1 Uncertainty and Risk Aversion

A farmer believes that there is a 50-50 chance that the next growing season will be abnormally rainy. His expected utility function has the form

\[ U(Y_{NR}, Y_R) = \frac{1}{2} Y_{NR}^{1-\theta} + \frac{1}{2} Y_R^{1-\theta} \]

where \( Y_{NR} \) and \( Y_R \) represent the farmer’s income in the states of "normal rain" and "rainy" respectively.

a) Let \( \theta = 1/2 \). Suppose the farmer must choose between two crops that promise the following income prospects

<table>
<thead>
<tr>
<th>Crop</th>
<th>( Y_{NR} )</th>
<th>( Y_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>$28,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Corn</td>
<td>19,000</td>
<td>15,000</td>
</tr>
</tbody>
</table>

Which of the crops will he plant?

b) Suppose the farmer can plant half his field with each crop. This means that in the \( Y_{NR} \) state he will earn \$14,000+9,500=23,500 and in the \( Y_R \) state he will earn \$5,000+7,500=12,500. Would he choose to do this?

c) What mix of wheat and corn would provide maximum expected utility to this farmer?

d) Let \( \theta = 3/4 \). How would your answer to (c) change in this case?

2 Pareto Optimality

Suppose individuals denoted by \( i \) have happiness functions that only depend on one’s own consumption, called \( u_i(x_i) \) where \( x_i \) is a vector of one’s own consumption. However, individuals are altruists in that they care about other people’s happiness also. Let \( I \) be the total number of people. Each person’s overall utility function is \( U_i(x_1, \ldots, x_I) \) where \( U_i \) has the form

\[ U_i(x_1, \ldots, x_I) = U_i(u_1(x_1), u_2(x_2), \ldots, u_I(x_I)) \]

Here, individual \( i \)’s utility depends on all the other people’s happiness functions \( u_1(x_1), u_2(x_2), \ldots, u_I(x_I) \).
a) Show that if consumption allocation $x = (x_1, \ldots, x_I)$ is Pareto optimal under altruistic utility function $U_i(\cdot)$, then allocation $x$ is also Pareto optimal under the individualistic utility function $u_i(\cdot)$ (where individuals only care about their own happiness).

b) Does this mean that a community of altruists can use standard competitive markets to attain Pareto optimality?

3 Dynamic Programming

Consider a farmer who lives for $T$ periods. In each period $t$, the farmer chooses how much to consume ($c_t$) and how many seeds to plant to be available in the next period ($K_{t+1}$). In particular, consumption at date $t$ is given by

\[ c_t = \alpha K_t - K_{t+1} \]

where $\alpha > 0$ is the yield to seed ratio. The farmer then solves the problem

\[
\max_{\{K_{t+1}, c_t\}_{t=1}^T} \sum_{t=1}^T \beta^{t-1} u(c_t) \tag{1}
\]

s.t.
\[
c_t = \alpha K_t - K_{t+1} \tag{2}
\]
\[
c_t \geq 0
\]

where $u(c_t)$ is the utility the farmer gets from consumption in period $t$, $\beta \in (0, 1)$ and $K_1 > 0$ is given exogenously (e.g. seeds the farmer inherited when he was born). Let $\{K_{t+1}^*, c_t^*\}_{t=1}^T$ solve the above problem. That is, the farmer picks a sequence of consumption and seed-planting decisions in each period to solve the above problem.

a) Argue that, regardless of the value of $T$, we will always have that $K_{T+1}^* = 0$.

b) Now let’s look at the case where $T = 2$. Use the fact that $K_3^* = 0$ and the constraints to eliminate $c_t$ and write the farmer’s problem as an optimization problem in one variable only, $K_2^*$. What constraint must $K_2^*$ satisfy?

c) Suppose the farmer’s per-period utility takes the form $u(c_t) = c_t$. Solve for the farmer’s optimal planting decision, $K_2^*$. (Hint: there will be two cases, depending on the values of $\alpha$ and $\beta$). What is consumption in each period?
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