

# 14.03/14.003 Problem Set 5: Solutions

## Fall 2014

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Due Tuesday, 11/25/2014 at 5pm EST via Class Website

### 1 Short Questions (25 points)

1. Consider 4 lotteries:

- (A) \$100 with certainty
- (B) \$1,000 with probability 0.1; \$100 with probability 0.89 and; \$0 with probability 0.01
- (C) \$100 with probability 0.11; \$0 with probability 0.89
- (D) \$1,000 with probability 0.1; \$0 with probability 0.9

Suppose Alex picks lottery A over lottery B. If Alex is an expected utility maximizer, he will always pick lottery C over lottery D. *True/False/Uncertain and why?* [Hint: Express preference relations in lotteries in terms of expected utilities and then you can rearrange terms by adding and subtracting terms to transform one set of lotteries to another] **(6 points)**

**Solution:** Alex picking Lottery A over Lottery B means that

$$\begin{aligned}u(100) &> 0.1u(1000) + 0.89u(100) + 0.01u(0) \\u(100) - 0.89u(100) &> 0.1u(1000) + 0.01u(0) \\0.11u(100) &> 0.1u(1000) + 0.01u(0) \\0.11u(100) + 0.89u(0) &> 0.1u(1000) + 0.01u(0) + 0.89u(0) \\0.11u(100) + 0.89u(0) &> 0.1u(1000) + 0.9u(0)\end{aligned}$$

Here, the second adds subtracts  $0.89u(100)$  from both sides (legit by the independence axiom after appropriate scaling). Similarly, the fourth line adds  $0.89u(0)$  to both sides (again legit by independence axiom). In the last line, note that the probabilities add up to 1, and represent lotteries (C) and (D).

Therefore, Alex will always pick lottery C over lottery D.

**Now consider the following lotteries**

- (A) \$100 with certainty
- (B) \$200 with probability 0.7; \$0 with probability 0.3
- (C) \$110 with certainty
- (D) \$1200 with probability 0.1; \$0 with probability 0.9
- (E) \$200 with probability 0.35; \$100 with probability 0.5; \$0 with probability 0.15
- (F) \$1200 with probability 0.05; \$110 with probability 0.5; \$0 with probability 0.45

2. Suppose Bob is indifferent between lottery A and lottery B. He is also indifferent between lottery C and lottery D. Is Bob risk-averse, risk-loving, risk-neutral or is it impossible to say. Explain your answer. **(4 points)**

**Solution:** The expected value to lottery B is \$140. Bob is indifferent between lottery A, that provides \$100 with certainty and lottery B, that provides a higher amount, \$140, in expectation but with uncertainty. This must mean that he is risk-averse in this choice. The same conclusion follow if we examine his choice between lottery C and D.

3. Cynthia has same choices as Bob. She is indifferent between lottery A and lottery B and between lotteries C and D. If Cynthia is an expected utility maximizer and her utility is increasing in money/wealth, which lottery will she pick when faced with a choice between lottery E and lottery F? **(5 points)**

**Solution:** Note that Lottery (E) a convex combination of lotteries (A) and (B): It offers lottery (A) with 50% probability and lottery (B) with 50% probabilities. Since Cynthia is indifferent between lotteries (A) and (B), Lottery (E) is equivalent to Lottery (A). Similarly, lottery (F) is a convex combination of lotteries (C) and (D) and is therefore equivalent to lottery (C). As Cynthia's utility is increasing in wealth, she prefers lottery (C) to (A). Therefore, she will pick lottery (F) to lottery (E).

More formally, Cynthia's choice between lotteries A and B and lotteries C and D imply that (Normalizing Cynthia's initial wealth level to 0)

$$\begin{aligned} u(100) &= 0.7u(200) + 0.3u(0) \\ \implies 0.5u(100) &= 0.35u(200) + 0.15u(0) \end{aligned}$$

and

$$\begin{aligned} u(110) &= 0.1u(1200) + 0.9u(0) \\ \implies 0.5u(110) &= 0.05u(1200) + 0.45u(0) \end{aligned}$$

Now, the expected utility from lottery E is given by

$$\begin{aligned} &0.35u(200) + 0.5u(100) + 0.15u(0) \\ &= 0.5u(100) + 0.5u(100) \\ &= u(100) \end{aligned}$$

The expected utility from lottery F is given by

$$\begin{aligned} &0.05u(1200) + 0.5u(110) + 0.45u(0) \\ &= 0.5u(110) + 0.5u(110) \\ &= u(110) \end{aligned}$$

Since Cynthia's utility is increasing in wealth

$$u(110) > u(100)$$

so she will pick lottery F.

