Midterm Exam 2 Solutions

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There are 80 points in total. You have 80 minutes to complete the exam. Do not spend too much time on any particular section.

1 Short Questions (30 points)

Explanations are needed for True/False questions to receive full points.

a) (6 pts) Suppose two agents have the same preferences and thus the same levels of risk-aversion. For them to accept a risk-sharing contract, what must be true about the correlation between their income realizations?

Solution: If the shocks are perfectly positively correlated (same shock), they will not risk-share since this would be simply transferring risk from one person to another. The shocks need to be somewhat idiosyncratic for agents to risk-share.

b) (6 pts) List the three major financial accounts and the differences among them.

Solution: The three types of financial accounts are income statement, balance sheet and statement of cash flow.

c) (6 pts) True or False: The Heckscher-Ohlin model of trade suggests that international trade is always Pareto-improving.

Solution: False. Owners of a country's relatively scarce factors lose as the country will export the goods that are intensive in the factors it is relatively abundant in.

d) (6 pts) What are the 3 conditions that define a Walrasian equilibrium?

Solution: (1) Profit maximization given prices. (2) Utility maximization given prices and endowments. (3) Feasibility of allocation.

e) (6 pts) What are the main findings in the paper "Risk and Insurance in Village India " (Townsend 1994)?

Solution: Household consumptions comove with village average consumption. More clearly, household consumptions are not much influenced by contemporaneous own income, sickness, unemployment, or other idiosyncratic shocks, controlling for village consumption (i.e. for village level risk).

2 Contracts and Moral Hazard (16 points)

Suppose a principal hires an agent to put in effort $e \in \{0, 1\}$ to output x. The probability of output levels $x \in \{5, 15\}$ depend on effort following

$$p(x = 5) = 1, \quad p(x = 15) = 0 \quad \text{if } e = 0$$
$$p(x = 5) = \frac{1}{2}, \quad p(x = 15) = \frac{1}{2} \quad \text{if } e = 1$$

The principal pays the agent wage w for their work. The agent chooses their effort level e. Profit for the principal and utility for the agent follow

$$\pi_p = x - w$$
$$U_a = w - e$$

Only output x is observed; effort e is not observed. The principal sets the wage contract $w(x)$ to depend on output $x \in \{5, 15\}$. The principal maximizes expected profit. The agent maximizes expected utility.

a) (4 pts) What is the expected profit for the principal when $e = 1$? Your answer should be in terms of $w(5)$ and $w(15)$.

Solution: Expected profit follows

$$E[\pi_p|e] = E[x - w(x)|e] = p(5|e)(5 - w(5)) + p(15|e)(15 - w(15))$$
$$\Rightarrow E[\pi_p|e = 1] = \frac{1}{2}(5 - w(5)) + \frac{1}{2}(15 - w(15))$$

b) (4 pts) What is the incentive compatibility constraint for the agent to choose effort $e = 1$ given a wage contract $w(x)$?

Solution: The incentive compatibility constraint is

$$E[U_a|e = 1] \geq E[U_a|e = 0]$$
$$\frac{1}{2}w(5) + \frac{1}{2}w(15) - 1 \geq w(5) - 0$$

c) (4 pts) The agent will turn down the contract if $E[U_a|e] < 0$. What "participation constraint" must $w(x)$ satisfy for the agent to accept a contract wherein the agent exerts effort $e = 1$?

Solution: The participation constraints are

$$E[U_a|e] = E[w(x) - e|e] = p(5|e)w(5) + p(15|e)w(15) - e \geq 0$$
$$\Rightarrow E[U_a|e = 1] = \frac{1}{2}w(5) + \frac{1}{2}w(15) - 1 \geq 0$$

d) (4 pts) Assume the principal wishes to induce $e = 1$. Set up the principal's profit maximization problem to find the optimal wage contract $w^*(x)$. (Remember, the agent must be willing to both accept the contract and exert effort. No need to solve.)

Solution: The profit max problem is

$$\max_{w(5),w(15)} \frac{1}{2}(5 - w(5)) + \frac{1}{2}(15 - w(15))$$
$$\text{s.t.} \quad \frac{1}{2}w(5) + \frac{1}{2}w(15) - 1 \geq w(5) - 0 \qquad \text{(IC constraint)}$$
$$\frac{1}{2}w(5) + \frac{1}{2}w(15) - 1 \geq 0 \qquad \text{(Participation constraint)}$$

3 Walrasian Equilibria (22 points)

Consider an exchange economy with two goods, fish f and chips c, and two agents denoted with $i \in \{1, 2\}$. Each agent has Cobb-Douglas preferences and endowments of each good following:

$$u_1(f, c) = \frac{1}{2}\ln f + \frac{1}{2}\ln c$$
$$u_2(f, c) = \frac{1}{4}\ln f + \frac{3}{4}\ln c$$
$$(\omega_{1f}, \omega_{1c}) = (4, 4)$$
$$(\omega_{2f}, \omega_{2c}) = (4, 2)$$

a) (4 pts) Without completing any calculations, which good do you expect will be more expensive and why?

