14.03/14.003 Problem Set 6 Solutions
Fall 2014

Professor David Autor

Due Thursday, 12/04/2014 at 5pm EST

- Attempt **any two** out of the three questions. Each question is worth 30 points.
- You are welcome to attempt all three. If you do so, the *average* score will be recorded. That is, there is no bonus for attempting all three questions (except, of course, it will be good practice for you and help you learn the materials)

1 Car talk

In the market for used cars, there are high and low quality cars. At a price P, the quantity of high and low quality cars supplied are (in thousands of cars): $\max(2P-8,0)$ and $\max(P-3,0)$ respectively. To buyers, a low quality car is worth 4.5 and a high quality car is worth 7. Buyers are risk neutral, and if they are uncertain about the quality of the car, their valuation is the expected value given their beliefs about the quality. The number of buyers far exceeds the number of sellers, so the demand for cars is infinite if the price is less than the expected value of a car.

1. Suppose that buyers are perfectly informed about the quality of each car. What is the market price and quantity of high and low quality cars? You may find it helpful to graph the supply and demand curves.

   In the market for low quality cars, supply is $P-3$ and consumers will purchase if $P \leq 4.5$. The equilibrium price is $P = 4.5$, with $Q = 1.5$.

   In the market for high quality cars, supply is $2P-8$ and consumers are willing to pay 7. At $P = 7$,$Q = 6$. Prices will not be higher than these because then demand goes to 0. Prices will not be lower because sellers have all of the market power. Any any lower price, other buyers will enter the market and offer to purchase at $P = 4.5$ and $P = 7$.

2. Suppose now that buyers have no information about what kind of car they are purchasing. If the prevailing market price for cars is P, what fraction of the cars available for sale are high quality? At that price P, what is the buyers' expected valuation for the cars?

   At price P, there are $2P-8$ high quality cars and $P-3$ low quality cars, for a total of $3P-11$ cars. To the buyer, any unknown car is high quality with probability $(2P-8)/(3P-11)$, the total fraction of high quality cars in the market. The car is low quality with probability $(P-3)/(3P-11)$.

   If the buyer believes a car to be high quality with probability λ, his valuation is $7\lambda + 4.5(1-\lambda)$. In this case, the probability is what we calculated above, and his expected value is $7(2P-8)/(3P-11) + 4.5(P-3)/(3P-11)$.

3. There is some price P such that the consumer's willingness to pay for a car is equal to price P. What is the quantity of high and low quality cars traded in this equilibrium? Make sure these answers are intuitive.

   We are looking for some price P such that $7(2P-8)/(3P-11) + 4.5(P-3)/(3P-11) = P$.

$$
\begin{aligned}
P &= 7\frac{2P-8}{3P-11} + 4.5\frac{P-3}{3P-11} \\
P(3P-11) &= 7(2P-8) + 4.5(P-3) \\
P &= \frac{59+\sqrt{145}}{12} \\
&= 5.92
\end{aligned}
$$

   Note that there is another solution to the quadratic at $P = 3.9$. However, at this price, the quantity of high quality cars is 0, but there is excess demand for low quality cars, so this is not an equilibrium.

4. Now suppose instead that the supply for high quality cars was $2P - 11$. If you were to solve part 3, you would find that there is no solution to the quadratic function. What is the equilibrium in this market? Why does the solution change so much?

   $P = 4.5$ *is now an equilibrium, where only low quality cars will be sold. This was not an equilibrium before because at the low price, there was positive supply of the high quality cars. now the supply for high quality cars is so low there is no solution where high quality cars are sold.*

2 Regression Discontinuity (30 points)

For policymakers, finding the right level of benefits for a social insurance program like disability insurance is tricky. Low benefits levels will not provide adequate insurance, while high levels of benefits may encourage people to withdraw from the labor market to seek benefits even if they are work-capable.

