

Lecture 22: Optimal Monetary Policy Design

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14.05, Fall 2025

Optimal monetary policy design

- This lecture note will dig deeper on (optimal) **monetary policy design**. Two parts:
 1. **Theory**
 - So far worked with simple Taylor rule as description of monetary policy. What monetary policy rule (robustly) implements the social optimum?
 - One more shock: cost-push shocks breaking “divine coincidence”. Which rules still work? Beyond our models: such “cost-push” pressures are exactly what tariffs induce.
 - Can we learn optimal policy rules from the data?
 2. **Application**: monetary policy during the post-covid inflation
 - Was monetary policy well-described by a Taylor rule?
 - What rationale can we give for observed monetary policy?
- At the end: **course summary** and wrap-up

Outline

1. Optimal Monetary Rules & Outcomes

Optimal Monetary Policy Problem

Cost-Push Shocks

Connection to Empirical Evidence

2. Application to Post-Covid Inflation

3. Course Summary

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Central bank objective

- So far we described monetary policy with a simple **Taylor rule**:

$$r_t^n = \phi_\pi \pi_t + \hat{m}_t \quad (1)$$

- Let's now instead suppose that interest rates are set to minimize a **loss function**.
- Natural candidate: “dual-mandate” objective function

$$\mathcal{L} = \frac{1}{2} (\lambda_\pi \pi_t^2 + \lambda_y (y_t - y_t^*)^2) \quad (2)$$

- Notation: λ_π, λ_y are the weights on inflation and output, $y_t^* \equiv \frac{\frac{1}{\varphi} + 1}{\frac{1}{\varphi} + \gamma} \hat{a}_t$ is the RBC (or “natural”) level of output, and $y_t - y_t^*$ is the output “gap”
- The loss function (2) can be justified either from the utility function of our rep. household [recall that the RBC model was efficient] or simply seen as the mandate of the Fed
- The policymaker minimizes (2) subject to constraints. What are the **constraints**?

Optimal monetary policy problem

- Optimal private-sector behavior imposes two restrictions—**demand & supply**:

$$-\gamma y_t = r_t^n - \theta_t \quad (3)$$

$$\pi_t = \kappa(y_t - y_t^*) \quad (4)$$

Note how I've re-written the supply shock as y_t^* . Beyond that it's just our usual NK model.

- Could impose further constraints on r_t^n , e.g., ZLB, or inability to move rates too abruptly. Will abstract from that here.
- The central banker's problem thus is to minimize (2) subject to (3) - (4). This is a simple constrained optimization problem!

Optimality conditions

- Let's compute the **FOCs** with respect to π_t, y_t, r_t^n :

$$\partial \pi_t : \lambda_\pi \pi_t + \varphi_{AS,t} = 0$$

$$\partial y_t : \lambda_y (y_t - y_t^*) - \gamma \varphi_{AD,t} - \kappa \varphi_{AS,t} = 0$$

$$\partial r_t^n : \varphi_{AD,t} = 0$$

where the $\varphi_\bullet$'s are the Lagrange multipliers.

- Combining and simplifying we get the following relationship:

$$\lambda_\pi \pi_t + \frac{\lambda_y}{\kappa} (y_t - y_t^*) = 0 \tag{5}$$

- The optimal policy equilibrium is the solution to equations (3) - (4) together with the optimality condition (5). Will compute this momentarily.

The optimal monetary policy rule

- First question: to what extent is the simplified **first-order condition**

$$\lambda_{\pi}\pi_t + \frac{\lambda_y}{\kappa}(y_t - y_t^*) = 0 \quad (5)$$

a **monetary policy rule**?

