Problem Set 2

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Prof: Robert Townsend

TA: Laura Zhang and Michael Wong

1 Uncertainty and Risk Aversion

A farmer believes that there is a 50-50 chance that the next growing season will be abnormally rainy. His expected utility function has the form

$$U(Y_{NR}, Y_R) = \frac{1}{2}\frac{Y_{NR}^{1-\theta}}{1-\theta} + \frac{1}{2}\frac{Y_R^{1-\theta}}{1-\theta}$$

where Y_{NR} and Y_R represent the farmer's income in the states of "normal rain" and "rainy" respectively.

a) Let $\theta = 1/2$. Suppose the farmer must choose between two crops that promise the following income prospects

Crop	Y_{NR}	Y_R
Wheat	\$28,000	10,000
Corn	19,000	15,000

Which of the crops will he plant?

b) Suppose the farmer can plant half his field with each crop. This means that in the Y_{NR} state he will earn \$14,000+9,500=23,500 and in the Y_R state he will earn \$5,000+7,500=12,500. Would he choose to do this?

c) What mix of wheat and corn would provide maximum expected utility to this farmer?

d) Let $\theta = 3/4$. How would your answer to (c) change in this case?

2 Pareto Optimality

Suppose individuals denoted by i have happiness functions that only depend on one's own consumption, called $u_i(x_i)$ where x_i is a vector of one's own consumption. However, individuals are altruists in that they care about other people's happiness also. Let I be the total number of people. Each person's overall utility function is $U_i(x_1, \ldots, x_I)$ where U_i has the form

$$U_i(x_1, \ldots, x_I) = U_i(u_1(x_1), u_2(x_2), \ldots, u_I(x_I))$$

Here, individual i's utility depends on all the other people's happiness functions $u_1(x_1), u_2(x_2), \ldots, u_I(x_I)$.

a) Show that if consumption allocation $x = (x_1, \ldots, x_I)$ is Pareto optimal under altruistic utility function $U_i(\cdot)$, then allocation x is also Pareto optimal under the individualistic utility function $u_i(\cdot)$ (where individuals only care about their own happiness).

b) Does this mean that a community of altruists can use standard competitive markets to attain Pareto optimality?

3 Dynamic Programming

Consider a farmer who lives for T periods. In each period t, the farmer chooses how much to consume (c_t) and how many seeds to plant to be available in the next period (K_{t+1}). In particular, consumption at date t is given by

$$c_t = \alpha K_t - K_{t+1}$$

where $\alpha > 0$ is the yield to seed ratio. The farmer then solves the problem

$$\max_{\{K_{t+1}, c_t\}_{t=1}^{T}} \quad \sum_{t=1}^{T} \beta^{t-1} u(c_t) \tag{1}$$

$$s.t.$$

$$c_t = \alpha K_t - K_{t+1}$$

$$c_t \geq 0 \tag{2}$$

where $u(c_t)$ is the utility the farmer gets from consumption in period t, $\beta \in (0,1)$ and $K_1 > 0$ is given exogenously (e.g. seeds the farmer inherited when he was born). Let $\{K_{t+1}^*, c_t^*\}_{t=1}^{T}$ solve the above problem. That is, the farmer picks a sequence of consumption and seed-planting decisions in each period to solve the above problem.

a) Argue that, regardless of the value of T, we will always have that $K_{T+1}^* = 0$.

b) Now let's look at the case where $T = 2$. Use the fact that $K_3^* = 0$ and the constraints to eliminate c_t and write the farmer's problem as an optimization problem in one variable only, K_2. What constraint must K_2 satisfy?

c) Suppose the farmer's per-period utility takes the form $u(c_t) = c_t$. Solve for the farmer's optimal planting decision, K_2^*. (Hint: there will be two cases, depending on the values of α and β). What is consumption in each period?

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