

Lecture 12: Technological Change and Inequality

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Technological change and income inequality

- Recall that technological change/growth creates winners & losers.
Some of the effects are idiosyncratic (like Apple-Blackberry). Some of the effects are more systematic.
- This lecture: discussion of the systematic effects on workers
 - Main focus: gap between **skilled and unskilled labor**. Discuss skill-biased technological change as one natural explanation.
The “skill” dimension here is educational attainment. High school vs. college.
 - Then: focus more narrowly on *top* income shares. Will require explanations that go beyond skill-biased technological change.
 - Finally: speculation and early evidence regarding the labor market effects of generative AI
- Overall objective: give you some sense of recent trends in inequality & likely explanations, and a framework to think about possible future developments

Outline

1. History of labor income inequality and skill premia
2. A theory of the history of labor income inequality
 - Skill-biased technological change
 - Measuring technical change
 - Endogenous bias in technological change
3. Recent inequality trends beyond the skill premium
4. The labor market effects of AI

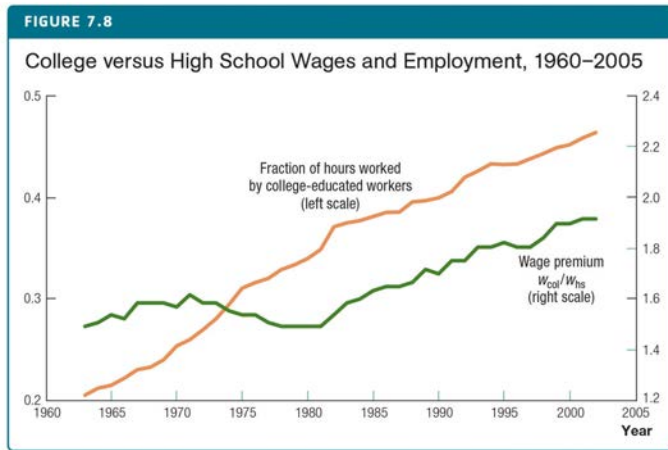
Wage inequality



Figure 2. Changes in the Indexed Value of the 90th, 50th, and 10th Percentiles of the Wage Distribution for White Males (1963 values normalized to 100). Data from March CPSs.

Fact 1A: wage inequality has been increasing over the past decades

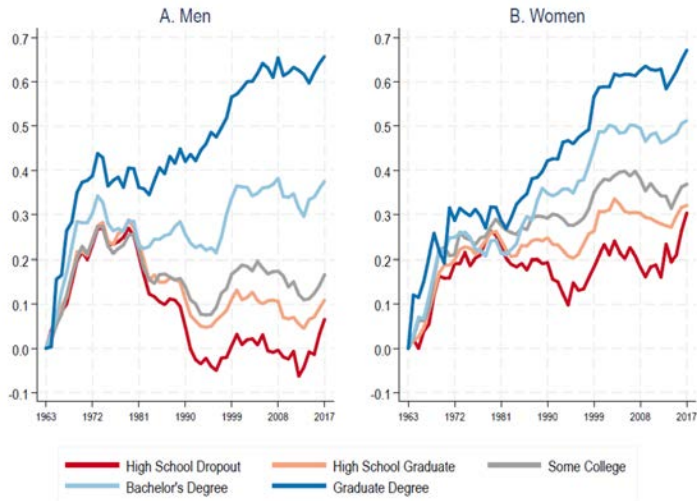
Earnings and educational attainment



Fact IB: wage inequality reflects college/non-college divide

Fact II: average educational attainment has been trending up

Earnings and educational attainment



Rising wage premium contributes to inequality

- Together these observations seem a bit puzzling.
 - The price of college (skilled) labor = wage has been rising. But the supply has also been rising at the same time.
 - Normally the price of a good with high supply will go down.
- This must mean: “demand” must have moved by even more. Today we’ll investigate one explanation: skill-biased technological change.
- Let’s formalize this argument using our tools.

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Skilled and unskilled factors

- Abstract from capital (for simplicity), and suppose there are two types of labor denoted by $H(t)$ and $L(t)$:
 - $H(t)$ denotes skilled (college educated) labor
 - $L(t)$ denotes unskilled (high-school educated) labor
- Furthermore suppose technological change comes in two varieties:
 - $A_H(t)$: skill-augmenting technology
 - $A_L(t)$: unskill-augmenting technology
- We can then produce as if we have $A_H(t) H(t)$ and $A_L(t) L(t)$ factors.

Aggregate production function

- We will consider an aggregate production function given by:

$$Y = F(A_H H, A_L L) = \left[(A_H H)^{\frac{\sigma-1}{\sigma}} + (A_L L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

Skilled and unskilled labor are now combined to produce total output. σ captures the **elasticity of substitution between the two factors**.

- Special cases:

1. If $\sigma = \infty$, then skilled-unskilled labor are perfect substitutes: $Y = A_H H + A_L L$

Can increase output by raising one skill or another.

2. If $\sigma = 0$, then skilled-unskilled labor cannot be substituted: $Y = \min(A_H H, A_L L)$

To raise output, need to raise both factors in proportion.

3. If $\sigma = 1$, then we have Cobb-Douglas function (in between): $(A_H H)^{1/2} (A_L L)^{1/2}$

- For high-low skill, empirical studies typically find $\sigma \in (1.5, 2)$, so skill seems somewhat substitutable (more than Cobb-Douglas).

