14.03/14.003 Exam 1 Solutions

Fall 2014

Professor David Autor Wednesday, 10/01/2014

Instructions

- Please do not open this exam until directed
- There are 80 points on this exam and you have 80 minutes to complete it.
- You may not use a calculator (there is no need for it).
- No other reference material is allowed.
- There are 3 parts to the exam: please use a separate exam book for each part.
- Correct answers without explanation do not receive credit.
- Part I is SHORT ANSWER. Answer 4 problems of 6. (5 points each, 20 points total.) You must explain your answers with one or two sentences or graphs. Correct answers without explanation do not receive credit. *If you answer more than 4 questions, we will only grade the first 4.*
- Part II has 1 long question that you must answer. (30 points)
- Part III has 1 long question that you must answer. (30 points)

Part I

Short Questions (Choose any 4 of 6: 5 points each)

Please answer (True/False/Indeterminate) about the statments in bold (assume the rest of the problem is true) and provide a brief explanation. *Correct answers without explanation will not receive credit.*

1. You learn that at the same time as the minimum wage was increased in New Jersey, the U.S. Congress passed a nationwide law that lowered food safety standards for fast food restaurants. **The existence of this legislation invalidates the causal inference strategy used by Card and Krueger.**

   False. This nationwide change should affect both New Jersey and Pennsylvania and hence should be absorbed into the time effect. It does not necessarily invalidate the causal inference strategy.

2. A researcher is testing whether a new health plan improves health. She selects 200 U.S. cities, and randomly assigns half of these cities to the treatment group and half to the control group. She advertises the health plan in the 100 treatment cities. For adults who sign up for the plan, she measures their health at the beginning and end of the calendar year. In the control cities, she randomly selects an equally-sized sample of adults and measures their health at the beginning and end of the calendar year. **A Difference-in-Difference design would provide a valid estimate of the causal effect of the health plan on participating adults in the 100 treatment cities.**

   False. The randomization was violated when the researcher recruited volunteers for the treatment while selecting controls at random. The correct way to do this analysis would be to first recruit volunteers for the experiment and then to assign them randomly to treatment and control groups.

3. The aggregate elasticity of labor supply to fast food restaurants in the state of Georgia is 0.50, meaning that a 10 percent increase in fast food wages would increase the supply of fast food workers by 5 percent. **This elasticity is consistent with a perfectly competitive labor market for fast food workers in Georgia.**

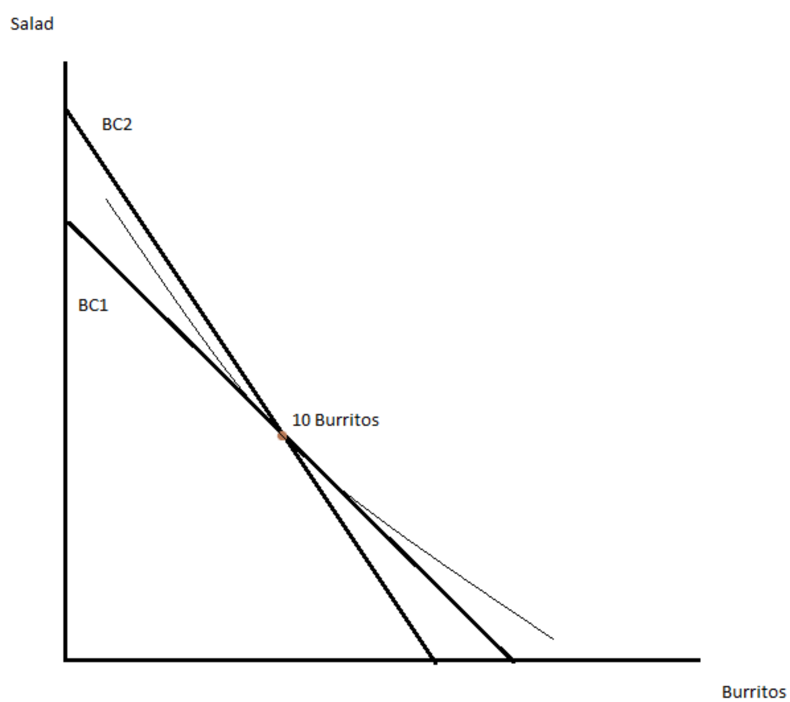
   True. The fact that the aggregate labor supply is upward sloping does not mean that any individual employer has market power (i.e., monopsony power). If all employers are 'small' relative to the market, it is plausible that no employer could individually affect the market wage. (But if all employers demanded more workers simultaneously, wages would of course rise.)