Denote by $Y_{B,i}$, the potential wage income individual i would earn when receiving disability benefits amount equal to B. You want to find the causal effect of reducing benefit size B_i from B to B' for the participants who were receiving benefit B.

1. **[2 points]** Using the potential outcome notation developed throughout the class, write the causal effect that you are trying to estimate. Call this estimate T^*

   Solution: *The causal estimate that you are interested is:*

$$T^* = E\left[(Y_{B,i} - Y_{B',i})\,|B_i = B\right]$$

2. **[3 points]** One way to estimate T^* is just comparing wage earnings of participants with wages B and B'. That is:

$$T_1 = E\left[Y_i|B_i = B\right] - E\left[Y_i|B_i = B'\right]$$

   Why is T_1 unlikely to be an unbiased estimate of T^*?

   Solution*: Those receiving higher benefits are different from those receiving lower benefits. For example, those who are receiving higher benefits might be more disabled and may have lower earning potential at the labor market.*

In Netherlands, a reform in 1993 effectively reduced the benefit of disability insurance received by existing beneficiaries.[1] However, a parliamentary motion made individuals that were aged 45+ years by August 1, 1993 exempt from the effect of the reform (they continued getting the same level of benefits as they would under the old scheme).

3. **[5 points]** Let X_i be the age of the individuals and let Z_i be a dummy variable that is equal to 1 if individual i is above 45 years of age on August 1, 1993. Supposed you wanted to use Z_i as an instrument of B_i and estimate

$$T_{IV} = \frac{E\left[Y_i|Z_i = 0\right] - E\left[Y_i|Z_i = 1\right]}{E\left[B_i|Z_i = 0\right] - E\left[B_i|Z_i = 1\right]}$$

   What are the identifying assumptions for T_{IV} to be an unbiased estimator of T^*. Are they met in this setting?

   Solution*: The two assumptions are:*

   First stage: The instrument Z_i has an effect on B_i. This would be hold if the policy was implemented properly (which it was)

   Exclusion. The instrument Z_i (being above 45) affects Y (earning potential) only through changes in benefit. This is not likely to be satisfied because age itself has a direct effect on earnings potential. For example, being over 45 means individuals have more experience and therefore a higher earnings.

4. Consider an alternative strategy. You observe month of birth of each disability participant, and you want to use the discontinuity at 45 years (as of Aug 1, 1993) to estimate the causal effect of interest.

   (a) **[2 points]** Explain the rationale of this regression discontinuity (RD) strategy relative to the IV strategy. Why would you prefer one or the other?

   Solution*: The IV strategy applies to the entire range of the variables of interest. In this setting the assumptions about the instrument applies to everyone who are above 45 or below 45. That is, it would require comparing individuals that are very young with individuals who are very old.*

[1] If you are interested in more details, you can refer to Borghans, Gielen and Luttmer (2014) article: http://pubs.aeaweb.org/doi/pdfplus/10.1257/pol.6.4.34

The RD strategy only focuses on individuals right around the point of discontinuity. Essentially, it compares outcomes of individuals that are just below the cut-off with individuals that are just above. The assumptions that individuals just above the cutoff are comparable to individuals just below the cutoff is more plausible. One disadvantage is that, the RD treatment effect applies only to those individuals who are right around the cutoff.

Essentially, the RD strategy is doing IV but only with individuals right around the point of discontinuity.

(b) [**3 points**] What are the identifying assumptions for the RD strategy?

***Solution**: Formally, the identifying assumptions are:*

$$\lim_{\varepsilon \to 0} E\left[Y_{B,i}|X_i = 45 + \varepsilon\right] = \lim_{\varepsilon \to 0} E\left[Y_{B,i}|X_i = 45 - \varepsilon\right]$$
$$\lim_{\varepsilon \to 0} E\left[Y_{B',i}|X_i = 45 + \varepsilon\right] = \lim_{\varepsilon \to 0} E\left[Y_{B',i}|X_i = 45 - \varepsilon\right]$$

(c) [**4 points**] Write down the RD estimator using formal causal notation. Note that this expression involves taking limits. You can follow the notation in the lecture note.

