14.03/14.003 Problem Set 2
Fall 2014

Professor David Autor

Due **Friday**, 9/26/2014 at 5pm EST

1 Short Answer Questions

1. Alice only consumes Sushi (S) and Kind Bars (K), and spends all of her income on the two goods. Under what conditions would a government gift of 2 S and 2 K (which cost a total of $10 on the market) be as good in terms of Alice's well being as a gift of $10?

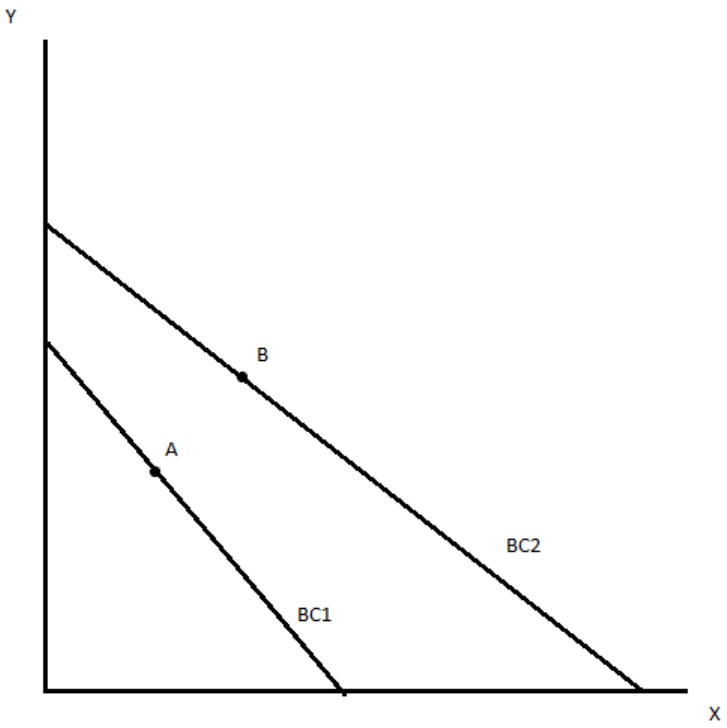
2. Bob also only consumes only Sushi and Kind Bars.

   (a) True, false, or indeterminate **and** why: If the price of S doubles, Bob's marginal rate of substitution at his original optimal level of consumption changes.

   (b) What is the MRS of S at the new optimal bundle compared to the old one?

   (c) In this scenario, you find that Bob's consumption of K decreased. What are the substitution and income effects on K that would explain this? What do we call this kind of good?

3. In the following graph, we have two price ratios and budget constraints, labeled BC1 and BC2. The optimal levels of consumption chosen on BC1 and BC2 are A and B, respectively. Describe the change in income I and the price ratio p_x/p_y that would move the budget from BC1 to BC2. Can we determine if Y is a normal or inferior good?.



4. Carol has utility function $U(X, Y) = \min(X, 2Y)$. Draw indifference curves for this utility function. Which of the 5 axioms of consumer preferences (if any) does Carol violate?

5. In response to widespread student malnutrition at MIT, President Reif implements a pizza subsidy program that reduces the price of a slice of a pizza from \$2 to \$1. Prior to the subsidy program, Carlos (an MIT student with standard consumer preferences that satisfy all five axioms) bought 2 slices of pizza every day and spent the rest of his money on Soda. Pizza is a normal good for Carlos. True, false, indeterminate **and** why: Carlos is indifferent between the subsidy program and a program that gives him 1 free slice of pizza each day. (Assume that Carlos can buy pizza in any increment, e.g., he can buy 10% of a slice for 10% of the cost of a whole slice).

2 Income and Substitution Effects

A consumer has utility function

$$U(x, y) = \left(\frac{x}{2}\right)^{\frac{1}{4}} y^{\frac{3}{4}}$$

1. Suppose prices are $P_x = 3$, $P_y = 1$ and income $I = 30$. Set up the consumer's constrained maximization problem. Solve for the optimal consumption bundle by first taking a log-transformation of the utility function. Why is it okay to take logs of the utility function for solving the optimization problem?

2. Find the Marshallian demand curves d_x and d_y for x and y given prices P_x, P_y and income I.

3. Given the demand curve you found for x, determine the uncompensated price effect, that is $\frac{\delta d_x}{\delta P_x}$.

4. Given the demand curve you found for x, determine the income effect, that is, $\frac{\delta d_x}{\delta I} \times \frac{\delta E}{\delta P_x}$ (where E is the expenditure function).

5. Using the Slutsky equation, find the substitution effect, $\frac{\delta d_x}{\delta P_x} \mid_{U=const}$.

6. In general, is x a normal good? Why or why not?

3 Indirect Utility and Expenditure Functions

In an economy with two goods, fish (f) and chips[1] (c), a consumer has the following indirect utility function

$$v\left(p_c,\, p_f,\, I\right) = \ln\left(I\right) - \quad \ln p_c - \left(1 - \quad\right) \ln p_f \tag{1}$$

where p_f and p_c are the prices of fish and chips, I denotes income, and $\quad \in (0, 1)$ is a parameter.