4. Donna is a farmer who receives \$500 in good state (plentiful rain) which occurs with probability 0.8. With probability 0.2, there is drought and she receives \$50. If her utility function is given by  $u(x) = \ln(x)$ , how much is she willing to pay for insurance that pays \$450 in case of drought and nothing in the case of plentiful rain. (You can assume that she does not have any other wealth). **(5 points)**

**Solution:** Without insurance, her expected utility is

$$0.8 \ln(500) + 0.2 \ln(50) = 5.75$$

With insurance, she pays premium  $p$  in both states. Her expected utility is

$$0.8 \ln(500 - p) + 0.2 \ln(50 + 450 - p) = \ln(500 - p)$$

She will buy insurance iff

$$\ln(500 - p) \geq 5.75$$

which implies  $p \leq 184.5$ . Therefore, her willingness to pay for the insurance is at most \$184.5

5. Assuming that all consumers are risk averse and that health insurance can be purchased by any consumer who is willing to pay the going market price, why might not everyone choose to buy insurance? Offer at least two possible explanations. **(5 points)**

**Solution:** Consumer are credit constrained. The cost of the policy would exceed current wealth. This is possible if the consumer has an expensive preexisting condition.

The market price of the policy could exceed its value to the consumer. This would occur if the market price is far above the fair-odds price, so that the cost of insurance exceeds the gain in surplus that the consumer would receive from being insured.

Consumers misperceive their risks. They erroneously believe that they have low odds of needing medical care and thus decide that buying insurance is a bad deal for them.

Consumers believe that if they don't buy insurance, they will be taken care of by public insurance programs (e.g., Medicaid) if they actually need acute care. Thus, they choose not to buy insurance to take advantage of the implicit insurance provided by public policies.

## 2 Data Mining (35 points)

You are interested in estimating the  $VSL$  by analyzing data on salaries and fatalities in coal mining.<sup>1</sup> You decide to compare mine workers to steel workers. Like mine workers, steel workers do physically demanding jobs under tough working conditions. But fatal injury rates in steel manufacturing are much lower.

For each worker  $i$ , imagine that they have two potential annual salary rates, one if employed in coal, one if employed in steel. Call these  $W_{1i}$  and  $W_{0i}$  respectively. Similarly, imagine that each worker has two potential annual fatality rates (annual probability of death), one if employed in coal, one if employed in steel. Call these  $M_{1i}$  and  $M_{0i}$ , and assume  $M_{1i} > M_{0i} \forall i$ .

1. Assume that all workers share the same  $VSL = V$  and have full knowledge of  $W_{1i}, W_{0i}, M_{1i}$  and  $M_{0i}$ . Characterize their choice problem of which sector to work in. **(5 points)**

**[Solution:** a worker  $i$  will choose to work in the coal sector if  $W_{1i} - M_{1i}V > W_{0i} - M_{0i}V$  which is if

$$W_{1i} - W_{0i} > V (M_{1i} - M_{0i})$$

i.e. only if the wage increment is sufficiently large to compensate for the additional risk of death. The worker will choose to work in the steel sector if the inequality is reversed, and if there is equality, the worker will be indifferent.]

2. Suppose that we observe these four parameters for all  $i$  (though of course this is impossible), along with each worker's revealed (i.e., chosen) preference of which sector to work in. [Assume indifferent workers flip a coin to determine their choice of sector.] How would you use this information to bound  $V$  (that is, to place a minimum and maximum value on  $V$ )? Will we ever be able to pin down  $V$  exactly? Explain. **(5 points)**

**[Solution:** the key here is that if we knew that any particular worker  $j$  was indifferent between the two sectors, we could easily calculate the  $VSL$  as  $V = \frac{W_{1j} - W_{0j}}{M_{1j} - M_{0j}}$ . But we only observe which sector was weakly preferred, without knowing whether any particular worker was indifferent. Still, we can use this information to bound  $V$ . First note that for any worker  $k$  in the coal sector, it must be the case that

$$\begin{aligned} W_{1k} - W_{0k} &\geq V (M_{1k} - M_{0k}) \\ \Rightarrow V &\leq \frac{W_{1k} - W_{0k}}{M_{1k} - M_{0k}} \forall k \in \text{coal sector} \\ \Rightarrow V &\leq \min_{k \in \text{coal}} \left[ \frac{W_{1k} - W_{0k}}{M_{1k} - M_{0k}} \right] = \bar{V} \end{aligned}$$

which gives us an upper bound for the  $VSL$ . We get this by finding the worker in the coal sector who had the least to gain from choosing that sector, and yet still chose it. We use his parameters to back out an upper bound for  $V$ . Similarly, we know that for any worker  $l$  in the steel sector it must be the case that

$$\begin{aligned} W_{1l} - W_{0l} &\leq V (M_{1l} - M_{0l}) \\ \Rightarrow V &\geq \min_{l \in \text{steel}} \left[ \frac{W_{1l} - W_{0l}}{M_{1l} - M_{0l}} \right] = \underline{V} \end{aligned}$$

which gives a lower bound for the  $VSL$ . Together we have that  $V \in [\underline{V}, \bar{V}]$ . The more workers in our sample, the more likely we are to get tight bounds. In addition, with enough indifferent workers so that we have at least one in each sector, we will get  $\underline{V} = \bar{V}$  and pin down the  $VSL$  exactly.]