***Solution**: The RD estimator looks similar to the IV estimator*

$$T_{RD} = \frac{\lim_{\varepsilon \to 0} E\left[Y_i|X_i = 45 - \varepsilon\right] - \lim_{\varepsilon \to 0} E\left[Y_i|X_i = 45 + \varepsilon\right]}{\lim_{\varepsilon \to 0} E\left[B_i|X_i = 45 - \varepsilon\right] - \lim_{\varepsilon \to 0} E\left[B_i|X_i = 45 + \varepsilon\right]}$$

(d) [**4 points**] Using the information in the two panels of Figure 1 below, what is the estimated increase (or decrease) in earnings in 1999 for one dollar reduction in DI benefits? (Note that the point estimates for benefits received and earnings levels are given in the top of each panel. Your task is to construct the Wald estimator. You do not need to calculate a standard error.)

***Solution**: The top-panel gives the denominators which is -1076. The bottom panel gives the numerator which is 624. The increase in earnings per unit reduction in benefits is 0.58*

5. [**4 points**] The authors who studied this reform also wanted to look at the effect of DI benefit reduction on income from other social assistance program. They used the same RD strategy to estimate this effect. Their result is shown in Figure 2. Find the RD estimate of a dollar reduction in DI benefits on social assistance income from other sources.

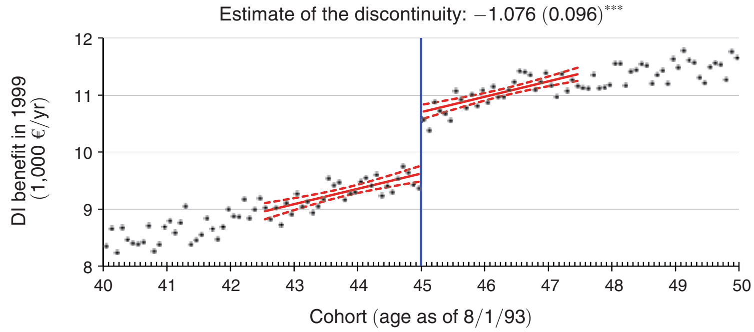
   ***Solution**: The RD estimate is calculated exactly like before. From the figure, the increase in income from other sources is 305. Therefore increase in other social assistance income per unit reduction in DI benefits is 0.28*

6. [**3 points**] Based on the information above, how many Euros in total transfer benefits would you anticipate the Dutch government would save per Euro reduction in disability benefits?

   ***Solution**: One Euro reduction in DI benefits saves Dutch government 1 Euro in DI benefits but they will have to payout 0.28 Euros in other social insurance programs, so the total savings is 0.72 Euros.*

