

14.03/14.003 Problem Set 4

Fall 2014

Professor David Autor

Due Thursday, 11/6/2014 at 5pm EST

1 Trade and production (30 points)

Walter White lives in the desert in relative isolation. He only produces (for personal amusement) red phosphorus (R) and lye (Y), and every day, he has 20 hours to spend making them. He produces (in grams)

$$R = \sqrt{L_R}$$
$$Y = \frac{\sqrt{L_Y}}{2}$$

and has utility $U = RY^2$. His total time spent cannot exceed 20: $L_R + L_Y = 20$.

1. Write Walter's constraint on amount of red phosphorus and lye he can consume. Set up his maximization problem with this constraint and solve. What is his utility at this optimum? On a graph, draw Walter's production possibility frontier (PPF) and his optimal level of production. **(7 points)**
2. Walter's former student Jesse invents the International (Recreational Chemicals) Trade Machine, which can take in any amount of phosphorus to produce $1/p$ times that amount of lye, or take any number of lye and produce p times that number of red phosphorus. On your graph, for a single p draw Walter's new budget constraint if he produces the previous optimal amount of R and Y but can also use International Trade and state the approximate value of p for your line. How does a change in p affect your budget constraint? **(4 points)**
3. Without doing any algebra, for what values of p will the International Trade Machine increase Walter's payoff at this level of production? On your graph indicate where a new optimal point might be for the p that you drew (without solving). **(4 points)**
4. Walter's old optimal production is no longer optimal if there is trade. Walter can increase his utility if he were to produce in a way that maximizes the value of his goods in the International Trade Machine. If the machine will trade 1 *tweaker* for each gram of red phosphorus and p *tweakers* for a gram of lye, setup Walter's maximization where he maximizes his number of tweakers using his time constraint. Solve the problem and draw this point and the budget constraint on your graph. What is his income (in tweakers)? **(5 points)**
5. With the tweakers that Walter now has, he can then trade them back to the International Trade Machine in exchange for red phosphorus and lye at the same ratio (1 and p respectively). What is his new budget constraint (Should be in terms of tweakers rather than hours)? Solve his new utility maximization problem. **(5 points)**
6. Assume now that $p = 2$. How much time does international trade save Walter? Calculate the number of hours he would have worked (without trade) in order to have utility equal to the optimal utility under trade. **(5 points)**

2 Quidditch and Congestion¹ [30 points]



Oliver Wood organizes quidditch practices each week, but ever since Harry Potter became famous, attendance has been too high. On Tuesday evenings, each student at Hogwarts decides whether to go to quidditch practice, getting utility $u(N) = h + N - \theta N^2$ where N is the number of people at the practice and $\theta > 0$. Students who don't attend quidditch practice ride unicorns in the Forbidden Forest instead, getting fixed utility $\bar{F}$ (which does not depend on how many people choose that option). There are 100 students in total. Assume throughout that $h + \frac{2}{4\theta} > \bar{F} > h$. [Note: you should ignore discreteness throughout this question.]

1. What exactly is the externality in this context? Is it positive, negative, or ambiguous? [Hint: sketch out the $u(N)$ function.] **(5 points)**
2. If there is no cost of attending quidditch practice, how many will attend quidditch practice? [Hint: there are three equilibria – explain each, and use a graph if it helps. Assume that individuals are ‘small’ in the sense that if they move from hide-and-seek to quidditch, they don't take into account their effect on N and thus on $u(N)$ (this is similar to assuming that firms are price-takers).] **(7 points)**
3. What N maximizes the combined welfare of Hogwarts students? Call this N^* . **(7 points)**
4. Suppose we start off in an equilibrium with $N > N^*$ (famous Harry Potter is leading to Quidditch congestion). What price can Oliver charge to dissuade enough people that only N^* attend in equilibrium? Assume utility is linear in the price, i.e. $u(N) = h + N - \theta N^2 - p$, and use that $\bar{F} = 10$, $h = 5$, $\frac{2}{4\theta} = 3$ and $\theta = \frac{1}{4}$. **(6 points)**
5. Suppose the revenue from charging a price is distributed back to students lump sum (i.e. each student receives $\frac{pN^*}{100}$ – again assume that this enters linearly into utility). Does Wood's pricing policy lead to a Pareto improvement (recall that we started off in the equilibrium with $N > N^*$)? Can any other price do better (i.e. be a further Pareto improvement)? **(5 points)**

3 Causal Inference using Instrumental Variables [30 points]

3.1 Theory

A key objective in empirical analysis is to credibly estimate the causal effect of a variable of interest X on an outcome Y . In this problem set, we will apply an important technique discussed in lecture for estimating causal relationships in (typically) non-experimental settings. That tool is instrumental variables (IV, also known as Two Stage Least Squares, or 2SLS). We will use instrumental variables here to study the classic economic question of whether and by how much schooling raises earnings. Concretely, we will measure the causal effect (in percentage terms) of an additional year of education on workers' earnings.