1. Recalling that the indirect utility function comes from

$$v\left(p_c,\, p_f,\, I\right) = \max_{c,f} u\left(c,\, f\right) \tag{2}$$

$$\text{s.t.} I = p_c c + p_f f$$

use the Envelope Theorem to obtain $\frac{\delta v}{\delta p_c}$, $\frac{\delta v}{\delta p_f}$ and $\frac{\delta v}{\delta I}$ in the general problem (2) and in our particular example (1) [Hint: set up the Lagrangian]. Use the results to obtain the Marshallian demands of this consumer.

2. Find the expenditure function.

3. With another application of the Envelope Theorem, derive the Hicksian demands from the expenditure function you have just obtained.

4. Suppose you learn that this consumer initially spends $\frac{1}{3}$ of her income on fish and $\frac{2}{3}$ of her income on chips. What would be your best estimate of $\quad$ in equation (1)? Use this value of $\quad$ for the next subquestion.

5. Assume that initial prices are $p_f = p_c = 1$, initial income is $I = 900$ and the price of fish increases to $p'_f = 1.75$ after a toxic waste spill kills some of the fish. How much additional income would be necessary to bring this consumer back to the welfare level achieved under the initial prices?

[1] In the British sense, not that this matters.

4 Cash for Clunkers

4.1 Theory

Matt and David are two consumers who love cars (C) and food (F) (and nothing else!). Both of them live for two periods, period 1 and period 2. They have same utility functions $U(C_t, F_t)$ in each period and have same income (I) each period. They both face same prices of cars and food: p_{F1} and p_{F2} for food and p_{C1} and p_{C2} for cars. The subscripts 1 and 2 indicate the period. Both Matt and David know all the prices in period 0, before period 1 begins.

But they are very different in how they go about living their lives. Matt spends his entire income each period. David, on the other hand, decides in period 1 how he will spend his total income ($2I$) across both periods. (Assume that he can borrow or save as needed between periods at no cost).

1. Write down Matt's optimization problem in period 1.
2. Write down Matt's optimization problem in period 2. You need to set up the Lagrangian but do not need to solve.
3. Explain whether his demand for car in period 2 depends upon the price of cars in period 1.
4. Assuming that David wants to maximize the sum total of utilities in two periods, write down his optimization problem and the associated Lagrangian.
5. True/false/uncertain **and** why: David's demand for car in period 2 does not depend upon the price of cars in period 1.
6. Further assume that cars bought in period 1 gives additional utility in period 2 (because you still haven't scrapped the car). How will David allocate income between two periods in light of this fact?
7. How will it affect David's demand for cars (in both periods) when the price of car goes down in period 1?

4.2 Empirics

Read the paper "The effects of fiscal stimulus: Evidence from the 2009 cash for clunkers program" by Atif Mian and Amir Sufi posted on the course website. The paper evaluates the impact of the 2009 Cash for Clunkers program that subsidized purchase of energy-efficient autos. Let $t = 0$ represent the period in which the policy is announced, $t = 1$ is the period in which the policy is in being implemented, and $t = 2$ as the period in which the policy stopped being implemented. Let $Y_{i,t}$ represents total car purchases in city i at time t.

1. Suppose you were in control of how the policy was implemented, and your goal was to evaluate the causal effect of the program on car consumption under credible assumptions. You choose to implement the subsidy in half of the US cities selected randomly, where the other half receives no subsidy.

(a) Let $X_i \in \{0, 1\}$ be an indicator of whether city i had the policy implemented, what comparison would you make to find the effect of the policy in period 1? What comparison would you make to find the effect of the policy in period 2?

(b) Show mathematically, along with clearly stated assumptions, that the comparison in (a) would be the causal effect of the policy

2. Unfortunately, that is not how the program was rolled out. In a simplified sense, the policy was implemented ($X_i = 1$) in cities with 'high' number of pre-existing clunkers (as of period 0) but was not implemented ($X_i = 0$) in cities with 'low' number of pre-existing clunkers (as of period 0). Assume that all cities in the US can be categorized into one of the two categories.

   (a) Suppose you estimated

   $$E\left[Y_{i,t=1}|X_i = 1\right] - E\left[Y_{i,t=1}|X_i = 0\right]$$

   using the data on auto sales in all the cities. Show mathematically, what assumptions are needed for this estimate to be a causal estimate of the policy in period 1.

   (b) Is the assumption in (a) likely to be satisfied? Answer yes, no, or indeterminate and explain why.

   (c) The authors estimate a variant of the following estimate

   $$E\left[Y_{i,t=1} - Y_{i,t=0}|X_i = 1\right] - E\left[Y_{i,t=1} - Y_{i,t=0}|X_i = 0\right]$$

   Show mathematically, the assumptions needed for this estimate to be a causal estimate of the policy in period 1. Also, state the assumption that you are making verbally. Is it likely to be satisfied?

   (d) Write down the estimate that you would use to find the effect of the policy in period 2. Are the identifying assumption different now?

   (e) If all consumers were like Matt, what would you find in (d)? If you found something different, what would that tell you?

   (f) If all consumers were like David, would you expect to find no effect of the policy in period 2? Why (not)?

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