Wages for high and low skill labor

- With competitive markets, factors are paid their marginal products. The two wages in this case are:

$$w_H = \frac{\partial Y}{\partial H} = \dots \text{ and } w_L = \frac{\partial Y}{\partial L} = \dots$$

- For present purposes, the key is the ratio of these wages. This is what we call **the skill premium** and this is what was plotted on the earlier graph from the data.
- The skill premium can be calculated as [do it!]

$$\underbrace{\omega = \frac{w_H}{w_L}}_{\text{skill premium}} = \overbrace{\left(\frac{A_H}{A_L}\right)^{\frac{\sigma-1}{\sigma}}}^{\text{skill-bias in technological change}} \underbrace{\left(\frac{H}{L}\right)^{-1/\sigma}}_{\text{relative supply of skill}}$$

Determinants of the skill premium

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- If we think of the firms choosing the ratio H/L , we can read the above as a downward sloping demand curve: gives *relative* demand for H/L as decreasing function of ω .
If $\sigma > 1$: demand curve shifts up with higher A_H/A_L , so skill-based technological change $\rightarrow$ relative demand for skilled labor goes up.
- In eq'm, H/L is given by the relative supply of the two factors. So, once we impose market-clearing, the same equation can be re-read as determining ω as:
 1. a decreasing function of the relative supply of skill
 2. an increasing function of the relative level of skill-based technology

Determinants of the skill premium

- Now imagine possible changes to supply and demand:
 - What happens to ω as the supply H/L increases, given A_H/A_L ?
 - What happens to ω as technology advances so as to augment skill relatively more, so A_H/A_L increases, taking H/L as given?

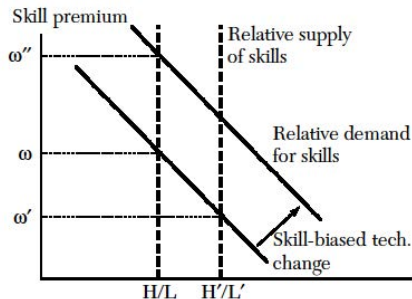


Figure 4. The Relative Demand for Skills

Determinants of the skill premium

- All else equal, greater supply of H/L reduces the premium ω .
- But: all else equal, **skill-biased technological change**, i.e., increase in A_H/A_L , increases the demand for skill and the skill premium ω .
- How can we use this analysis to explain the comovement of ω and H/L in the 20th and early 21st century?
 - From the lens of this model, as H/L was increasing (e.g., due to greater schooling), A_H/A_L must also be increasing alongside sufficiently fast to overturn the supply effect.
 - This explanation seems intuitive, but raises two further questions:
 1. Was A_H/A_L really increasing? Is there any direct evidence?
 2. Why was A_H/A_L increasing? By chance? Or by purpose?

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Effect of computerization on tasks

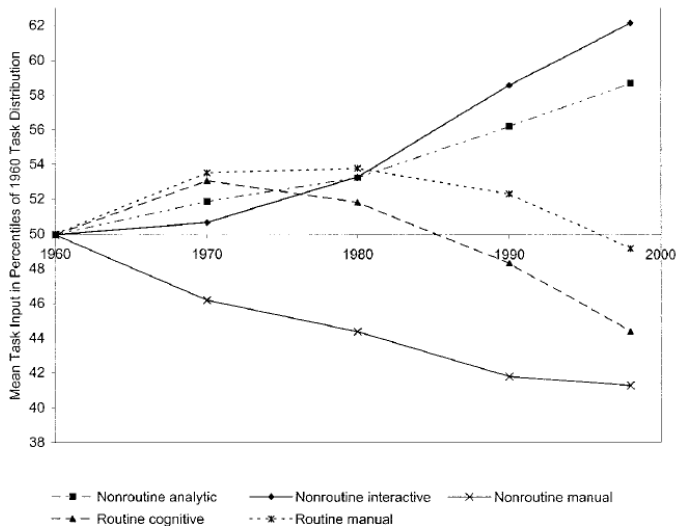
- A major recent technological change is computerization.
- Autor, Levy, and Murnane [QJE, 2003] explore effect on **tasks**.
 - Computers are likely to substitute/replace certain tasks, and complement/enhance others.
 - This in turn will tend to increase the wage of labor doing the complementary tasks relatively more than the wage of labor doing substitutable tasks.
 - Note the distinction between jobs and tasks – a job consists of many tasks. Will return to this when thinking about labor market effects of AI.
- But what are those complementary and substitutable tasks?

Effect of computerization on tasks

PREDICTIONS OF TASK MODEL FOR THE IMPACT OF COMPUTERIZATION ON FOUR CATEGORIES OF WORKPLACE TASKS

	Routine tasks	Nonroutine tasks
	Analytic and interactive tasks	
Examples	• Record-keeping • Calculation • Repetitive customer service (e.g., bank teller)	• Forming/testing hypotheses • Medical diagnosis • Legal writing • Persuading/selling • Managing others
Computer impact	• Substantial substitution	• Strong complementarities
	Manual tasks	
Examples	• Picking or sorting • Repetitive assembly	• Janitorial services • Truck driving
Computer impact	• Substantial substitution	• Limited opportunities for substitution or complementarity

Effect of computerization on tasks



Effect of computerization on tasks & wages

- The available evidence on tasks suggest that computers have shifted labor away from routine tasks toward abstract nonroutine tasks.
 - So workers who were better in doing nonroutine tasks may have benefited, and workers who were better in doing routine tasks may have lost.
 - I.e., the "nonroutine premium" has gone up.
- But how does this translate to the skill premium?
 - Abstract nonroutine (high-skill) and manual nonroutine (low-skill) jobs are at the opposite ends of the skill spectrum.
 - This suggests computerization might lead to polarization: U-shaped effect on wages.
- Evidence: Autor (2015) finds that there was indeed a U-shaped effect on wages, but with a smaller rise at the bottom of the skill distribution.