- The Taylor rule we considered before is an example of an *explicit* rule. Set the instrument as an explicit function of macro aggregates.
 - (5) is an *implicit* rule: set the instrument r_t^n so that outcomes [y and π] are at optimum
- Such implicit rules are how the Fed actually communicates its policymaking:

“The Fed has a rule. The Fed’s rule is that we will go for a 2 percent inflation rate. We will go for the natural rate of unemployment. We put equal weight on those two things. We will give you information about our projections about our interest rates. That is a rule and that is a framework that should clarify exactly what the Fed is doing.”—Bernanke (2015)

Optimal outcomes

- **Optimal outcomes** are easy to characterize:

1. Following **demand shocks** θ_t :

$$y_t = \pi_t = 0, \quad r_t^n = \theta_t$$

2. Following **supply shocks** y_t^* :

$$y_t = y_t^*, \quad \pi_t = 0, \quad r_t^n = -\gamma y_t^*$$

This is just **divine coincidence**: perfectly stabilizing inflation attains optimal outcomes.

- Note that we could have equivalently attained these outcomes through the **Taylor rule**

$$r_t^n = \phi_\pi \pi_t + \hat{m}_t$$

with $\phi_\pi = \infty$ and $\hat{m}_t = 0$. But is this robust? Does it always work?

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Adding cost-push shocks

- Let's add a further wedge to the **Phillips curve**:

$$\pi_t = \kappa(y_t - y_t^*) + \psi_t \quad (6)$$

- This is “stagflationary” shock: even if output is at the right frictionless level (y_t^*), inflation is not equal to target
- Such shocks thus break divine coincidence and will make the central bank's life harder. Face a **trade-off** between stabilizing output and stabilizing inflation
- What are examples of such stagflationary “cost-push” disturbances?
 - The classical example are the oil price shocks of the 1970s. Distortions in the labor market [e.g., income taxes] would also have similar effects.
 - More recently: in open-economy extensions of our basic NK model, tariffs can look like such cost-push shocks [If you're interested: Iván Werning (MIT) et al. “Tariffs as Cost-Push Shocks: Implications for Optimal Monetary Policy.”]

Optimal monetary policy

- Optimal policy FOCs are unaffected by the addition of the cost-push shock ψ_t ! Still have

$$\lambda_\pi \pi_t + \frac{\lambda_y}{\kappa} (y_t - y_t^*) = 0 \quad (5)$$

Verify this. Intuition: the shocks to the constraints don't change what policy can achieve.

- **Optimal outcomes** following a cost-push shock are

$$y_t = -\frac{\lambda_\pi}{\lambda_\pi \kappa + \frac{\lambda_y}{\kappa}} \psi_t, \quad \pi_t = \frac{\frac{\lambda_y}{\kappa}}{\lambda_\pi \kappa + \frac{\lambda_y}{\kappa}} \psi_t, \quad r_t^n = \frac{\lambda_\pi \gamma}{\lambda_\pi \kappa + \frac{\lambda_y}{\kappa}} \psi_t$$

- Note that in eq'm there is *stagflation*: inflation is high and output low
- The central bank optimally hikes rates, depressing the economy and lowering inflation. If κ is small, then can't fight inflation, so just accept 1-1 pass-through of ψ_t to π_t .
- How to implement this optimum?
 - Note that a Taylor rule with $\phi_\pi = \infty$ does *not* generate these outcomes!
 - The rule (5) instead is robustly optimal *independently of the shocks hitting the economy*

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Learning about optimal rules

- Recall again our **optimal policy rule**:

$$\lambda_{\pi}\pi_t + \frac{\lambda_y}{\kappa}(y_t - y_t^*) = 0 \quad (5)$$

- λ_{π} and λ_y are given to us from the objective function. Is there any way we can empirically estimate the remaining parts of this optimal rule from data?
 - Here this just means estimating the PC slope κ
 - More generally we'd want an empirical strategy that is able to recover the optimal policy rule largely independently of the underlying model, not just for the NK model
- Next few slides: causal effects of **monetary shocks** $\hat{m}_t$ pin down the optimal rule!
 - Remember that we can get these causal effects from regressions of the form

$$x_t = \alpha_x + \beta_x \hat{m}_t + \varepsilon_{x,t}, \quad x_t = y_t, \pi_t$$