4. Sergio consumes food and alcohol and has income of \$50. If he receives \$50 of food stamps, Sergio spends \$50 on food and \$50 on alcohol. With an unrestricted \$50 gift, Sergio spends \$30 on food and \$70 on alcohol. **The deadweight loss associated with food stamps is equal to \$20.**

   False. The deadweight loss from food stamps is less than or equal to \$20. If Sergio's value for moving from \$30 food and \$50 alcohol to \$50 food and \$50 alcohol is equal to zero, then the deadweight loss from food stamps is exactly equal to \$20. Otherwise, if Sergio's value from that change is equal to X between 0 and 20, then the deadweight loss associated food stamps is equal to \$20-X.

5. Lily consumes burritos and salads and has standard consumer preferences. Each day, she spends \$10 on \$1 burritos. In an effort to promote healthy eating, the city of Cambridge increases the price of burritos to \$1.1, but also gives \$1 to Lily every day to compensate her loss. **This policy will decrease the number of burritos Lily eats.**

   True. BC1 is Lily's budget constraint before the change, and BC2 is her budget after the change. Lily's optimal point on BC1 is exactly on BC2. Lily's indifference curve tangent to BC1 is in the interior of BC2, and her new optimal will be a point with lower consumption of burritos.



6. In Hunan province, households spend all of their income on two goods: Rice and Pork. For some very poor households, rice is a Giffen good. **For the households for whom Rice is a Giffen good, an increase in *either* the price of Rice *or* the price of Pork will**

strictly increase consumption of Rice.

True. We know that $\partial d_R/\partial p_R > 0$ by the definition of a Giffen good. We can establish that $\partial d_R/\partial p_P > 0$ by noting that $\partial h_R/\partial p_P > 0$ due to the substitution effect, and of course $\partial d_r/\partial I < 0$ since Rice is strongly inferior (it's Giffen after all) and a rise in the price of Pork corresponds to a reduction in real income. You might ask: what if the household is already spending its full income on Rice, in which case, a rise in the price of Pork cannot cause the household increase in consumption of rice. But this case would be inconsistent with Rice being Giffen: if a household is already spending its full income on Rice, this would imply that $\partial d_R/\partial p_R = -\frac{I}{p_r^2} < 0$, which means that the demand curve for rice slopes downward. For a household's demand for a good to be Giffen, there must be some other good that the household reduces its consumption of as the price of the Giffen good rises.

Part II

Long Question 1: Consumer Theory

Matt divides his time between grading 14.03 problem sets and research. Since there are only T hours in Matt's day, he is subject to constraint

$$x + y \leq T$$

where x is hours on grading problem sets and y is hours on research. He has some utility $u(x, y)$ and splits his time 50-50 between grading and research at the optimum (i.e. $x^* = y^* = \frac{T}{2}$).

1. **What utility function satisfying all five axioms of consumer theory could result in this behavior?** [there are many possible functions, pick one]

   Solution [3 points]: $u(x, y) = x^{\frac{1}{2}} y^{\frac{1}{2}}$ or a positive monotonic transformation of this, e.g. $\frac{1}{2}\ln x + \frac{1}{2}\ln y$ work. So does $u(x, y) = x + y$ but in this case Matt is indifferent between any division of time among x and y.

Suppose instead that Matt has utility $u(x, y) = x + \sqrt{y}$ and that Matt's friend, Donghee, built a form of time machine. For every hour of problem set writing the time machine gives $1-p_x$ additional hours of time. For every hour of research the time machine gives $1 - p_y$ additional hours of time, with $p_x, p_y < 1$. It follows that Matt faces new time constraint

$$x + y \leq T + (1 - p_x)x + (1 - p_y)y$$

2. **Write the utility maximization problem and find Marshallian demands for x and y. Show that time on research (y) does not depend on T. [You may assume that Matt is at an interior solution where he consumes both x and y.] Can you provide any intuition for why time on research (y) does not depend on T?**

   Solution [8 points]: the budget constraint can be re-written as

$$p_x x + p_y y \leq T$$

   which looks like the usual constraint in choice-over-goods problems. Lagrangian is

$$\mathcal{L} = x + \sqrt{y} + \lambda \left(T - p_x x - p_y y\right)$$

   FOCs are

$$\frac{\delta \mathcal{L}}{\delta x} = 1 - p_x \lambda = 0$$

$$\frac{\delta \mathcal{L}}{\delta y} = \frac{1}{2} y^{-\frac{1}{2}} - p_y \lambda = 0$$

$$\frac{\delta \mathcal{L}}{\delta \lambda} = T - p_x x - p_y y = 0$$

   taking the ratio of the first two we have

$$\frac{p_x}{p_y} = 2y^{\frac{1}{2}} \Rightarrow y^* = \left. \frac{p_x}{2p_y} \right)^2$$

   which doesn't depend on T. This is because the marginal utility of x is constant (at 1) so once the marginal utility of y has dropped to a certain level (which depends on prices), it makes sense to devote all remaining time to x (problem set grading). It follows that

$$x^* = \frac{T - p_y y^*}{p_x} = \frac{T}{p_x} - \frac{p_x}{4p_y}$$

3. **Write the expenditure minimization problem (you do *not* need to solve it). Explain in words what the expenditure minimization problem is. What does it mean to minimize expenditure in this context?**