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Reasons for skill-biased technological change

- Available evidence supports skill-biased change.
- But **why** has technological change been skill-biased?
 - Perhaps by chance, e.g., scientific developments made computerization possible in late 20th century but not earlier.
 - Part of the story but not the full story.

Reasons for skill-biased technological change

- As we discussed before, some technological change in practice is purposeful, e.g., driven by costly research.
 - Even if the fundamental insights come from basic science, commercial applications likely driven by profit incentives.
- What are some important forces, when we think about whether technology complements high or low skill?
 - Consider a tool (e.g., software) that makes skilled labor more productive.
 - Where would this tool make more profits: in the US with a lot of educated people, or in another country with less education?

Market size effect

“The development of skill-biased technologies will be more profitable when they have a larger market size—i.e., when there are more skilled workers.” —Acemoglu (2002)

- Intuition: develop skill-biased technologies if there are lots of workers to adopt them
 - This may help explain why a lot of the skill-biased technological change has been happening in the US and other rich countries
 - Or why international trade (by enlarging the relevant markets) may have further accelerated the trend
- In fact this argument even works for past historical episodes
 - 19th century: artisan shop was replaced by the factory
 - Background: large migration of unskilled workers from villages and Ireland to English cities

Taking stock

- So the argument in a nutshell:
 - Supply of skill in 20th century increased due to greater schooling. All else equal, this would drive down the wage premium for skill.
 - The comovement of ω and H/L —as well as micro evidence—suggest that technology has been biased toward skill.
 - Endogenizing the direction of technology—due to the market size effect—can explain some of this bias toward skill.
- Returning to our initial question:
 - Skill-biased change—rising ω —arguably contributed to rising inequality by increasing the income of college graduates vs. non-college.
 - But inequality has also been rising at the very top of the income distribution, where pretty much everyone has a college degree ...

Outline

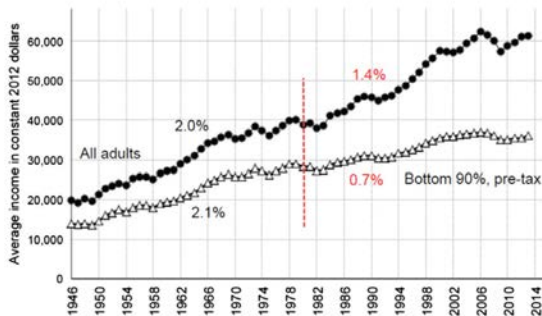
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Inequality beyond the skill premium

- So far we explained how technological change may generate winners and losers, thus contributing to inequality
- Other forces—international trade, immigration, regulation ...—are other examples of forces that may be good on average (help boost growth/TFP on the aggregate level) but may also be contributing to inequality.
- Let's dig a bit deeper into trends in inequality at the top ...

Inequality in the US: top 10% income share

Growth in U.S. Real Average National Income: Full Adult Population versus Bottom 90%

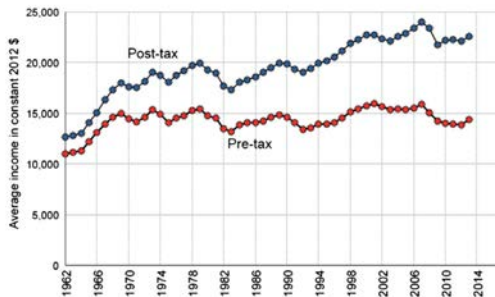


Note: Pre-tax income and post-tax income match total national income and are divided equally among spouses.
Source: Piketty, Saez, and Zucman (2016). Reprinted with permission.

- There has been a “regime change” around the early 1980’s:
 - Before: incomes of top 10% and bottom 90% growing at similar rates
 - After: incomes of top 10% have been growing considerably faster

Inequality in the US: bottom 50% income

Stagnating U.S. Real Income of Bottom 50% Adults: Pre-tax vs. Post-tax



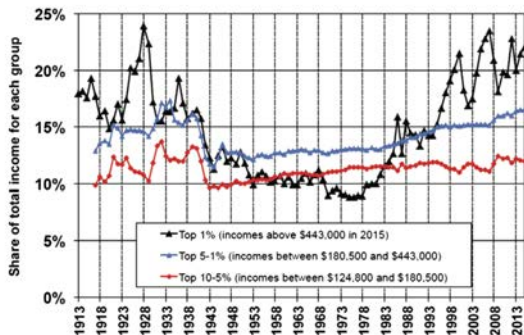
Note: Real values are obtained by using the national income deflator and expressed in 2012. Income is divided equally among spouses. Post-tax is income net of all taxes and adding all transfers and government spending.

Source: Piketty, Saez, and Zucman (2016). Reprinted with permission.

- In fact, bottom 50% has been stagnating for even longer time, despite the fact that the US as whole has been doing quite well
- This is a worrisome fact, and also an example of how just looking at averages could be hiding important trends.

