Problem Set 1

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1 Problem 1: Preference Relations and Utility Functions

a) Let $X = \mathbb{R}^2_+$ and there be two points $x = (x_1, x_2)$, $y = (y_1, y_2)$.
Suppose $x \succeq y$ if $x_1 > y_1$ or if $x_1 = y_1$ and $x_2 \geq y_2$.
Is the preference relation $\succeq$ complete? Transitive? Why or why not?

b) John has preferences over consumption bundles $(A, B) \in \mathbb{R}^2_+$ characterized by utility function $U(A, B) = A^{\frac{1}{3}} B^{\frac{2}{3}}$. Show that John's preferences satisfy strict monotonicity, local non-satiation, strict convexity, and continuity.

c) Consider the following constrained maximization problem using the utility function from part b)

$$\begin{aligned} \max \quad & U(A, B) = A^{\frac{1}{3}} B^{\frac{2}{3}} \\ \text{s.t.} \quad & p_A A + p_B B \leq I \\ & A \geq 0 \text{ and } B \geq 0 \end{aligned}$$

where $p_A, p_B, I > 0$. Let A^*, B^* denote the solution to the above problem.

i. Can we ever have $A^* = 0$ or $B^* = 0$? Why or why not?

ii) Can we ever have $p_A A^* + p_B B^* < I$? Why or why not?

iii) Set up the consumer's Lagrangian and find the first-order conditions. How do you know that these first-order conditions are sufficient to characterise the solution to the consumer's problem? For what values of p_A, p_B will the consumer consume twice as much A as B?

2 Problem 2: Income and Substitution Effects

A (potential) worker has utility over consumption c and leisure l given by

$$U(c, l) = \alpha \frac{c^\delta}{\delta} + \beta \frac{l^\delta}{\delta}$$

where $\delta < 1$. She has T hours to allocate between leisure and work. For each hour she works, she earns a wage w to spend on consumption c, which we normalize the price of c to one. However, because her wife works, she receives an additional 'non-labor income' Y regardless of how much she works. Assume she takes Y as given (i.e. her own decisions do not affect her wife's labor supply). She therefore maximizes utility subject to the following constraints:

$$\begin{aligned}&c \leq w(T-l)+Y\\&c \geq 0\\&0 \leq l \leq T\end{aligned}$$

a) Without writing down the Lagrangian or solving the optimization problem, identify which constraints above will always bind (hold with equality) at the optimum, and which constraints will always be slack (not hold with equality). Are there any constraints which fall into neither category?

b) Set up the Lagrangian and write out all the relevant conditions for a solution, using your answer to a) to help simplify things.

c) Assume now that the solutions are at an interior point. How do c and l change as non-labor income Y increases? What does this tell us about whether c, l are normal goods?

d) How do c and l change as the wage w increases? Show that your result can be interpreted as income and substitution effects. Note: An intuitive answer will get you most of the points.

3 Problem 3: Production Functions and Feasible Allocations

Recall the Leontief input-output model from lecture 4. Suppose we have two commodities and input-output matrix given by

$$A = \begin{bmatrix} .2 & .7 \\ .6 & .1 \end{bmatrix}$$

Specifically, producing one unit commodity 1 costs .2 units of commodity 1 and .7 units of commodity 2, and producing one unit commodity 2 costs .6 units of commodity 1 and .1 units of commodity 2

a) Suppose John has a demand vector given by $D = [3, 1]$. Find the production vector $X = [X_1, X_2]'$ that satisfies this demand.

b) Now suppose John has a utility function given by $U_J(Y_1, Y_2) = \alpha Y_1 + \beta Y_2$ where $\alpha, \beta > 0$. Characterize the set of production vectors X that gives John a utility of $V > 0$. (Hint: this will be a linear equation of X_1 and X_2 in terms of α, β, and V)

c) Suppose Sally does not like it when X_2 is produced in either too much or too little quantity. Specifically, Sally's utility is given by $U_S(X_2) = -\gamma|X_2 - \overline{X}|$

where $\gamma > 0$. Find the production vector X^* that maximizes Sally's utility subject to keeping John's utility constant at V. (Hint: you should not use any calculus to solve this problem)

4 Problem 4: Giffen Good

See the compressed folder on the Giffen Good exercise. Please recreate the results and include screenshots of your Stata output in your submission.