3. Suppose for simplicity that all workers are indifferent between the two sectors, and maintain the assumption that all workers have the same  $VSL$ . For this question we only observe the *actual* earnings and fatality rates of workers in their chosen sector, and not counterfactual earnings and fatality rates. Write the simple ratio estimate of  $V$ . Is it likely to provide an unbiased estimate of  $V$ ? Explain why or why not. [Hint: are steel workers similar in demographics or human capital to miners?] Offer one plausible scenario under which your proposed calculation would *underestimate*  $V$ . **(5 points)**

**[Solution:** The ratio estimate is  $\frac{\bar{W}_1 - \bar{W}_0}{\bar{M}_1 - \bar{M}_0}$  where  $\bar{W}_1$  ( $\bar{W}_0$ ) is the sample mean of the wages of those who are actually employed in the coal (steel) industry, and  $\bar{M}_1$  ( $\bar{M}_0$ ) is the sample mean of the annual fatality rate of those in the coal (steel) industry. Given that all workers are indifferent and share the same  $V$ , we know that true  $V = \frac{W_{1i} - W_{0i}}{M_{1i} - M_{0i}}$  for any  $i$ . But since this object involves unknown counterfactuals (e.g. the fatality rate in steel for a worker in the coal industry), we have to find an estimate for it. In the ratio estimate then we are using the average wages of those in steel to serve as the counterfactual wages for

<sup>1</sup>From the US Dept. of Labor: "From 1880 to 1910, mine explosions and other accidents claimed thousands of victims. The deadliest year in U.S. coal mining history was 1907, when an estimated 3,242 deaths occurred...While metal and nonmetal (non-coal) mining was less deadly than coal mining, available records for the era show that it, too, was highly hazardous...Total deaths in all types of U.S. mining, which had averaged 1,500 or more per year during earlier decades, decreased on average during the 1990s to under 100 per year, and reached historic lows of 35 total deaths in 2009 and 2012." Elsewhere, Wikipedia estimates that 1049 workers were killed in Chinese coal mines during 2013.

those who work in mining (and same for fatality rates). There are several reasons why these may not be good estimates of the counterfactual, and thus why this estimate is likely to be a biased estimate of  $V$ .

Firstly, miners may well be less educated than steel workers. This would lead us to overstate the counterfactual wages for miners had they been in the steel industry, understating the risk premium ( $W_1 - W_0$ ) which leads to a downward-biased estimate of  $V$ . We conclude that the value of statistical life is low since miners seem to be receiving very little in additional wages to compensate for a large difference in fatality risk. But really the story is that miners receive little in additional wages because they differ on other dimensions – e.g. human capital.

Another story is that those who select into mining may have a different  $M_0$  than those who select into steel. In particular, those who choose mining may be those that have high injury risk in any occupation (i.e. they would also have high  $M_0$ ). In this case we would overstate the difference in fatality risk between the two occupations, leading us again to a downward-biased  $VSL$ .]

4. You discover that a coal mine suddenly introduced stringent safety regulations in 2000, bringing down fatality rates enormously. Data exists on fatality rates for this particular coal mine since 1990, as well as other coal mines in the area. Data also exists on mine-specific wages since 1990. How would you use this quasi-experiment to estimate the  $VSL$ ? Is this estimate likely to be unbiased? **(5 points)**

**[Solution:** we expect that wages fell in the mine that introduced regulations relative to the other mines in the area. We can construct an estimate of the  $VSL$  in a similar way to Ashenfelter and Greenstone (2004) – using two diff-in-diff estimates. First we find the effect of the safety regulation on fatality rates, our estimate is

$$\hat{\alpha}_1 = (M_{t,post} - M_{t,pre}) - (M_{c,post} - M_{c,pre})$$

where  $M$  is the fatality rate, *post* (*pre*) indicates after (before) 2000,  $t, c$  indicate treated (the mine that introduced regulations) and control (other mines in the area). Next we find the effect on annual wages:

$$\hat{\alpha}_2 = (W_{t,post} - W_{t,pre}) - (W_{c,post} - W_{c,pre})$$

And our estimate of  $V$  is then  $\hat{V} = \frac{\hat{\alpha}_2}{\hat{\alpha}_1}$ .