Inequality in the US: top 1% income

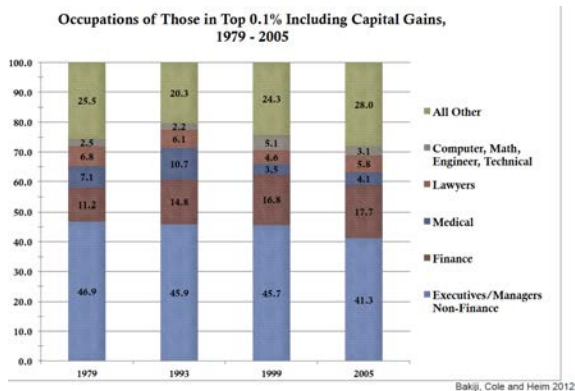
Decomposing the U.S. Top 10% Pre-tax Income into Three Groups, 1913–2015



Note: Series based on pre-tax cash market income including realized capital gains and excluding government transfers.
Source of data: Alvaredo et al. (2016). An earlier version of the data and figure appears in Piketty and Saez (2003).
Source of figure: Atkinson, Piketty and Saez (2011). Updated to 2015. Reprinted with permission.

- Vast majority of top-10% gains are concentrated among the top 1%.
- And if we look even deeper, we will see an even more dramatic increase in the share of income (and wealth too) of the top 0.1%.

Inequality in the US: who are the top 0.1%?



- The decomposition of the top 0.1% in terms of occupations has been relatively stable.
- The most highly paid people within these occupations have been experiencing a large increase in their labor income relative to both the less productive ones with the same occupation, as well as relative to people in other, less skilled, occupations.

Explanation: skill-biased technology

- The increase is broad-based, so the explanations that we propose should arguably also be broadly relevant.
- Narrow explanations that focus on a particular profession (CEO salaries) or particular industries (finance) will not be fully satisfactory.
- One such explanation is the “superstar” phenomenon (Rosen, 1982).
 - Basic idea: technology complements skilled labor, as we noted before, but the effects are much more powerful at the top.
 - Technology enables skilled people to scale their activities more easily, which creates “superstars.” What are some examples?

Examples of “stars”

Table 1
Average Pay of Top Hedge Fund Managers and Law Partners

<i>Year</i>	<i>Average pay of top 25 hedge fund managers (Millions of \$2010 [and relative to average adjusted gross income of top 0.1%])</i>	<i>Average profit per partner at top 50 law firms (Millions of \$2010 [and relative to average adjusted gross income of top 0.1%])</i>
1994		\$0.704 [0.268]
1996		\$0.784 [0.219]
1998		\$0.997 [0.200]
2000		\$1.084 [0.167]
2002	\$133.7 [34.6x]	\$1.099 [0.285]
2004	\$289.5 [55.7x]	\$1.286 [0.247]
2006	\$616.2 [90.3x]	\$1.491 [0.218]
2008	\$469.8 [82.1x]	\$1.449 [0.253]
2010	\$882.8 [177.6x]	\$1.557 [0.313]
2012	\$537.2 [115.7x]	

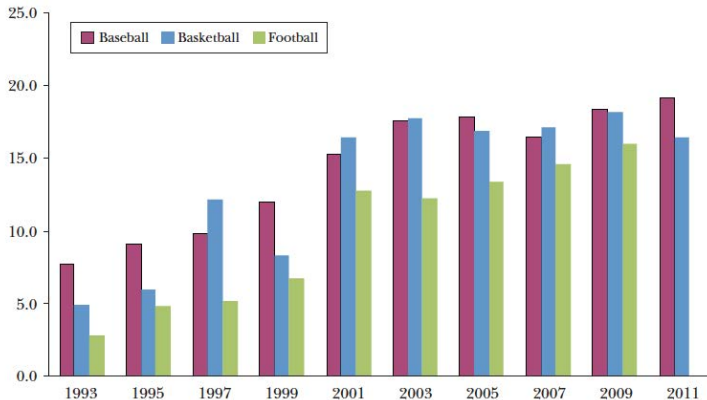
Source: Authors using data from *Absolute Return + Alpha* magazine, *American Lawyer*, and Piketty and Saez (2013).

Examples of “stars”

Figure 3

Average Top 25 Salaries in Professional Baseball, Basketball, and Football

(in millions of 2010 dollars)



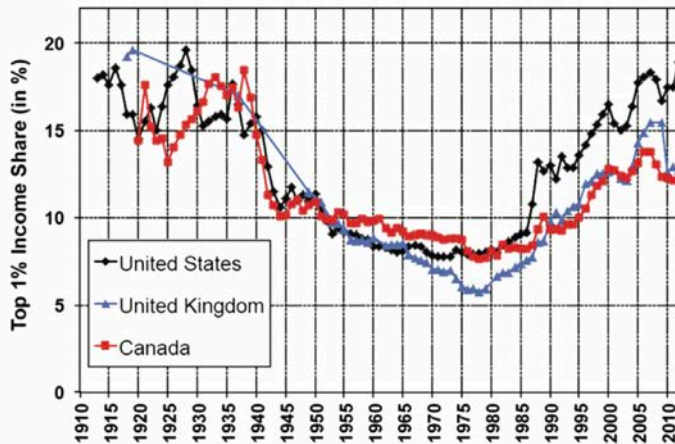
Source: USA Today Sports Salary Database.