   Solution [6 points]: problem is

$$\min T = p_x x + p_y y \;\text{ s.t.} u\left(x, y\right) = x + \sqrt{y} \geq U_0$$

   and yields solution $T\left(p_x, p_y, U_0\right)$. The problem is to choose x and y to minimize expenditure subject to utility being above a certain level (U_0). Graphically this is like picking the budget set closest to the origin given the constraint that we must lie on or above a given indifference curve. In this context, the problem is to allocate time between activities to minimize total time spent subject to getting a minimum level of utility. How efficiently time is spent on each activity depends on how good the time machine at translating that particular activity into additional units of time.

4. Matt's boss Rotua Divad broke the time machine (as punishment for not writing exam questions on time) so it can't be used anymore. **What are Matt's demands for y and x now?**

   Solution [3 points]: it is as if $p_x = 1$ and $p_y = 1$. We don't need to re-solve, just need to substitute in. Follows that $y^* = \frac{1}{4}$ and $x^* = T - \frac{1}{4}$.

5. Matt's nephew William is interested in buying Matt's time machine. William's preferences follow the five axioms of consumer theory. At William's existing consumption bundle, Time machines are Giffen goods. He has Hicksian demand $h_t(p_t, p_z, U_0)$ and Marshallian demand $d_t(p_t, p_z, I)$ for time machines where p_t is the price of time machines, p_z is the price of all other goods and I is income. **What is the sign of $\frac{\delta h_t}{\delta p_t}$? What is the sign of $\frac{\delta d_t}{\delta p_t}$?**

   Solution [5 points]: sign of Hicksian derivative is negative (this is always the case), sign of Marshallian derivative is positive (since the good is a Giffen good).

6. William's close friend Miss Meager does not consume any time machines. **Can time machines be a Giffen good from her point of view? Explain using the Slutsky equation.**

   Solution [5 points]: the answer is no. A good t (time machines) is Giffen if $\frac{\delta d_t}{\delta p_t} > 0$ where d_t is Marshallian demand for time machines. By the Slutsky equation we have

$$\frac{\delta d_t}{\delta p_t} = \frac{\delta h_t}{\delta p_t} - \frac{\delta d_t}{\delta I} d_t$$

   where the first term is the Hicksian derivative/substitution effect (always negative) and the second term is the income effect, with d_t being the existing consumption of time machines. For the good to be Giffen it is necessary that $\frac{\delta d_t}{\delta I} < 0$ (inferior) and $d_t > 0$ (for the income effect to kick in, it must be that the individual starts off by consuming some of the good). Since $d_t = 0$ in this case, time machines cannot be Giffen from Miss Meager's point of view.

Part III

Long Question 2: Causal Inference

Malaria is one of the biggest killer of infants in Sub-Saharan Africa. In Kenya, insecticide treated bed nets are sold at prenatal clinics where pregnant women come for medical checkups. bed nets are often sold at subsidized prices. You conduct an experiment to measure the causal effect of the price of bed nets on the consumption (purchase) of bed nets. You choose 100 clinics to participate in this experiment and randomly select 50 to receive a price subsidy. They sell the bed nets at price $p = p_L$. The remaining 50 clinics continue selling the bed nets at the prevailing price $p = p_H$. (Note that because of the subsidy $p_L < p_H$).

Let $X_i = 1$ represent the clinics that were chosen to receive the subsidy and $X_i = 0$ represent the clinics that were not chosen to receive the subsidy. Let Y_i represent the total sales of bed nets in clinic i. The sales data are collected 6 months after the subsidy is rolled out.

1. **Using the notation of the Rubin Causal Model framework, write the expression for the causal effect of the subsidy on bed net sales in clinic i. Call this T_i^*. Explain why you cannot compute T_i^*.**

   Using the notation from class where Y_{1i} is the potential outcome if clinic i receives the subsidy and Y_{0i} as the potential outcome if clinic i does not receiving the subsidy. Then, for clinic i, the causal effect of the subsidy is

   $$T_i^* = Y_{1i} - Y_{0i}$$

   For each clinic i only one of the potential outcomes are realized. If clinic i receives the subsidy, Y_{1i} is realized whereas Y_{0i} is realized if the clinic does not received the subsidy. Hence, we cannot compute the difference. This is the fundamental problem of causal inference which is expressed mathematically as

   $$Y_i = Y_{1i} \cdot X_i + Y_{0i} \cdot (1 - X_i)$$

2. Instead, you choose to estimate $T^* = E\left[T_i^* | X_i = 1\right]$. Using the data collected, you estimate

   $$T_1 = E\left[Y_i | X_i = 1\right] - E\left[Y_i | X_i = 0\right]$$

   What is T^* called? Show mathematically what assumption(s) you need to make in order to interpret T_1 as the causal effect. Is the assumption likely to be satisfied in this context? Explain.