- My claim: we can use the β_x 's to get the coefficients of the optimal rule

Preliminaries: a re-written policy problem

- Note that, stacking coefficients, we can re-write the private-sector block (3) & (6) as

$$A \times \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = B_r r_t^n + B_\theta \theta_t + B_{y^*} y_t^* + B_\psi \psi_t$$

where A is 2×2 and the $B_\bullet$'s are 2×1 . Pre-multiplying by A^{-1} to solve the system:

$$\begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \Omega_r r_t^n + \Omega_\theta \theta_t + \Omega_{y^*} y_t^* + \Omega_\psi \psi_t \quad (7)$$

- The Ω 's in the solved-out system (7) are *impulse responses*: the causal effects of interest rates and macroeconomic shocks on the outcomes y_t and π_t
- We will later in detail study Ω_r . For now the exact expressions are not important.
- Since (7) is just a re-cast version of the constraint set, we can equivalently consider the problem of minimizing (2) subject to (7). What does that give?

Preliminaries: a re-written policy problem

- Denoting the two Lagrange multipliers by $\varphi_{\pi,t}$ and $\varphi_{y,t}$, the FOCs are now

$$\begin{aligned}\lambda_{\pi}\pi_t + \varphi_{\pi,t} &= 0 \\ \lambda_y(y_t - y_t^*) + \varphi_{y,t} &= 0 \\ \Omega_{\pi,r}\varphi_{\pi,t} + \Omega_{y,r}\varphi_{y,t} &= 0\end{aligned}$$

where $\Omega_r = (\Omega_{\pi,r}, \Omega_{y,r})'$. Re-arranging this, we get our **new optimal rule**

$$\lambda_{\pi}\Omega_{\pi,r}\pi_t + \lambda_y\Omega_{y,r}(y_t - y_t^*) = 0 \quad (8)$$

- Key seems to be Ω_r , the causal effects of rates. What can we say about this object?

The form (8) applies for general linear models. Now we use the NK model to specialize it.

- Solving the original system (3) & (6), we have

$$\Omega_{y,r} = -\frac{1}{\gamma}, \quad \Omega_{\pi,r} = -\frac{\kappa}{\gamma}$$

- It thus follows that the rules (5) and (8) are identical, as they should be

Connecting to empirical evidence

- Point of this detour: will show now that empirical work on the effects of **monetary policy shocks**—our $\hat{m}_t$ —estimates $\Omega_{\pi,r}, \Omega_{y,r}$ and thus delivers the optimal policy rule!
- To see this, solve the system (1) + agg. demand and supply (w/o other shocks) to get

$$y_t = -\underbrace{\frac{1}{\gamma + \phi_\pi \kappa}}_{\Omega_{y,m}} m_t, \quad \pi_t = -\underbrace{\frac{\kappa}{\gamma + \phi_\pi \kappa}}_{\Omega_{\pi,m}} m_t, \quad r_t^n = \underbrace{\frac{\gamma}{\gamma + \phi_\pi \kappa}}_{\Omega_{r,m}} m_t$$

- The $\Omega_{\bullet,m}$'s here are the causal effects of monetary policy shocks, estimable through simple regressions on $\hat{m}_t$, as we have seen.
- Crucially, they also recover the causal effect of interest rates on macro outcomes: $\Omega_{y,r} = \Omega_{y,m}/\Omega_{r,m}$, $\Omega_{\pi,r} = \Omega_{\pi,m}/\Omega_{r,m}$, so we have pinned down the optimal rule
- This is actually really general, and works across many models: regression on $\hat{m}_t$ gives causal effects of rates, and those effects pin down the optimal monetary policy rule