Taking stock

- Significant increase in inequality in the last 30-40 years
- Very likely that a large part of it has to do with technological change (computers, robots, ...) and market forces
- But is this the whole story? What about taxation and government policies aimed at “taming” market forces?
 - Now: look at some other advanced countries
 - Why? likely subject to similar market forces as the US but have different degrees of government intervention

Other advanced countries

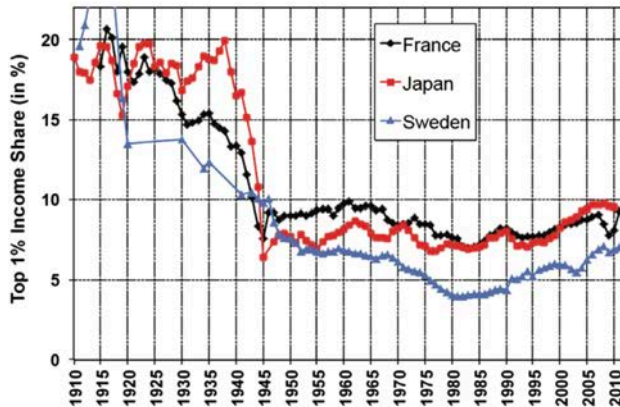
Top 1% Income Share: English Speaking Countries (U-shaped)



Source of data: Alvaredo et al. (2016).

Other advanced countries

Top 1% Income Share: Continental Europe and Japan (L-shaped)



Source of data: Alvaredo et al. (2016).

Top income inequality in some other countries

- Skill-biased technical change/superstars likely to be applicable to both sets of countries.
- This suggests that these global effects might be interacting with local features (policies?) to determine the ultimate outcomes on inequality.
- Recent research emphasizes the role of taxes and more generally of regulation and government intervention.

US tax policy

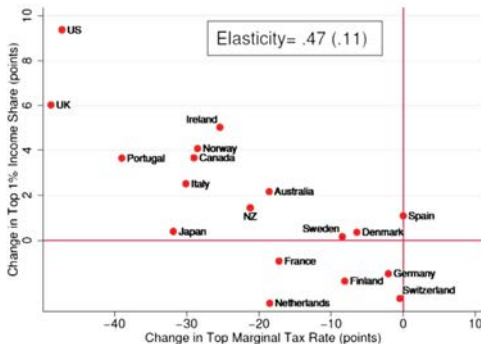


Source of data: Alvaredo et al. (2016). An earlier version of the data and figure appears in Piketty and Saez (2003). Top MTR data source is the Tax Policy Center.

- In the US, there has been a significant reduction in the tax rates of the top income earners (Reagan's tax cuts, more recently Trump's tax cuts).

Tax changes and income share changes

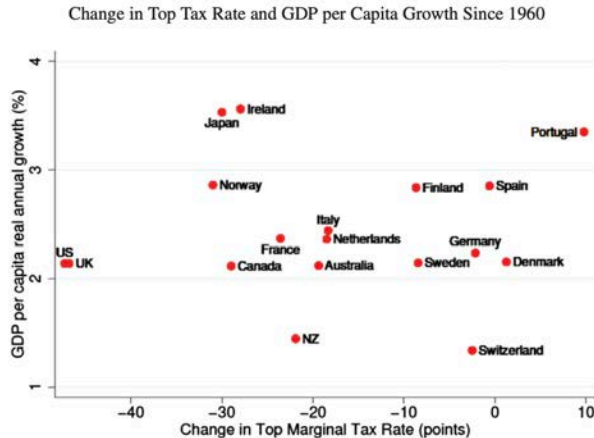
Change in Top 1% Pre-tax Income Share and Change in Marginal Tax Rates
from 1960–1964 to 2005–2009



Source: Piketty, Saez, and Stantcheva (2014). Reprinted with permission.

- Relation extends to cross section of countries
- Countries that experienced larger drop in tax rates of the top earners also experienced an increase in pre-tax income inequality

Tax changes and growth rate changes



Source: Piketty, Saez, and Stantcheva (2014). Reprinted with permission.

- Interestingly, no relation between changes in taxes and growth rates.

Taking stock

- Labor (and overall) income inequality is rising in recent years.
 1. Skill-biased technological change provides one explanation.
 - Part of the skill-bias could be endogenous (market size effect).
 - Skill-biased technology/superstars also relevant at the top.
 2. But other factors, including taxation, government intervention and market concentration, have also played a role.
- Lively research agenda at the intersection of macro & labor.
- Next frontier: labor market & inequality effects of **AI**

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The labor market effects of AI

- Question of the day: what will AI do to labor markets?

Background on the next few slides: evidence on fast adoption of AI in industry.

- Our approach

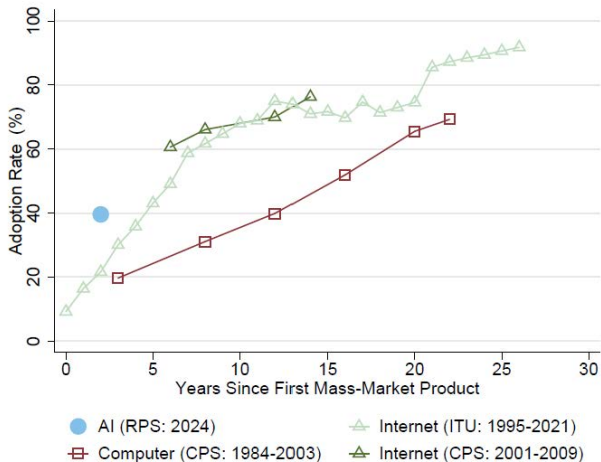
1. **Theory**

- View jobs as bundles of tasks: what tasks are exposed to AI?
- From task exposure to economic implications, role of general equilibrium effects

2. **Evidence:** go through four recent papers that try to evaluate the (very early) labor market impacts of AI