Solution: We should expect chips to be more expensive since agent 2 really likes chips but has less of chips compared to fish to start off with. Therefore agent 2 will be willing to trade a lot of fish for some chips. At the same time, agent 1 does not have asymmetric preferences for one or the other, and starts with equal amounts of both.

b) (4 pts) Outline how to solve for the agents' demand functions $(x^*_{1f}, x^*_{1c}, x^*_{2f}, x^*_{2c})$ as a function of prices p_f, p_c. (Solution is provided below, so no need to actually solve).

Solution: The Lagrangians of the two agents are

$$\mathcal{L}_1 = \frac{1}{2}\ln x_{1f} + \frac{1}{2}\ln x_{1c} - \lambda_1(p_f x_{1f} + p_c x_{1c} - 4p_f - 4p_c)$$
$$\mathcal{L}_2 = \frac{1}{4}\ln x_{2f} + \frac{3}{4}\ln x_{2c} - \lambda_2(p_f x_{2f} + p_c x_{2c} - 4p_f - 2p_c)$$

Taking FOCs (or using the classic Cobb-Douglas form tricks), we can rearrange to get the demand functions.

c) (6 pts) The demand functions are

$$x^*_{1f} = \frac{4p_f + 4p_c}{2p_f} \quad x^*_{1c} = \frac{4p_f + 4p_c}{2p_c}$$
$$x^*_{2f} = \frac{4p_f + 2p_c}{4p_f} \quad x^*_{2c} = \frac{3(4p_f + 2p_c)}{4p_c}$$

Solve for the Walrasian equilibrium price ratio p_f^*/p_c^* and allocations in this economy. *Hint*: Normalize the price of chips to $p_c^* = 1$.

Solution: From Walras' Law, we only need to solve one of the market clearing conditions for the price ratio. We solve for fish. From market clearing, we then have

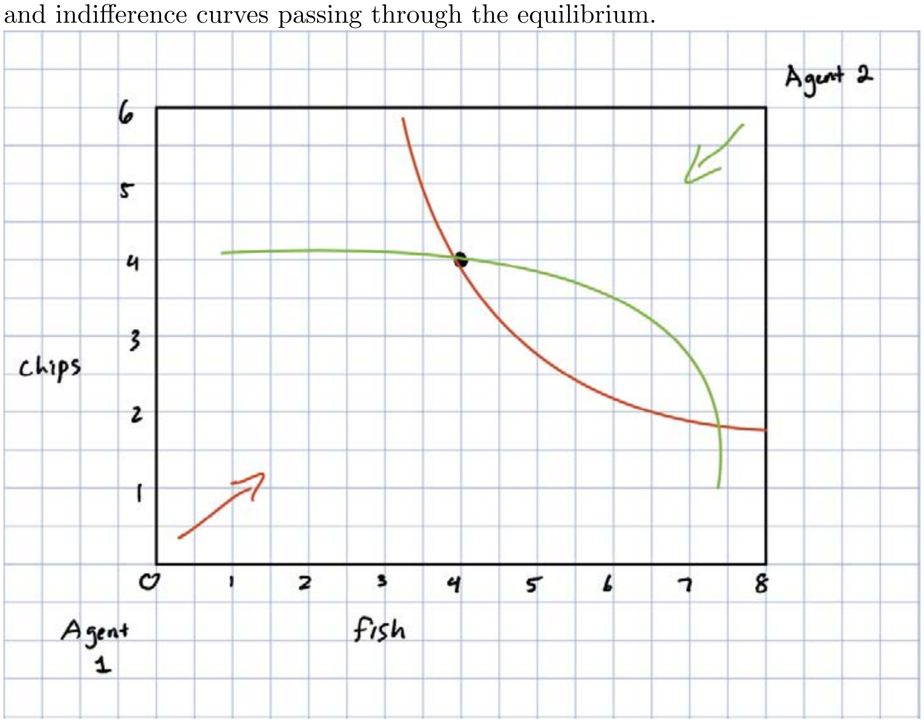
$$x_{1f}^* + x_{2f}^* = \omega_{1f} + \omega_{2f} = 4 + 4 = 8$$
$$\Rightarrow \frac{4p_f + 4p_c}{2p_f} + \frac{4p_f + 2p_c}{4p_f} = 8$$

$$2 + \frac{4p_c}{2p_f} + 1 + \frac{2p_c}{4p_f} = 8$$
$$\Rightarrow \frac{p_c}{p_f} = 2$$

