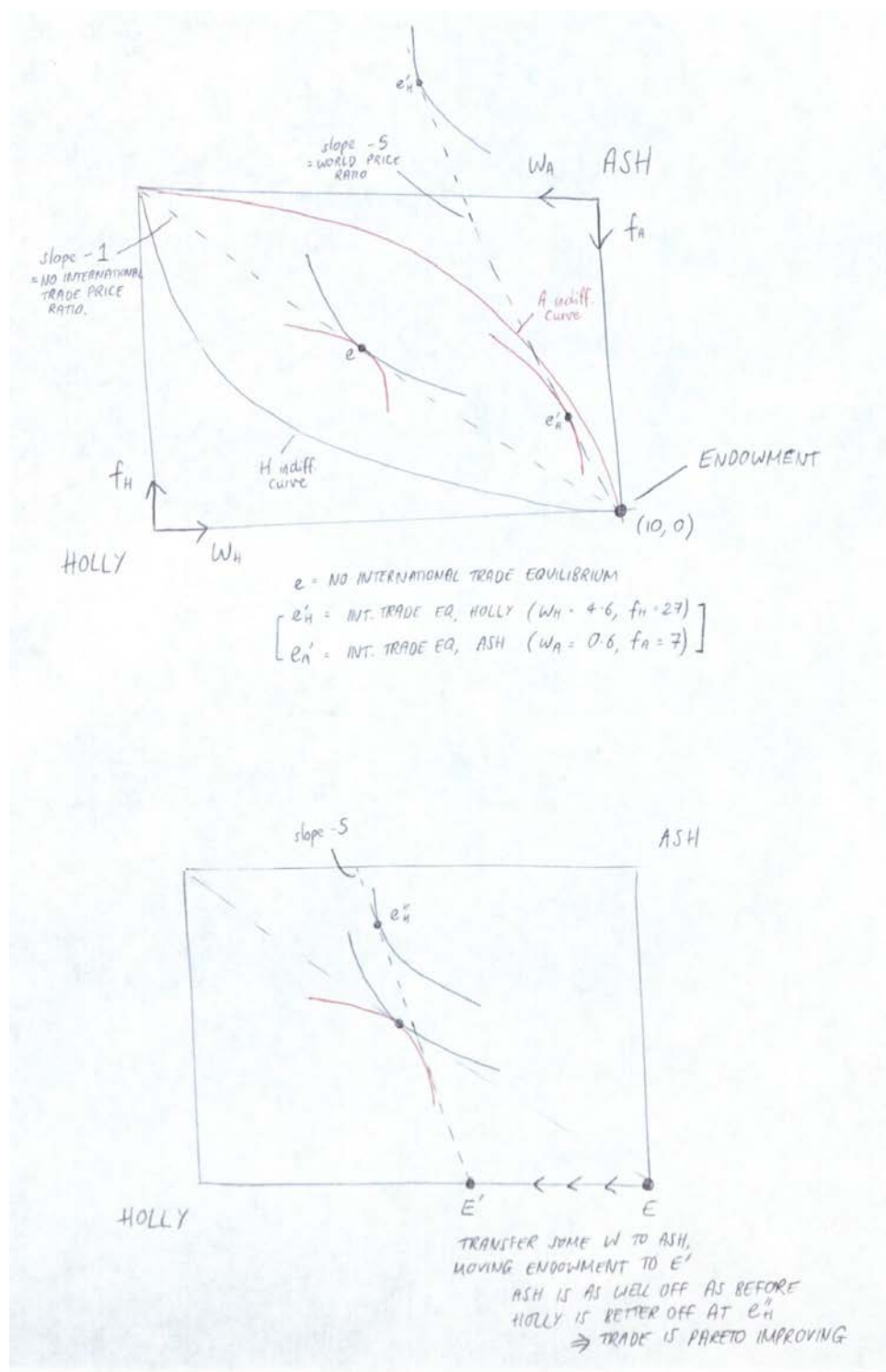


Figure 1: Trade is Pareto-Improving with Transfers

Part III

Causal Inference with Instrumental Variables (30 points)

In the slums of India, poor patients often seek medical help from Bengali doctors. Bengali doctors are untrained and largely unqualified primary healthcare providers. The government wants to train Bengali doctors to identify life-threatening conditions such as heart disease so that they can refer patients to hospitals when needed.