Figure 1: RD estimates of DI Benefit and Earnings

Panel A. Effect on DI benefit amount



Panel A. Effect on earnings in 1999

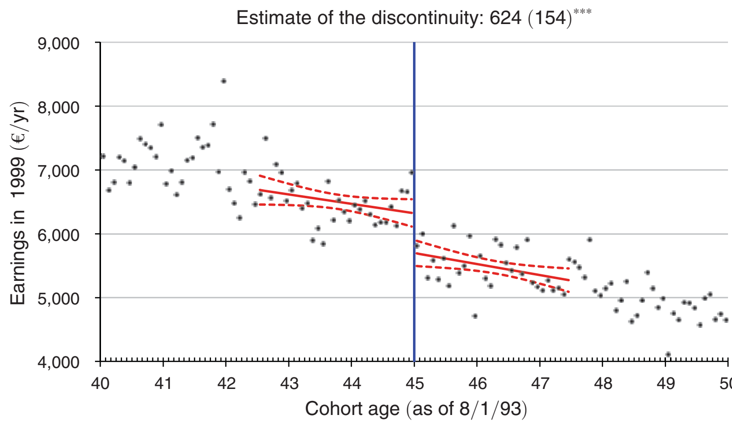
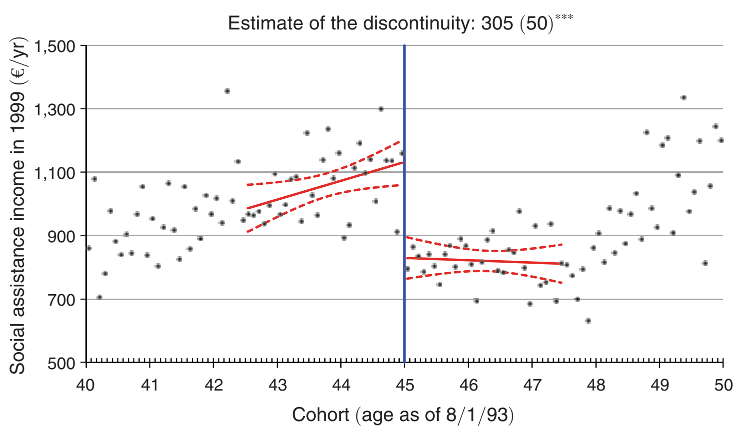


Figure 2: RD estimates of other social assistance program

Panel A. Other social assistance income in 1999



3 Signaling Spontaneity [30 points]

Robin would like a spontaneous boyfriend. Men have a level of spontaneity (known to them) s_i distributed uniformly on interval $[0,1]$. Robin doesn't observe spontaneity directly, but knows its distribution in the population. You can imagine that $s_i = 1$ corresponds to the type of person who would take you to Paris out of the blue, whereas a person with $s_i = 0$ wouldn't even take you to Somerville with less than 3 weeks' notice.

Robin goes on dates to a restaurant "Carmichaels" with randomly chosen men from the zero-one interval. Unfortunately, men cannot reveal that they are spontaneous since talk is cheap.[2] That said, these potential partners *can* take a costly action that may signal that they are spontaneous: they can choose whether or not to steal the blue French horn that hangs on the restaurant wall, and give it to Robin.[3] Stealing the French horn costs $1 - s_i^2$ for a person with level of spontaneity s_i (i.e. the action is costless for those with $s_i = 1$ who would go to Paris on a whim).

Robin decides which dates to accept as boyfriends [she can have more than one]. She gets utility $4s_i - 3$ from accepting a person with spontaneity s_i, and zero from not accepting. All candidates want to be Robin's boyfriend, which gives them utility $\frac{2}{3}$.

1. Find a separating equilibrium, i.e. an equilibrium where some dates steal the blue French horn and others don't bother. You should take the following steps to solve the problem: **(12 points)**

   (a) Find which potential partners will steal the blue French horn if only those that steal it become Robin's boyfriends.

   (b) Determine which people Robin will accept as a function of her beliefs about their level of spontaneity (assuming again that only those who steal the French horn are eligible).

   (c) Confirm that if potential partners act according to (a), then Robin will indeed accept them if and only if they steal the blue French horn.

   Solution: *(a) The benefit of stealing the blue French horn is $\frac{2}{3}$ (utility of being Robin's boyfriend) while the cost is heterogeneous, and equal to $1 - s_i^2$, so a person should steal if*

   $$\frac{2}{3} \geq 1 - s_i^2 \Rightarrow s_i^2 \geq \frac{1}{3} \Rightarrow s_i \geq \sqrt{\frac{1}{3}} \approx 0.58$$

   so the cut-off $\tilde{s} = \sqrt{\frac{1}{3}}$.