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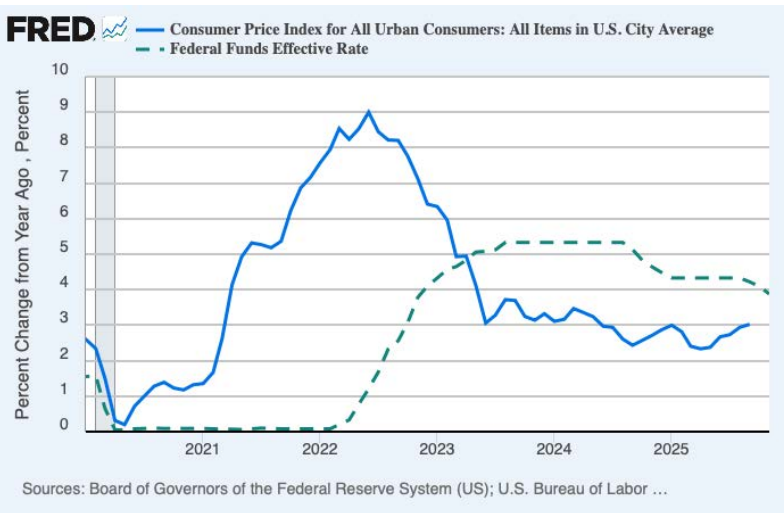
3. Course Summary

Monetary policy after 2021

- Let us now use our framework to evaluate monetary policy after 2021
- Broad consensus: Fed “**behind the curve**” in late 2021 and early 2022
 - Federal funds rate only started rising in mid/late 2022, while inflation was already high
 - In particular much focus on the **Taylor principle** being violated: interest rates did not rise one-to-one with inflation
- Next few slides:
 1. Historical Fed policy conduct through the lens of the Taylor rule
Is the Taylor rule descriptive?
 2. Does the fact that rates don't rise more than 1-1 with inflation necessarily signal a problem?
Normatively, are violations of the Taylor principle a problem?

Background reading: “Beyond the Taylor Rule” by Nakamura-Riblier-Steinsson.

Inflation and the federal funds rate



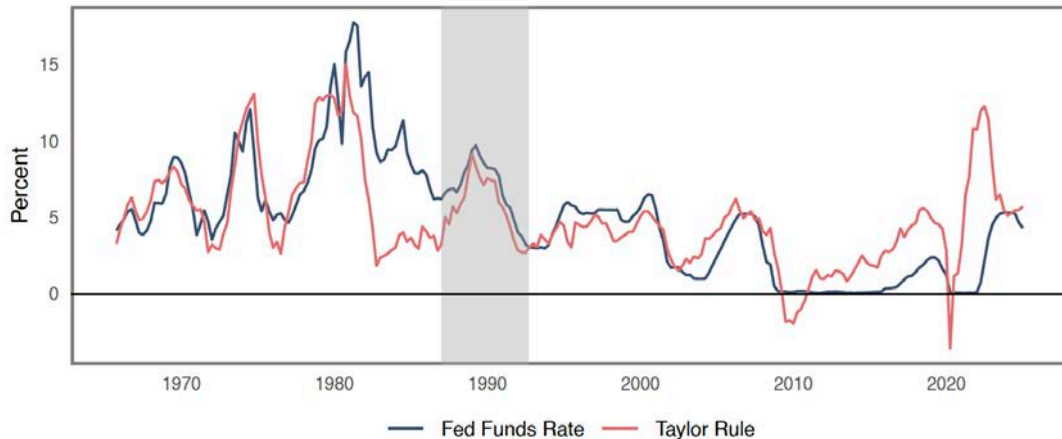
The Taylor rule as a description of monetary policy

- Originally estimated in **Taylor (1993)** as a good positive description of monetary policy in 1987-1992. Precise form:

$$i_t = 2 + \pi_t + 0.5(\pi_t - 2) + 0.5(y_t - y_t^*)$$

- Next slide: notable deviations in quality of fit over time
 1. Rates too high under Volcker (early 1980s). Depress economy to achieve inflation-fighting credibility.
 2. Rates too low in build-up of Great Recession. Contribute to financial excesses?
 3. Rates should go negative in Great Recession (but can't). Afterwards too low for too long.
 4. Rates much too low as inflation picked up post-2021 [our focus]
- Takeaway: historically Taylor rule has not been particularly descriptive, but deviation post-2021 does stand out