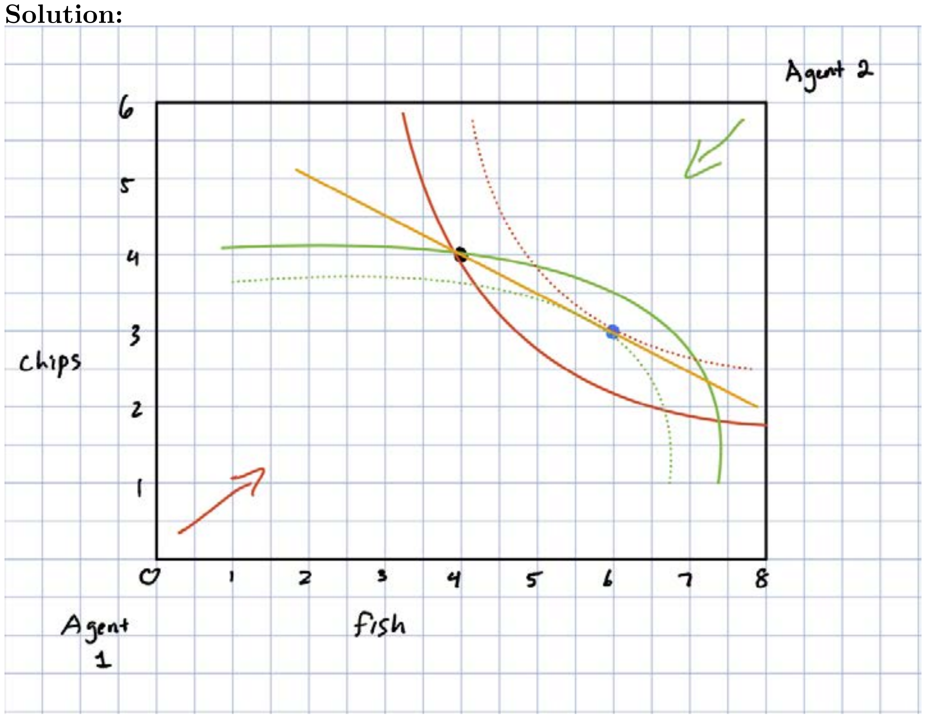
Plugging this back into the demand functions, we can find the allocation values.

$$x_{1f}^* = 2 + \frac{4p_c}{2p_f} = 6 \quad x_{1c}^* = 2 + \frac{4p_f}{2p_c} = 3$$
$$x_{2f}^* = 1 + \frac{2p_c}{4p_f} \approx 2 \quad x_{2c}^* = \frac{3}{2} + \frac{3p_f}{p_c} = 3$$

d) (4 pts) Add to the Edgeworth box below the equilibrium point, price vector, and indifference curves passing through the equilibrium.



Solution:



e) (4 pts) Are both agents made better off by exchange? How do you know?

Solution: Yes the indifference curves in the Edgeworth Box are higher for both agents.

4 Optimal risk-sharing (12 points)

There is an economy with two consumers 1 and 2, and one consumption good c. There are two states of the world: boom B, which happens with probability π_B, and recession R which happens with probability $\pi_R = 1 - \pi_B$.

Consumer i's endowments of consumption good in each state s, denoted ω_s^i, are as follows:

	State	
Consumer	B	R
1	10	5
2	5	1

The aggregate endowment in each state s is denoted $\overline{\omega}_s = \omega_s^1 + \omega_s^2$.

Both consumers maximize expected utility with a von Neumann-Morgenstern utility function $u(c) = \ln c$. In other words, each consumer i maximizes

$$U(c_B^i, c_R^i) = \pi_B \ln c_B^i + \pi_R \ln c_R^i$$

where c_s^i denotes i's consumption in state $s \in \{B, R\}$.

a) (4 pts) Set up the planner's problem for the Pareto optimal allocation of consumption given Pareto weights λ_1, λ_2 such that $\lambda_1 + \lambda_2 = 1$.

Solution:

$$\max_{c_s^i} \sum_i \lambda_i U(c_B^i, c_R^i)$$

subject to

$$c_B^1 + c_B^2 = \overline{\omega}_B$$
$$c_R^1 + c_R^2 = \overline{\omega}_R$$

b) (6 pts) Solve for the Pareto-optimal risk-sharing rule. In other words, derive the Pareto-optimal individual consumption levels in each state s as a function of λ, π and ω.

Solution: Use Lagrange method. Take FOCs. Find that $c_s^i / c_s^j = \lambda_i / \lambda_j$. Combining with resource constraints, we get that $c_s^i = \lambda_i \overline{\omega}_s$.

c) (2 pts) The optimal risk-sharing rule insures against aggregate risk. True or false? Briefly explain.

Solution: False. The optimal consumption bundle depends on aggregate endowment in the realized state of the world. The optimal risk-sharing rule insures only against idiosyncratic risk, since the optimal bundle does not depend not individual endowment after accounting for the aggregate endowment.