Pace of adoption

(a) Overall Adoption

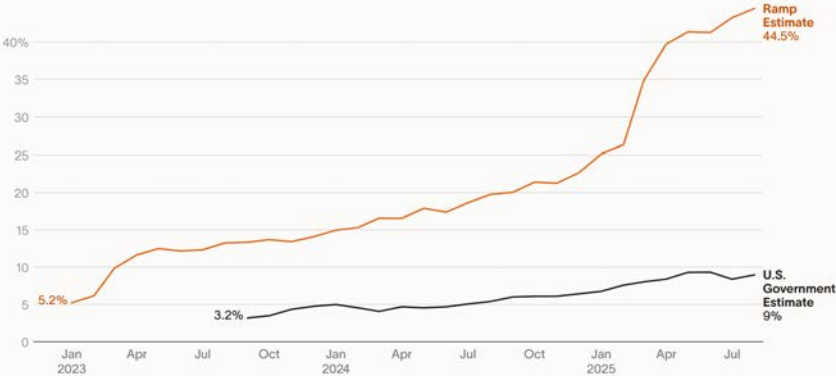


Corporate usage

Ramp AI Index: Overall Adoption Rate

Share of U.S. businesses with paid subscriptions to AI models, platforms, and tools

View by Overall Sector Size Model



Many more studies of usage and productivity

- **Kobak et al. (2025)**

- As of September 2024, 18% of financial consumer complaints, 24% of press releases, 15% of job postings and 14% of UN press releases showed signs of LLM writing, at least 14% of 2024 biomedical research abstracts used LLMs

- **Daniotti et al. (2025)**

- By December 2024 AI wrote 30% of functions in US Github Python commits, 11-24% in other countries
- Analysis suggests “moving to 30% AI use raises quarterly commits by 2.4%”

- **Hartley et al. (2025)**

- By July 2025 45.9% of workers report “using generative AI for their job”, for on average 1/3 of work tasks

Some theory

- Can think of the **direct effect** of AI in terms of a task-based view of jobs
 - Job = bundle of tasks: routine cognitive (e.g., bookkeeping), non-routine cognitive (e.g., statistical analysis), routine manual (e.g., packaging), non-routine manual (e.g., plumbing)
 - AI is different from prior technologies: can handle certain non-routine cognitive tasks.
 - Suggests simple exposure measure: jobs with large non-routine cognitive tasks that can be solved by AI. But misses that task mix in the economy is *endogenous*.
- **General equilibrium**
 - Autor (2015): *“Journalists and even expert commentators tend to overstate the extent of machine substitution for human labor and ignore the strong complementarities between automation and labor that increase productivity, raise earnings, and augment demand.”*
 1. Task complementarity: ↑ labor demand in non-automated tasks in adopting sectors
 2. Automation of low-productivity tasks bundled with other tasks
 3. Goods complementarity: ↑ demand for labor in other sectors

Job transformation in the recent past

- What happened in the past? How did jobs change?
- Finding: shift from routine tasks toward non-routine analytic tasks arose predominantly from changes in task content within occupations rather than shifts in occupations

	Total	Between	Within	Within-share
1986 level	0.252			
1986-1992	0.025	0.002	0.022	0.906
1992-2006	0.298	0.057	0.241	0.809
2006-2012	0.019	0.002	0.017	0.890
2012-2018	0.053	0.028	0.025	0.476
Total change	0.395	0.089	0.306	0.775

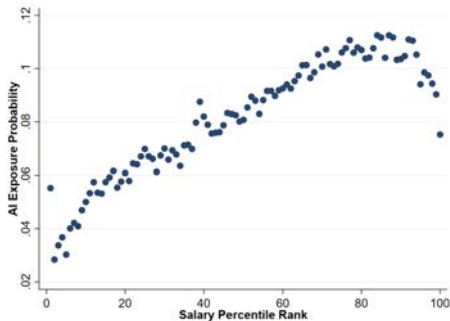
Notes. Freund (2023): “Superstar Teams.” Worker-level surveys for Germany (BIBB). This table reports the within-between occupation decomposition of the change in the share of complex (non-routine analytical) tasks over time.

Job transformation in the distant past

Period	Preparatory tasks		Tasks while machine running							Tasks while power loom stopped							
	Prepare warp	Dress warp	Let off warp	Pick shuttle	Beat reed	Take up cloth	Adjust warp tension	Replace empty bobbin	Monitoring	Fix smashes	Adjust temples	Back up loom	Replace empty shuttle	Fix broken weft	Fix broken warp end	Remove cloth, cleaning	Replace warp
Handloom	•	•	•	•	•	•	•		•		•		•	•	•	•	•
Early power loom (~1820)							•	•	•	•	•	•	•	•	•	•	•
1833							•	•	•	•		•	•	•	•	•	•
1883							○	•	•	•			•	•	•	•	○

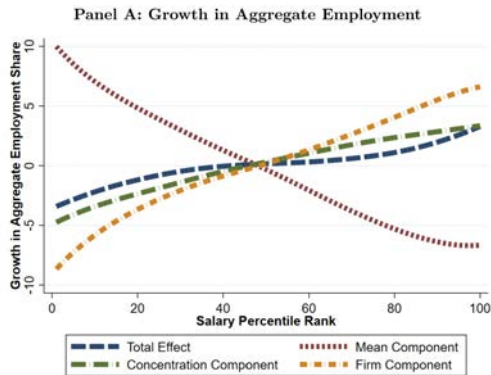
Notes. • = Task performed; ○ = Reduced frequency; Empty = Task not performed.