The Taylor rule as a description of monetary policy



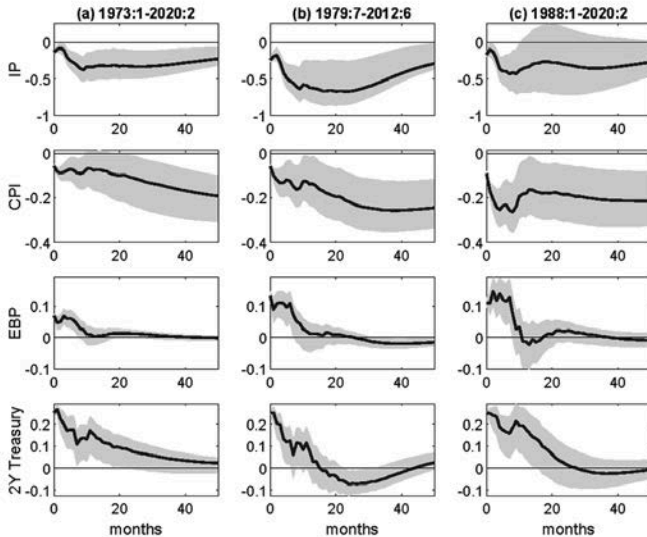
Was post-2021 monetary policy necessarily suboptimal?

- We've seen: following a **Taylor rule** is neither necessary nor sufficient for **optimal monetary policy**—depends on the shock hitting the economy
- What happened post-2021? two obvious candidates:
 1. Fiscally-led **demand shock** [stimulus checks]
 2. **Cost-push disturbances** [supply-chain disruptions, shock to relative prices of goods ($\uparrow$) vs. services ($\downarrow$), labor market tightness]
- Want strong monetary offset for 1., less clear for 2. Recall the optimal response to ψ_t :

$$y_t = -\frac{\lambda_\pi}{\lambda_\pi \kappa + \frac{\lambda_y}{\kappa}} \psi_t, \quad \pi_t = \frac{\frac{\lambda_y}{\kappa}}{\lambda_\pi \kappa + \frac{\lambda_y}{\kappa}} \psi_t, \quad r_t^n = \frac{\lambda_\pi \gamma}{\lambda_\pi \kappa + \frac{\lambda_y}{\kappa}} \psi_t$$

- Do rates respond more or less than 1-1 to inflation? ratio is $r_t^n / \pi_t = \frac{\lambda_\pi \gamma \kappa}{\lambda_y}$, which can be smaller than 1 for a very flat Phillips curve or if mostly concerned about output
Can also interpret small κ as rate effects on inflation being *later* than the initial shock (next slide).

Reminder: long lags of monetary policy



Was post-2021 monetary policy necessarily suboptimal?

- Fed initially decided to “**look through**” [i.e., respond little to] the elevated level of inflation. We can now make sense of this decision:
 - Interpret the shock as cost-push. Inflation is high, but output is below or at target.
 - Low κ : either literally flat NKPC, or expect inflation to be transitory while monetary policy acts with a lag [so effective κ is small]
 - Given these two ingredients, a subdued interest rate response can be justified.
- *Ex post* of course this assessment looks less good:
 - Economy was at or above potential, fiscal policy-induced **demand** contributed meaningfully to inflation
 - Inflation overall was highly **persistent**, necessitating eventual monetary adjustment
- Real-time policymaking is hard!

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Course summary

This concludes 14.05 – Intermediate Macroeconomics. What did we learn?

- Class split into two overarching questions:
 1. **Long-run**: What are the origins of long-run growth? How much should a nation save? What kind of policies can encourage growth?
 2. **Short-run**: Why do economies undergo ups and downs? What can monetary and fiscal policy do about it?
- We studied these questions through a unified **methodological approach**:
 - Stylized, abstract **models** that helped us elucidate core mechanisms & trade-offs, but always disciplined with (micro and macro) **data**
 - Key frameworks that we saw for our two questions:
 1. **Long-run**: Solow growth model, Ramsey problem, welfare theorems & invisible hand
 2. **Short-run**: permanent-income and hand-to-mouth consumers, real business cycle model, New Keynesian model

Thank you!

Good luck with all your future classes. I had a great time teaching you and hope you picked up something useful!

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