Among a group of 1,000 Bengali doctors, the government randomly invites 500 to participate in medical training. You wish to measure the causal effect of government-provided training to Bengali doctors on their patients' survival. For each doctor, you identify one patient who suffered from heart disease *prior to* the study. You define the following variables

$$\begin{aligned} Z_i &= \begin{cases} 1 & \text{if doctor } i \text{ was invited to the training} \\ 0 & \text{if doctor } i \text{ was not invited to the training} \end{cases} \\ D_i &= \begin{cases} 1 & \text{if doctor } i \text{ came to the training} \\ 0 & \text{if doctor } i \text{ did not come to the training} \end{cases} \\ Y_i &= \begin{cases} 1 & \text{if patient } i \text{ survives for one year after the program} \\ 0 & \text{if patient } i \text{ does not} \end{cases} \end{aligned}$$

Note that i refers to both *doctor and patient* since you have only one patient per doctor. (This is meant to *simplify* notation, not to confuse you.)

1. [3 points] Using the notation developed throughout the class, write down the causal effect that you are interested in. Explain your notation carefully. Call this estimate T^* .

Solution: *The estimate is*

$$T^* = E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1]$$

where Y_{1i} represents the potential outcome of patient i if her doctor received the training and Y_{0i} represents the potential outcome of patient i if her doctor did not receive the training.

2. [4 points] A government official proposes estimating the effect of training on patient health by contrasting the outcomes of patients whose doctors attended the training with the outcomes of patients whose doctors did not attend the training:

$$T_1 = E[Y_i|D_i = 1] - E[Y_i|D_i = 0]$$

Would you expect T_1 to be an unbiased estimate of T^* ? Explain.

Solution: Formally

$$E[T_1] = T^* + E[Y_{0i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

No, T_1 is not likely to be unbiased. Since the doctors who **choose** to attend the training are not a random sample of the control group, one cannot assume that $E[Y_{0i}|D_i = 1] = E[Y_{0i}|D_i = 0]$. Suppose that, among the invited doctors, only the doctors who cared deeply for their patients showed up. These doctors would likely have had better patient outcomes even if they did not receive the training. Thus, T_1 would probably provide an upward-biased estimate of T^* .

You decide to apply an instrumental variable (IV) strategy to estimate T^* . You use the random assignment of the invitation to training Z_i as an instrument for attending training D_i .

3. [6 points] State formally *and explain* the conditions necessary for the invitation Z_i to be a valid instrument for training D_i . Are these assumptions likely to be met in this setting?

Solution: The two main assumptions are:

- (a) First stage: The invitation has a causal effect on whether a doctor receives training or not. That is:

$$E[D_i|Z_i = 1] - E[D_i|Z_i = 0] \neq 0$$

If the invitations were sent out properly and some doctors responded to it, this condition is likely to be satisfied. (I am assuming here that the doctors who did not get an invite did not know of the training and do not attend). Since Z_i was randomly assigned, the first stage is causal.

- (b) Exclusion restriction: The invitation does not affect survival of the patients by channels other than the training itself. Formally

$$E[Y_i|D_i = 1, Z_i = 1] = E[Y_i|D_i = 1, Z_i = 0]$$

This condition is likely to be fulfilled here as invitations (without training) is unlikely to affect patient behavior

- (c) Optional: You could stipulate as a third condition “balance of treatment and control” as in the lecture notes. That should be guaranteed by the randomization here, but no harm in noting it. (Up to 2 points can be added for this answer, but the total for this question cannot exceed 6 points.)
4. [5 points] Write down the equation for the instrumental variables estimate. Call this T_{IV} . Of what two casual effects is T_{IV} the ratio?

