14.03/14.003 Problem Set 1
Fall 2014

Professor David Autor

Due Monday, 9/15/2014 at 5pm EST

1 Implicit Function Theorem and Envelope Theorem

Consider the function:

$$y = f(x; a) = 1 - ax^2 - a^2 x$$

where a is a parameter.

1. Find the critical point of this function. Denote it with $x^*(a)$.
2. Is this point a maximum, a minimum, or neither? (Show why)
3. For the rest of the problem, assume that $a > 0$. Using the critical point of the function $f(x; a)$ found in part 1, find $y^*(a)$, the maximized value of y as a function of a.
4. Find $\frac{dx^*}{da}$ and $\frac{dy^*}{da}$.
5. Now use the FOC from (1) and the implicit function theorem to find $\frac{dx^*}{da}$.
6. Use the envelope theorem to find $\frac{dy^*}{da}$. Why does this theorem allow you to simplify your calculations with respect to point 4.?

2 Constrained Optimization

Consider the problem

$$z(x, y) = x^a y^{(1-a)}$$
$$\max_{x,y} \; z(x, y)$$
$$s.t. \;\; x + \frac{4}{3}y = 30$$

1. Set up the Lagrangian and find the FOCs.

2. Find z^* the maximum, x^* and y^* where the maximum is achieved, and λ the Lagrange multiplier. What is the interpretation of the Lagrange multiplier in this problem?

3. For the rest of the problem, assume that $a = 0.75$. Set up the dual problem, i.e., minimize the primal constraint function subject to the primal objective function constrained to be equal to the value of z^* obtained in

4. Set up the dual Lagrangian and find the FOCs.

5. Find x^D and y^D where the maximum is achieved and λ^D the Lagrange multiplier in the dual problem. What is the interpretation of the Lagrange multiplier in the dual problem?

6. What is the relationship between λ and λ^D? Why?

3 Consumer Theory

1. Susan has standard consumer preferences over two goods, Kind Bars (KB) and Red Bull (RB), that satisfy all five axioms of consumer theory. She is indifferent between the bundles: A=(2KB, 1RB) and B=(1KB, 2RB). She is offered a third bundle C=(1.5KB, 1.5RB). Based on the axioms of consumer theory, what can you infer about Susan's preferences for C relative to A or B. Explain using the axioms.

2. Like Susan, Mahesh has standard consumer preferences over two goods, glasses of orange juice (OJ) and Dunkin bagels (DB). Mahesh's marginal rate of substitution between OJ and DB is 2: that is, at the margin, he would be willing to give up exactly 2 glasses of OJ to get exactly one DB. Thus, we can say that the total utility of OJ is half the total utility of DB for Mahesh. True or false? Explain your reasoning.

3. Consider the preference relation over (x, y) defined by $(x, y)^p(x', y') \iff x > x'$ and $y > y'$. For any pair (x, y) and (x', y'), if it is not the case that $(x, y)^p(x', y')$ or $(x', y')^P(x, y)$ then $(x, y)^I(x', y')$. Does this relation satisfy the first three axioms of consumer preferences?

4. Charles has utility function $u = x^{0.5}y^{0.5}$. Does the utility function $v = (xy + 2)^2$ represent his preferences? What about $\mu = \frac{xy+2}{\sqrt{xy}}$? Prove or give counterexamples.

4 Minimum Wages and Employment in the US

Please read the New York Times article (Dube) and the Wall Street Journal article (Neumark). Each writer takes different sides of the minimum wage debate in the US.

1. Which model of the labor market from the ones discussed in class do you think best fit the empirical results on employment effects referred to by both Dube and Neumark? Explain why.

[Hint: do the results suggest labor markets for restaurant workers in the US are competitive, monopsonistic, or something else?] Which assumptions does the model you chose rely on?

2. If both Dube and Neumark basically agree on the effect of the minimum wage on employment, what is the basis for their disagreement about the desirability of an increase in the U.S. minimum wage?

3. Dube discussed two empirical strategies in particular: one comparing employment in adjacent counties, one using all states in the US. Dube prefers the first – why? Can you give a reason why we might prefer the second approach instead?