   T^* is the average treatment effect on the treated (ATT). In this context, this is the average

causal effect of the subsidy on those clinics that receive the subsidy. We need to assume that the selection bias is zero in order for T_1 to be the ATT.

$$
\begin{aligned}
T_1 =& E\left[Y_i|X_i=1\right]-E\left[Y_i|X_i=0\right] \\
=& E\left[Y_{1i}|X_i=1\right]-E\left[Y_{0i}|X_i=0\right] \\
=& E\left[Y_{1i}|X_i=1\right]-E\left[Y_{0i}|X_i=1\right] \\
&+E\left[Y_{0i}|X_i=1\right]-E\left[Y_{0i}|X_i=0\right] \\
=& \underbrace{E\left[\left(Y_{1i}-Y_{0i}\right)|X_i=1\right]}_{ATT}+\underbrace{E\left[Y_{0i}|X_i=1\right]-E\left[Y_{0i}|X_i=0\right]}_{Bias}
\end{aligned}
$$

Yes, since X_i is randomly assigned, it does not depend upon the potential outcomes Y_{0i} in expectation. Hence

$$E\left[Y_{0i}|X_i=1\right]=E\left[Y_{0i}|X_i=0\right]$$

which eliminates selection bias.

3. A government official in Kenya objects, "You did not collect baseline data on sales in each village prior to your price subsidy. Why do you assume that any difference in bed net sales in the treatment relative to the control villages is due to the subsidy?" **How would you answer this official?**

   In expectation, randomization balances observable and unobservable characteristics across the treatment and control group. The only systematic difference between the groups should therefore be the subsidy itself. All other forces acting to change outcomes should affect both groups equally. Therefore, even without baseline data, the comparison above gives the causal effect of the subsidy.

4. Suppose that instead of random assignment, the clinics located in the most malaria-ridden parts were provided with the subsidy. You proceeded to compute similar estimate as above. Call this estimate T_2. **Why is T_2 likely to be biased? Do you think this source of bias will lead to an overestimate of the causal effect ($T_2 > T^*$) or an underestimate of the causal effect, ($T_2 < T^*$). Explain.**

   Since X_i is now non-random, $E\left[Y_{0i}|X_i=1\right]-E\left[Y_{0i}|X_i=0\right]$ will not be zero. In absence of the subsidy, the sales of bed nets in malaria-low region is a poor counterfactual for the sales of bed nets in malaria-high region. Even without any subsidy, the sale of bed nets in malaria-high region would likely have been higher as more people would purchase bed nets to protect themselves from malaria. (To take an extreme case, no one purchases bed nets in the U.S. because malaria was eradicated in the U.S. in the mid-twentieth century.)

Following the reasoning above $E[Y_{0i}|X_i=1] > E[Y_{0i}|X_i=0]$. *Therefore*

$$\begin{aligned} T_2 =& T^* + E[Y_{0i}|X_i=1] - E[Y_{0i}|X_i=0] \\ >& T^* \end{aligned}$$

Intuitively, the comparison will attribute pre-existing regional differences in bed net sales to the effect of subsidy as well.

Continue to assume that you have random assignment. But instead of just one level of subsidy as described above (that reduces price to $p = p_L$), suppose you had given different levels of subsidies to the different clinics. For example, among the 50 clinics in the control group, 10 were randomly selected for a 40% subsidy, 10 for a 50% subsidy, 10 for a 60% subsidy, etc.

5. **Using the data from this experiment, you find the following relationship between randomly allocated prices p and the sale of bed nets**

$$\ln(Bednets) = \quad -0.51 \cdot \ln(p)$$

This equation represents a demand curve. Which demand curve is it? Interpret the coefficient -0.51.

This is the Marshallian demand curve since income but not utility is held constant.

The coefficient is the uncompensated (Marshallian) price elasticity of bed net demand. When price of bed nets go up by 1 percent, bed net sales go down by 0.51 percent. To see this, note that

$$\begin{aligned} -0.51 =& \frac{d\ln(bednets)}{d\ln(p)} \\ =& \frac{d(bednets)}{dp} \cdot \frac{p}{(bednets)} \end{aligned}$$

Discretizing the derivatives, this is the same as

$$\left(\frac{\Delta bednets}{bednets}\right) / \left(\frac{\Delta p}{p}\right)$$

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