Solution: The IV estimate is a Wald Estimate, which is the ratio of the **reduced form** to the **first stage**. The first stage is the causal effect of the invitation to training on the probability

of receiving training. The reduced form is the causal effect of invitation to training on patient mortality.

$$T_{IV} = \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[D_i|Z_i = 1] - E[D_i|Z_i = 0]}$$

5. [6 points] You discover that at a recent meeting of the Bengali Doctors' Association, doctors from the treatment group gave medical training to some of the doctors in the control group. Will the IV estimate in part (4) produce an unbiased estimate of T^* ? Explain why or why not.

Solution: The second-hand training received by the control group **would** violate the exclusion restriction. The exclusion restriction requires that the invitation only affect patient outcomes through its effect on their doctors' participation in the government-provided training. Violation of the exclusion restriction would invalidate the IV estimate. Additionally, we don't know what fraction of doctors in the control group received second-hand training, so we could not rescale our Wald estimate appropriately **even if** we believed (implausibly) that government-provided training and second-hand training were identical.

6. [6 points] Assume that the Bengali Doctors Association meeting in part (5) did *not* occur. But, analyzing your data, you discover that some doctors from the control group *also* attended the training: $E[D_i|Z_i = 0] > 0$. However, a much larger fraction of treatment than control group doctors attended the training: $E[D_i|Z_i = 1] > E[D_i|Z_i = 0]$. Will the IV estimate in part (4) produce an unbiased estimate of T^* ? Explain why or why not. Note that you observe D_i for all doctors, so you can calculate $E[D_i|Z_i = 1]$ and $E[D_i|Z_i = 0]$. [This is a challenging question. Save it for last as time permits.]

Solution: Yes the Wald estimator would still be valid. The increase in the probability of attending the training between treatment and control groups is smaller than would be the case if no one from the control group had attended. This dilution also carries over to the reduced form: the contrast in patient outcomes between treatment and control groups is reduced because some of the control group doctors also receive the treatment. However, the non-compliance in the control group reduces the size of the first stage and reduced form estimates **proportionately**. Rescaling the reduced form by the first stage (i.e., calculating the Wald estimate) will still recover a valid estimate of T_{IV} . Formally, imagine that $E[D_i|Z_i = 1] = 0.8$ and $E[D_i|Z_i = 0] = 0.2$, so the invitation to the training raises the probability of attendance by 0.6. The first stage is

$$\pi'_1 = E[D_i|Z_i = 1] - E[D_i|Z_i = 0] = 0.6$$

The reduced form contrast will be:

$$\pi'_2 = E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0] = 0.6 \times T_{IV} = \pi'_1 \times T_{IV}.$$

Hence, the Wald estimator is

$$\frac{\pi'_2}{\pi'_1} = \frac{\pi'_1 \times T_{iv}}{\pi'_1} = \frac{0.6 \times T_{iv}}{0.6} = T_{IV},$$

which is identical to above. Intuitively, the invitation to training Z_i is still a valid instrument for D_i because it exogenously increases the probability that doctor in the treatment group attends the training **relative to** a doctor in the control group. Meanwhile, the treatment and control groups remain balanced due to the experiment. And the exclusion restriction is still likely to be valid: the instrument only plausibly improves patient outcomes through its effect on doctor training. Thus, the invitation to training remains a valid instrument and the Wald estimate is still a valid causal effect estimate. **Nothing** in this analysis would change so long as $0 \leq E[D_i|Z_i = 0] < E[D_i|Z_i = 1]$. This inequality includes the conventional case that we've studied in class where $E[D_i|Z_i = 0] = 0$.

Partial credit given if you recognize that non-compliance affects either (or both) the first stage and reduced form contrasts, even if you do not recognize that non-compliance 'cancels out' in the Wald estimate due to its proportional effect on the first stage and reduced form estimates.

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