4. Now suppose that the US labor market for restaurant workers is perfectly competitive. The labor supply curve is

$$ln\,(w) = \alpha_s + \beta_s ln\,(L)$$

where $\beta_s > 0$ (upward-sloping) and the labor demand curve is

$$ln\,(w) = \alpha_d + \beta_d ln\,(L)$$

where $\beta_d < 0$ (downward-sloping) and $\alpha_s < \alpha_d$ to ensure that an equilibrium exists. What is the equilibrium ln(wage), $ln\,(w^*)$? What is equilibrium ln(employment), $ln\,(L^*)$? [Hint: you need to set supply of labor equal to demand – solve the 2 equations in 2 unknowns]

5. Now suppose that a minimum wage of $1.5w^*$ is introduced. What condition needs to hold for total earnings ($= wL$) to increase? [Hint: you now need to find the equilibrium wage and employment *after* the minimum wage is introduced, w_2^* and L_2^*, and show what is needed for $w_2^* L_2^* > w^* L^*$ (which is the same as $ln\,(w_2^*) + ln\,(L_2^*) > ln\,(w^*) + ln\,(L^*)$)] Give some intuition for why this condition is needed.

5 Causal Inference

An estimated 2 billion people in the world are iron deficient, most of whom are in the developing countries. Iron plays an essential role in energy production in human bodies. Iron deficiency causes anemia (indicated primarily by low levels of hemoglobin) which is associated with greater susceptibility to disease and fatigue. This, arguably, reduces their productivity and may even fetch them lower wages.

The government in your favorite developing country is thinking of introducing a policy of distributing supplemental iron tablets to the iron-deficient population. The government (for some reason) also wants to learn the causal effect of distributing iron on productivity (measured by wages) of the individuals who are iron-deficient.

Denote as $Y_{1i,t}$ as the outcome if the individual i gets the supplement in time t (that is, $X_{i,t} = 1$) and $Y_{0i,t}$ as the outcome if the individual i does not get the supplement in time t (that is, $X_{i,t} = 0$).

For this exercise, $t = 0$ represents the time before program implementation and $t = 1$ represents the time after a year of program implementation.

1. Using this notation, and the framework developed in class, write down what the government wants to learn (in terms of the causal effect of interest). Call this effect T^*.

2. What is the fundamental problem of causal inference in this setting?

3. Suppose that the government did a small pilot study among a randomly chosen sample of 2000 adult (aged 15-45) males. The government conducted a hemoglobin screening program (at $t = 0$) and found that 1200 of them were iron-deficient (had hemoglobin levels lower than 12 grams per deciliter (g/dl)). Among the iron-deficient population, the government ran a lottery and picked 600 to receive weekly supplemental iron pills. The other 600 received a placebo drug that looked and tasted exactly like the iron pills but did not have any iron supplements. The program continued for a year and at the end of the year, outcomes (hourly wages) were measured for all individuals in the study.

   Suppose the government hired a few consultants to help them analyze the data.

   Denote the post intervention period by $t = 1$. For questions (a) - (c), ignore the 800 that were not-iron deficient.

   (a) The first consultant calculated the following statistic:

   $$E[T_1] = E[(Y_{i,t=1} - Y_{i,t=0}) | X_{i,t=1} = 1]$$

   That is, for the individuals treated at $t = 1$, subtract the average outcome at the beginning of the program from the end of the program.

   i. Clearly state (either mathematically, or in very precise words) the assumption(s) under which $E[T_1]$ is an unbiased estimate of T^*.

   ii. Are these assumptions likely to be satisfied in this context?

   iii. Give at least one reason this might not be a valid estimate. Explain whether this reason leads to an overestimate ($E[T_1] > T^*$) or an underestimate ($E[T_1] < T^*$)

   (b) The second consultant calculated a different statistic:

   $$E[T_2] = E[Y_{i,t=1} | X_{i,t=1} = 1] - E[Y_{i,t=1} | X_{i,t=1} = 0]$$

   i. Explain verbally what the estimate is computing.

   ii. State precisely (ideally, mathematically) the conditions under which $E[T_2]$ is an unbiased estimate of T^*

   iii. Are the conditions satisfied in this context? Explain.

(c) The first consultant wanted to improve on her first estimate. She now computed the following statistic:

$$\begin{aligned} E[T_3] =& E[(Y_{i,t=1} - Y_{i,t=0}) | X_{i,t=1} = 1] \\ & - E[(Y_{i,t=1} - Y_{i,t=0}) | X_{i,t=1} = 0] \end{aligned}$$

i. Explain verbally what the estimate is computing.

ii. What is this estimate called?

iii. State precisely (ideally, mathematically) the conditions under which this estimate will be an unbiased estimate of T^*?

iv. Are the conditions satisfied in this context? Explain

v. How do you expect this estimate to compare with the one computed in 3b?

vi. If you found out that this estimate is slightly different from the one in 3b, what would you infer? Which would you prefer and why?

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