

14.05 – INTERMEDIATE MACROECONOMICS

Problem Set 4, Fall 2025

Exercise 1

Consider households that live for three periods, $t \in \{0, 1, 2\}$. There are two types of households, which are denoted by superscript $j \in \{inc, dec\}$ and which are identical except for their income profile. In particular, half of the population consists of type *inc* households, which have an increasing income profile over their life,

$$y_0^{inc} < y_1^{inc} = y_2^{inc}. \quad (1)$$

The other half of the population consists of *dec* households who have a decreasing income profile over their life,

$$y_0^{dec} > y_1^{dec} = y_2^{dec}.$$

Households start with no initial assets, that is $a_0^j = 0$ for each $j \in \{inc, dec\}$.

Each household solves a standard consumption-savings problem, with discount rate $\rho = 0$ and interest rate $r = 0$. In particular, a type $j \in \{inc, dec\}$ household's optimization problem can be written as

$$\max_{\{c_t^j \geq 0, a_{t+1}^j\}_{t=0}^2} u(c_0^j) + u(c_1^j) + u(c_2^j)$$

subject to

$$\begin{aligned} c_t^j + a_{t+1}^j &= a_t^j + y_t^j \quad \text{for each } t \in \{0, 1, 2\} \\ a_3^j &= 0 \quad (\text{no assets are left behind}) \\ a_1^j &\geq 0. \end{aligned} \quad (2)$$

Here, a_{t+1}^j denotes the household's next period asset choice at date t . Condition (2) says that the household is subject to a severe borrowing constraint at date 0. For simplicity, the

household can choose a_2^j without constraints.

- a) Characterize the optimal consumption level for each type of household, c_0^{inc} and c_0^{dec} . Is the borrowing constraint binding for any type? Why or why not?
- b) Suppose each agent's current income, y_0^{inc} and y_0^{dec} , increases by exactly 1 dollar, while their future incomes remain unchanged. What happens to average consumption, $c_0 = \frac{c_0^{inc} + c_0^{dec}}{2}$?²
- c) Now suppose agents are allowed to borrow up to a limit at date 0, that is, the constraint in (2) is relaxed to $a_1^j \geq -b_1$, for some borrowing limit $b_1 > 0$. Suppose b_1 is sufficiently small so that any agents that you identified to be constrained in part a) continue to be constrained also in this part. Characterize the households' average consumption, $c_0 = \frac{c_0^{inc} + c_0^{dec}}{2}$. Suppose b_1 increases by 1 dollar. What happens to c_0 and why?
- d) In the run-up to the recent subprime crisis in the US, house prices dramatically increased. Briefly (not to exceed one sentence) explain how rising house prices could have increased some households' borrowing limit, b_1 . Using the empirical evidence discussed in lectures, briefly (not to exceed two sentences) describe whether the evidence suggests rising house prices increased, decreased, or didn't affect those households' borrowing and consumption in the run-up to the crisis.

Exercise 2

Consider an economy populated by a unit mass of identical households. Each of them solves the following labor supply problem.

$$\max_{c, \ell} u(c) - v(\ell)$$

subject to the budget constraint

$$c = w\ell$$

where w is wage, c is consumption and ℓ is labor. For the questions below, assume that $u(c) = c$ and $v(\ell) = \frac{\gamma}{2}(\ell)^2$.

- a) Determine the optimal level of labor supply, ℓ^S , for a household.
- b) Suppose that firms in this economy produce using the following production function: $y = A\ell^{1-\alpha}$ with $0 < \alpha < 1$. All markets are competitive. First determine the demand for

²Suppose the parameters are such that type *inc* households continue to have an increasing income profile after the change, as in (1).

labor, ℓ^D , as a function of the wage. Then calculate the equilibrium employment ℓ^* and the wage w^* . How does ℓ^* depend on γ ? Why?

- c) Consider a benevolent social planner who chooses the optimal level of labor to maximize households' utility function. Write down the social planner's problem. Show that the equilibrium level of labor is the same as above.
- d) We now ask how wages and employment respond to changes in labor demand and labor supply.
 - i) We begin with labor demand. Suppose that firm productivity A declines, i.e., workers are less productive and so less desirable for firms. What happens to ℓ^* and w^* ?
 - ii) Now consider labor supply. For this change household preferences to

$$u(c) - \xi v(\ell)$$

where $\xi > 0$ captures the relative desirability of work. Suppose that ξ increases, i.e., work becomes less desirable (or equivalently, leisure becomes more desirable). What happens to ℓ^* and w^* ?

- iii) Consider a Covid shock. We can think of this shock as reducing both labor demand—because certain firms need to shut down—and labor supply—because workers are unwilling to go to work, out of fear of getting sick. What does this shock do to equilibrium employment and wages?

Exercise 3

Provide short answers to the following questions.

- a) Consider two economies, A and B , which share the same fundamentals (preferences, productivity, ...) except that in economy A half the households are borrowing-constrained, while in economy B everybody is unconstrained. Suppose now that both economies experience the same drop in current income (and no change in expectations of future income). Which economy, if any, experiences a larger drop in aggregate demand?
- b) Do aggregate demand shocks generate realistic business cycles under the prism of the RBC model? Why/why not?
- c) According to the Neoclassical/RBC model, an increase aggregate productivity *necessarily* increases aggregate employment. True or false?
- d) According to the New Keynesian model, aggregate supply is an increasing function of inflation. Briefly explain why this is the case.

Exercise 4

Consider the following slightly altered version of the static New Keynesian model with demand shocks. The static demand relation is

$$-\gamma y_t = r_t^n - \pi_t - \hat{\theta}_t \quad (3)$$

Note the change relative to the model in class: now *contemporaneous* inflation, π_t , shows up in the aggregate demand equation (3). This change in the model allows it to more closely mimic an interesting property of the full dynamic New Keynesian model. The other two equations are unchanged: there is an aggregate supply relation,

$$\pi_t = \kappa y_t \quad (4)$$

and a policy relation,

$$r_t^n = \phi_\pi \pi_t. \quad (5)$$

Note that there are no supply and policy shocks in (4) and (5), as this problem focuses on the propagation of demand shocks. Throughout this problem you should assume that $\gamma > \kappa$.

- a) Suppose that $\phi_\pi > 1$. Provide a pictorial AS-AD representation of the equilibrium, with y on x -axis and π on y -axis. What are the slopes of the demand and supply curves?
- b) Continue to assume that $\phi_\pi > 1$. Combine equations (3) - (5) to solve for the effect of demand shocks $\hat{\theta}_t$ on output y_t . When does a demand shock move only prices? When only quantities? Relate your conclusions to the slopes of the demand and supply curves.
- c) Now suppose instead that $\phi_\pi = 0$, e.g., because nominal interest rates are at the zero lower bound, and suppose that demand declines, $\hat{\theta}_t < 0$. Again solve for the equilibrium response of output. How does the response of output to the demand contraction change as you make prices more flexible, i.e., as κ increases (though still below γ)? What is going on? [*Hint: You should find that the lower inflation induced by the demand shock lowers demand even further, leading to a vicious cycle.*]
- d) Provide a pictorial AS-AD representation of the equilibrium studied in c), with y on x -axis and π on y -axis. What's the slope of the demand equation? Discuss.

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