- **Data:** Revelio Labs job resume data (Linkedin), Compustat firm-level data
- **Steps**
 1. Construct firm-and-task specific measures of (2010-2023 vintage) AI adoption based on AI applications listed on employees' LinkedIn profiles
 2. Task-based model → approximate occupational employment effect of task-level AI exposure based on some measurable “sufficient statistics”
 3. Construct empirical proxies for these sufficient statistics
 4. Estimate impact of firm-level AI adoption on occupation-level labor demand



Note: This figure plots the average task-level probability of exposure to a given AI application by salary rank. We use imputed salaries for each job position to compute the ranks. See Section 2.2 for details.

(a) Exposure

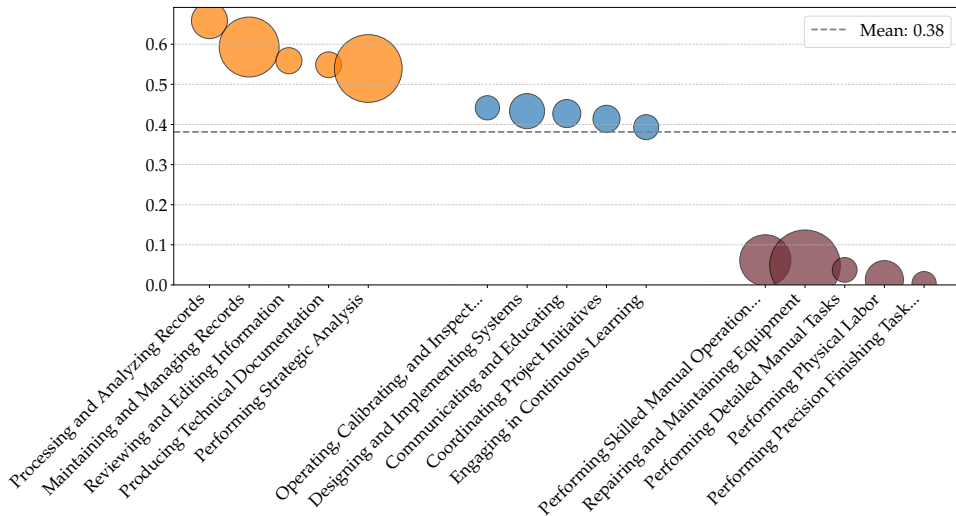


(b) Employment effects

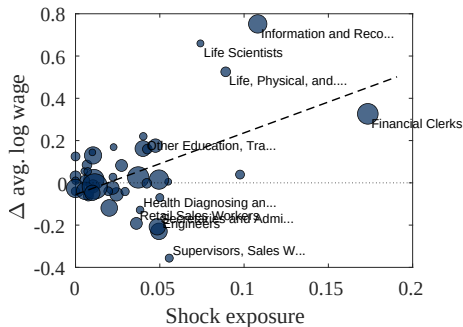
“Overall employment effects are modest, as reduced demand in exposed occupations is offset by productivity-driven increases in labor demand at AI-adopting firms.”

- **Core idea:** automation leads to job *transformation* $\Rightarrow$ who will win and who will lose from AI-induced job transformation?
- **Steps**
 1. Develop task-based model where (i) occupations *bundle* tasks, (ii) workers have heterogeneous portfolios of task-specific skills, (iii) choose occupations
 2. Estimate distribution task-specific skills using NLP tools and structural model
 3. Project LLM-induced job transformation effects given measures of job exposure

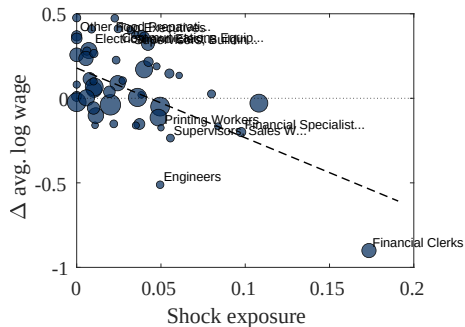
Exposure scores for task clusters



Freund and Mann (2025)