¹Happy Halloween

Let Y_i represent $\log(\text{earnings})$ of individual i . To simplify matters, assume that schooling is binary (people either go to school, or not go to school). Let S_i represent whether individual i went to school or not. That is, $S_i \in \{0, 1\}$ with $S_i = 1$ indicating that i went to school and $S_i = 0$ indicating that i did not go to school.

1. **[1 point]** Using the notation developed throughout the semester, write down the average treatment effect on the treated. Call this T^*

2. **[1 point]** Suppose you estimated

$$E[Y_i|S_i = 1] - E[Y_i|S_i = 0]$$

Explain, along with mathematical notation, that this estimate is unlikely to be causal.

Suppose you have an “instrument” Z_i that is randomly assigned but affects schooling S_i .

3. **[2 points]** You estimate

$$FS = E[S_i|Z_i = 1] - E[S_i|Z_i = 0]$$

Is this estimate a causal estimate of the instrument on schooling S_i ? Explain your answer using formal notation.

4. **[2 points]** Then you estimate

$$RF = E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]$$

Is this estimate a causal estimate of the instrument on earnings? Explain your answer using formal notation.

5. **[3 points]** Then you use the estimates in parts 3 and 4 to compute the following

$$\frac{RF}{FS} = \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[S_i|Z_i = 1] - E[S_i|Z_i = 0]}$$

- (a) What is this estimate called?
- (b) Can you interpret this as an unbiased estimate of T^* ? What *additional* assumption(s), if any, do you have to make in order to do so?

3.2 Empirical

1. **[3 points]** Load **census70.dta** into Stata. You will notice that all the relevant variables have been labeled. Regress $\log(\text{weekly wages})$ on completed education.

- (a) Is the coefficient statistically different from zero? How do you know?
- (b) Interpret the coefficient on education.

Joshua Angrist and Alan Krueger noticed that in the US children born in different months of the year start schooling at different ages. This is because school districts require a student to have turned age 6 by January 1 of the year in which he or she enters the school. Compulsory schooling laws require children to remain in school until their 16th (or 17th, depending on the state of residence) birthday. The interaction between the school entry requirement and compulsory schooling laws compel children born in certain months to attend school longer than students born in other months.

For example, consider two children: child A born on January 2, 1920 and child B born on December 2, 1920. Both children have to start school in September 1926 because on January 1, 1927, both A and B will be 6 years old. Note that A is 11 months older than B while starting school. Child A can quit school in January 2, 1936 when he turns 16 and would be in the middle of his 10th year at school. But child B cannot quit school until December 2, 1936 when he turns 16 and would be in the middle of his 11th year at school.

2. **[3 points]** Let's check whether this pattern holds true in the data. Load **census70.dta** into Stata. Compute average education by each year-quarter of birth. You can use the command **collapse** to do so. Make sure that you have both YOB and QOB inside the **by()** option. Now generate a single variable which will have both year and quarter of birth information. Plot average education against this variable. Comment on the shape of the plot. For which quarters are completed education the lowest?

[Hint: the new variable could be $YOB + (QOB - 1)/4$]

3. **[2 points]** Load the **census70.dta** again (as your dataset has changed after the **collapse** command). Find the average education and average log weekly wages for each quarter of birth.
4. **[3 points]** Generate a variable **first** that is 1 if the individual is born in the first quarter and 0 otherwise.

- (a) What is the average education attained by those born in the first quarter,

- (b) by those born in other quarters?
 - (c) What is the difference?
 - (d) What is the t-statistic of the difference? Is this difference statistically significant?
 - (e) Assuming that QOB is random, which estimate from section 3.1 does this correspond to?
5. **[3 points]** Repeat the same exercise as in part 4 with $\log(\text{weekly wages})$.
- (a) What is the average $\log(\text{wages})$ for those born in first quarter?
 - (b) for those born in other quarters?
 - (c) What is the difference?
 - (d) Is this difference statistically significant?
 - (e) Assuming that QOB is random, which estimate from section 3.1 does this correspond to?
6. **[2 points]** Now, compute the Wald estimator. Is the Wald-estimator statistically significant? [Hint: You will need to use Stata command `ivregress 2sls` to compute the standard errors and t-statistics]
7. **[2 points]** What are the assumptions needed to interpret the Wald estimator as the causal effect of schooling on wages. Are all the assumptions satisfied? Explain.
8. **[1 point]** Suppose that your assumptions in 7 are satisfied. Interpret the coefficient given by the Wald-estimator.
9. **[2 points]** Propose an economic hypothesis on why your estimate from 1 is different from that in 6

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14.03/14.003 Microeconomic Theory and Public Policy
Fall 2025

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