This estimate is more likely than the last to be unbiased since we are making comparisons only within coal mining (mitigating the concern that steel workers are ‘different’) and taking differences across time. The regulations were also introduced suddenly, and so it is unlikely that wages began to adjust even before its introduction. That said, we still may worry that the mine that introduced safety regulations was different to other mines (e.g. particularly high profits to afford regulations), and so may have already had wages trending upward for instance (a failure of parallel trends). If this the case, we would understate the wage drop following the new regulations, leading to a downward-biased  $VSL$ .]

5. Miners and steel workers are aware of  $M_{1i} = M_1$  and  $M_{0i} = M_0$  (you can assume common risk here) due to reporting by the Occupational Safety and Health Administration. But miners in Kentucky do not know that at the Upper Big Branch Mine, the mine operator has been covertly violating mine safety rules to increase profits. If you were to calculate the  $VSL$  (as in (3)) using wage and fatality data from the UBB Mine, would you tend to over or underestimate the  $VSL$ ? **(5 points)**

**[Solution:** we will tend to underestimate the  $VSL$ . What matters for workers is the *perceived* fatality risk. If these perceptions systematically differ from what we observe in the data, our estimates of the  $VSL$  will be incorrect since they are inferred from the rational choices of workers. Really then we have

$$V = \frac{W_{1i} - W_{0i}}{\tilde{M}_1 - \tilde{M}_0}$$

where  $\tilde{M}_i$  is perceived fatality risk in sector  $i$ . Since we estimate the  $VSL$  using  $M_1 > \tilde{M}_1$ , we will underestimate the  $VSL$  (since the denominator is larger). Intuitively, since the miners do not know that the true fatality risk is much higher, they will be willing to accept a lower wage than otherwise. This is reflected in a lower estimated  $VSL$ .]

6. A technological innovation in mining reduces average fatality rates by half. How would you expect this to affect  $E[W_1 - W_0]$ ? How would you expect it to affect the  $VSL$ ? **(5 points)**

**[Solution:** the reduction in fatality rates should lower the mine wage premium, but not affect the  $VSL$ . The  $VSL$  is essentially a preference parameter, unaffected by wages and fatality rates, even though it is these that we use to estimate the  $VSL$ . Since the  $VSL$  should not change, we can pin down exactly how much the wage premium should fall, since it means we should have (assuming here that our ratio estimate of the  $VSL$  is unbiased)

$$VSL_{pre} = \frac{\Delta^{pre} \bar{W}}{\bar{M}_1 - \bar{M}_0} = \frac{\Delta^{post} \bar{W}}{\frac{1}{2} \bar{M}_1 - \bar{M}_0} = VSL_{post}$$

$$\Rightarrow \frac{\Delta^{post}\bar{W}}{\Delta^{pre}\bar{W}} = \frac{\frac{1}{2}\bar{M}_1 - \bar{M}_0}{\bar{M}_1 - \bar{M}_0} < 1$$

]

7. Mine fatality rates vary greatly over time and across countries. According to Wikipedia, “In 2003, the death rate per million tons of coal mined in China was 130 times higher than in the United States, 250 times higher than in Australia (open cast mines) and 10 times higher than the Russian Federation (underground mines)...in 2007 China produced one-third of the world’s coal but had four-fifths of coal fatalities.”

Provide an economic hypothesis for why mine fatality rates may be higher in China and Russia than in the U.S. and other high income countries. This explanation should assume that markets are operating efficiently in all countries. Provide a non-economic hypothesis for why mine fatality rates may be higher in China and Russia than in the U.S. and other high income countries. This explanation should not assume that markets are operating efficiently in all countries. **(5 points)**

**[Solution:** one economic hypothesis is that safety is a normal good, so WTP for safety and longevity (and therefore the *VSL*) is increasing in income – and so lower in low income countries, even relative to other goods. One might argue that this is due to the shorter life expectancy in low income countries (and arguably lower quality of life). With a low *VSL* it follows that workers do not demand a large wage premium in return for taking on large risks at work, and from this it follows that employers do not have strong incentives to invest in safety.

Another (less plausible) economic hypothesis is that that risk preferences are different in China – aversion to fatality risk is weaker, leading to a lower *VSL*, and a smaller wage premium for miners to compensate for the fatality risk.

A non-economic hypothesis is that mining workers are unaware of the greater risk in China (similarly leading to a small wage premium in equilibrium), and that regulation of the industry is light so that safety standards are not enforced. Another is that there is restricted movement of workers across sectors (e.g. due to internal migration restrictions, which were common historically in China), preventing workers from ‘voting with their feet’ and moving from high-risk to low-risk jobs. This kind of movement could lead to higher wages or lower fatality rates in high-risk jobs in equilibrium.]

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