Problem Set 5

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1 Walrasian Equilibrium and Welfare Theorems

There are two consumers, both with Leontief preferences for two goods x, y in an exchange economy. The consumers' utility functions are

$$U_1(x_1, y_1) = \min\{x_1, y_1\}$$

$$U_2(x_2, y_2) = \min\{4x_2, y_2\}$$

and the endowments are such that consumer 1 has only 30 units of good x, and consumer 2 has only 20 units of good y. Assume that prices $p_x, p_y \in [0, \infty)$ and that agents can only have non-negative consumption.

a) Draw an Edgeworth box with the endowment point and the two agents' indifference curves going through the endowment point.

b) What are the competitive equilibria in this economy? *Hint:* prices can be 0. Redraw these competitive equilibria on your Edgeworth box. Are the equilibria also Pareto optimal allocations?

c) Suppose that consumer 1 instead has an endowment of 10 units of good x and again no units of good y. What are the competitive equilibria in this case? Draw these competitive equilibria on a new Edgeworth box. Be sure to find all the equilibrium allocations.

d) Now suppose that there is a social planner who can pick an endowment allocation vector $\omega_i = (\omega_{xi}, \omega_{yi}) \geq 0$ for agents $i \in \{1, 2\}$. The social planner is restricted by the economy's resource constraints. In particular, suppose we have the total resources as in (c) so that $\omega_{x1} + \omega_{x2} = 10, \omega_{y1} + \omega_{y2} = 20$. On your Edgeworth box from part c) show the set of all allocations that can be supported as a competitive equilibrium for some endowment allocation.

2 Existence

Consider an exchange economy with three individuals and two goods. Each individual has the utility function

$$U^i(x, y) = x^2 + y^2$$

and has an endowment $(1, 1)$.

a) Are these preferences convex?

b) Calculate the demands of each individual when they face prices $(1, p)$.

c) Show that there cannot be a Walrasian equilibrium with $p \neq 1$.

d) Show that in fact, there does not exist any Walrasian equilibrium for this economy[1]. Where does Negishi's proof break down?

3 Quasi-Linear Utility and Gorman Form

Consider a consumer with quasi-linear preferences over two consumption goods X and Y given by $U(X, Y) = X + \ln(Y)$. Assume that the consumer has wealth w and that consumption of both goods must be weakly positive, i.e. $(X, Y) \in \mathbb{R}^2_+$.

a) Write down the utility maximization problem for the consumer and the first-order conditions for the optimal consumption bundle (X, Y).

b) Show that the consumer's demand functions are given by

$$\begin{aligned} X(p_X, p_Y, w) &= \frac{w}{p_X} - 1 \\ Y(p_X, p_Y, w) &= \frac{p_X}{p_Y} \end{aligned}$$

You can assume an interior solution.

c) Compute indirect utility $V(p_X, p_Y, w)$.

d) What does it mean for an individual's preferences to satisfy Gorman form? Does the consumer's utility satisfy Gorman form?

[1] Assume lotteries are not possible - each consumer consumes a non-random amount.