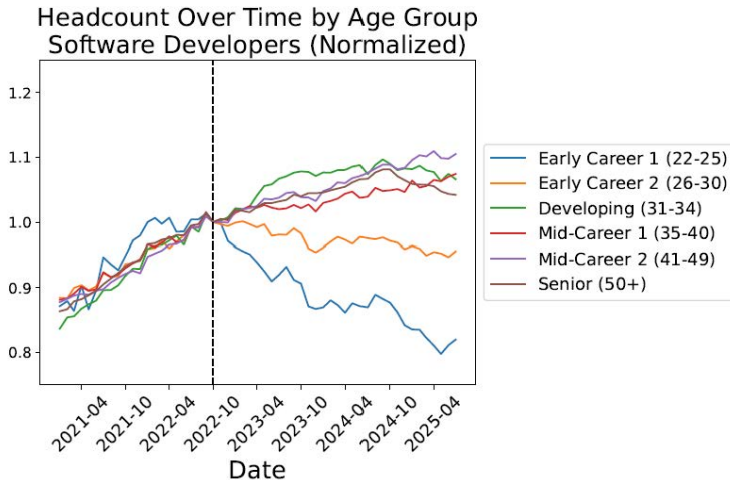


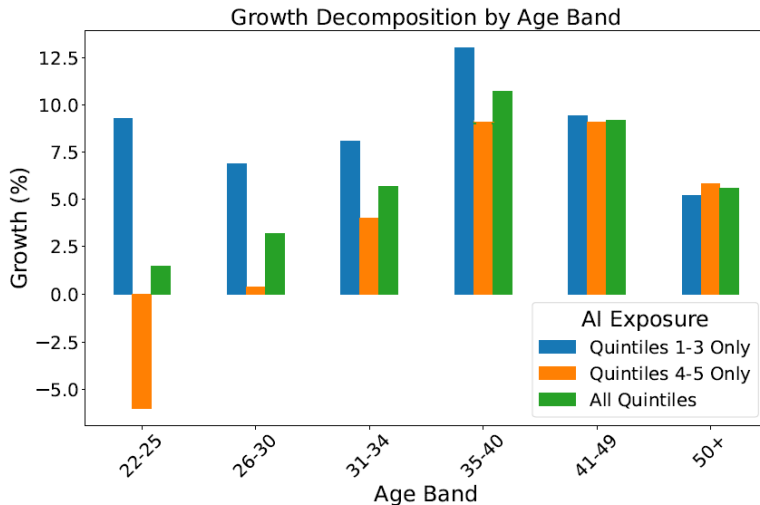
(a) Incumbent stayers



(b) Incumbent leavers

- LLM automation creates large+ heterogeneous wage effects through job *transformation*
 - Workers specialized in information-processing tasks leave & suffer wage losses. Those specialized in customer-facing/coordination tasks stay & enjoy wage gains
- We thus see that task *automation* can actually *augment* workers





Humlum and Vestergaard (2025)

- **Q:** what are the early labor market impacts of AI chatbots based on actual usage?
- **Large-scale data from Denmark**
 - Large-scale, representative adoption surveys of workers in 11 highly exposed occupations
 - Linked to administrative records on monthly earnings, hours worked, and occupations
- **Main findings**
 1. Estimates for earnings, recorded hours, and wages are all precisely estimated zeros
 2. Adopters are more likely to have transitioned into new occupations after the arrival of AI chatbots
 3. Workplaces that encourage chatbot use show no differential changes in employment
- **Conclusion:** “Overall, our findings challenge narratives of imminent labor market disruption from Generative AI. ”

Some takeaways

- Generative AI is the latest automation technology and is being adopted rapidly ...
- ... but, unlike previous technologies, mostly affects white-collar jobs/tasks
 - Exposure measures are useful indicators of “potential for change” but we should not naively interpret them as implying adverse impact
 - Early evidence on labor market impacts paints a mixed picture
- Caveat: “forecasting is hard, especially about the future”

The Internet's impact on the economy has been no greater than the fax machine's...ten years from now, the phrase “information economy” will sound silly.

(Paul Krugman, 1997)

Correction for Lecture Note 12

Christian Wolf

October 22, 2025

Consider the production function

$$Y = \theta(A_H H)^{\frac{\sigma-1}{\sigma}} + (1-\theta)(A_L L)^{\frac{\sigma-1}{\sigma}} \quad \frac{\sigma}{\sigma-1}, \quad \theta \in (0, 1).$$

Relative to the production function considered in the lecture notes, I have added the weighting coefficient θ . In the discussion below I will pay particular attention to $\theta = \frac{1}{2}$.

Let us now consider what happens as $\sigma \rightarrow 1$. Taking logs and re-writing, we get

$$\ln Y = \frac{\sigma}{\sigma-1} \ln \left(\theta(A_H H)^{\frac{\sigma-1}{\sigma}} + (1-\theta)(A_L L)^{\frac{\sigma-1}{\sigma}} \right) = \frac{\ln \left(\theta(A_H H)^{\frac{\sigma-1}{\sigma}} + (1-\theta)(A_L L)^{\frac{\sigma-1}{\sigma}} \right)}{\frac{\sigma-1}{\sigma}}.$$

As $\sigma \rightarrow 1$, numerator and denominator both go to 0, so apply L'Hôpital in σ :

$$\lim_{\sigma \rightarrow 1} \ln Y = \lim_{\sigma \rightarrow 1} \frac{\frac{d}{d\sigma} \ln \left(\theta(A_H H)^{\frac{\sigma-1}{\sigma}} + (1-\theta)(A_L L)^{\frac{\sigma-1}{\sigma}} \right)}{\frac{d}{d\sigma} \frac{\sigma-1}{\sigma}}.$$

Computing derivatives:

$$\lim_{\sigma \rightarrow 1} \ln Y = \lim_{\sigma \rightarrow 1} \frac{\frac{\theta(A_H H)^{\frac{\sigma-1}{\sigma}} \ln(A_H H) + (1-\theta)(A_L L)^{\frac{\sigma-1}{\sigma}} \ln(A_L L)}{\theta(A_H H)^{\frac{\sigma-1}{\sigma}} + (1-\theta)(A_L L)^{\frac{\sigma-1}{\sigma}}} \cdot \frac{1}{\sigma^2}}{\frac{1}{\sigma^2}} = \theta \ln(A_H H) + (1-\theta) \ln(A_L L).$$

Exponentiating yields $\lim_{\sigma \rightarrow 1} Y = (A_H H)^\theta (A_L L)^{1-\theta}$. We thus for $\theta = \frac{1}{2}$ in particular recover

$$Y = (A_H H)^{\frac{1}{2}} (A_L L)^{\frac{1}{2}},$$

as claimed in the lecture notes. Finally note that, for $\sigma \rightarrow 0$ or $\sigma \rightarrow \infty$, the conclusions of the lecture notes apply unchanged, modulo an additional scaling by the factor $\frac{1}{2}$.

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14.05 Intermediate Macroeconomics
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