   (b) Robin only observes whether a date steals the horn or not. She will accept those who steal only if her beliefs are that the average spontaneity among that groups is greater or equal to the one that solves for indifference in $4\hat{s} - 3 = 0$. Therefore she accepts all who steal iff

   $$E[s \mid steal] \geq \frac{3}{4}$$

   (c) In equilibrium if individuals steal iff. $s \geq \tilde{s} = \sqrt{\frac{1}{3}}$, Robin is acting optimally in accepting all potential partners that steal since $E[s \mid steal] = \frac{1+\tilde{s}}{2} \approx 0.79 > \frac{3}{4}$. Similarly we have $E[s \mid not\,steal] = \frac{\tilde{s}}{2} \approx 0.29 < \frac{3}{4}$ so it is optimal not to accept these as boyfriends. This is a separating equilibrium.

2. Suppose Robin can only go on one date, with a randomly chosen man from the zero-one interval. What is Robin's expected utility of the date given this separating equilibrium? **(5 points)**

   Solution: *A date in which the horn is not stolen gives zero utility (since Robin doesn't accept), whereas a date where the horn is stolen gives Robin expected utility*

   $$4E[s \mid steal] - 3 = 4(0.79) - 3 = 0.16$$

[2] I recently heard a guy in a coffee shop tell his date the words "I am actually a very spontaneous person". I thought that this would be meaningless since anyone can say that, spontaneous or not. But it does seem that the rest of the date went pretty well.

[3] https://www.youtube.com/watch?v=FDHi-Qe0DR0

It follows that expected utility of the entire date is

$$P\,[not\,steal] \cdot 0 + P\,[steal] \cdot 0.16$$
$$= P\,[s < \tilde{s}] \cdot 0 + P\,[s \geq \tilde{s}] \cdot 0.16$$
$$= (1 - \tilde{s}) \cdot 0.16 \approx 0.067$$

3. The company Spontaneous Combustion, Inc. has developed a clever app that reveals a potential partner's exact level of spontaneity by scanning the iris of the person. The company sells use of the app for price p per person tested [assume that all utilities are measured in money, so buying the test also costs p units of utility]. What is the maximum price Robin would be willing to pay for use of the app? [You can again consider that she goes on only one date.] **(6 points)**

   Solution: *without the app, Robin gets expected utility* 0.067 *(per date) in the separating equilibrium. With the app, Robin can use the efficient cutoff rule, only accepting as boyfriends those with* $s \geq \frac{3}{4}$. *This gives her per-date expected utility of*

$$P\left[s < \frac{3}{4}\right] \cdot 0 + P\left[s \geq \frac{3}{4}\right]\left(4E\left[s \mid s \geq \frac{3}{4}\right] - 3\right) - p$$
$$= \frac{1}{4}\left(4\left(\frac{7}{8}\right) - 3\right) - p = 0.125 - p$$

   The maximum Robin is willing to pay is then $\bar{p}$ *such that* $0.125 - \bar{p} = 0.067$ *(making her indifferent between using the app or not). This gives us* $\bar{p} = 0.058$.

4. Now find a pooling equilibrium. What expected utility does Robin get per date? Is this less or more than in the separating equilibrium without the app? **(7 points)**

   Solution: *a pooling equilibrium has all potential partners taking the same action – either stealing, or not stealing. Consider the former first: all candidates steal the blue French horn. All dates look the same to Robin (they all steal), and so* $E\,[s \mid steal] = \frac{1}{2}$ *– expected spontaneity is equivalent to the population average. Since* $E\,[s \mid steal] < \frac{3}{4}$, *Robin's optimal response is to reject all dates. It follows that this cannot be an equilibrium – if Robin is rejecting all dates, it is not optimal for potential partners to take the costly action by stealing. They would be better off not stealing (except for* $s_i = 1$ *who would be indifferent).*

   Consider the other possible pooling equilibrium: no candidates steal. Again we have $E\,[s \mid not\,steal] = \frac{1}{2}$ *and Robin's optimal response is to reject all dates. This is an equilibrium since potential partners are responding optimally to Robin's strategy (rejecting all dates) by not stealing. Robin gets zero utility in this equilibrium. This is less than in the separating equilibrium where Robin gets expected utility of* 0.067 *from each date.*