

Lecture 10: The Neoclassical Growth Model

Part III: Dynamics

Christian Wolf

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Roadmap

- So far: characterize eq'm up to a system of equations. Can we say more? I.e., how the economy is going to evolve over time?
- This lecture:
 1. Set up the system in continuous time
Why? same economics, but allow much easier graphical analysis.
 2. Introduce convenient tool for graphical analysis: phase diagrams
 3. Apply to study how the economy responds to: higher taxes, more gov't spending, ...
- Big picture: main payoff of moving to the neoclassical growth model is that we can talk about effects of changes in policies.
 - Predicting steady-state (= long-run) effects is always easy and can be done analytically. Phase diagrams are our tool of choice for the transitions.
 - Note: Solow growth model can't do any of that because s is exogenous

Outline

1. Warm-up: Ramsey problem in continuous time

2. Characterizing equilibrium dynamics

The phase diagram

Comparative statics & dynamics

3. Taking Stock

4. Midterm discussion

Planner's problem

- The planner's problem in the continuous-time version of the Ramsey problem is:

$$\max_{c, k: [0, T] \rightarrow \mathbb{R}_+} \int_0^T e^{-\rho t} u(c(t)) dt \quad (1)$$

$$s.t. \quad c(t) + i(t) = f(k(t)) \quad (2)$$

$$\dot{k}(t) = i(t) - \delta k(t) \quad (3)$$

$$k(0) = k_0 > 0 \quad (4)$$

k and c are functions of time (not discrete-time sequences), $\beta = e^{-\rho}$ and $\rho > 0$ is the discount rate.

- (1) is social welfare (discounted present value of utility of the representative household), with u being the per-period utility function
- (2) is the resource constraint at date t , with i denoting gross investment
- (3) is the law of motion of capital, with $\dot{k} \equiv dk/dt$ being net investment
- (4) is the initial condition, with $k_0 > 0$ given by history.

Planner's problem

- The planner's problem is an optimal-control problem with k being the “state variable” and (c, i) being the “control variables”.
- There are at least two types of mathematical techniques that you can use to solve this kind of problem:
 1. optimal control (Hamiltonians)
 2. continuous-time dynamic programming (Hamilton-Jacobi-Bellman equations)
- I will not review these methods and will not “properly” solve the model. Instead, I will use a heuristic, which leads to the correct solution.
 - Objective: get a differential equation (rather than a difference equation) that characterizes eq'm dynamics, then study its properties
 - We'll get the “right” differential equation, but not formally prove that it is the right one

Heuristic solution

- Go back to the discrete-time neoclassical growth model. Recall that the Euler condition of the planner's problem is given by

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta(1 + f'(k_{t+1}) - \delta)$$

- Suppose that preferences exhibit constant elasticity of intertemporal substitution (CEIS), which means that

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \quad \theta > 0$$

- The Euler condition can then be restated as

$$\left(\frac{c_{t+1}}{c_t}\right)^\theta = \beta(1 + f'(k_{t+1}) - \delta)$$

Heuristic solution

- Taking logs of the FOC on both sides, we get

$$\log c_{t+1} - \log c_t = \frac{1}{\theta} [\log \beta + \log(1 + f'(k_{t+1}) - \delta)]$$

- Next, recall that, by a first-order approximation, $\log(1 + x) \approx x$ whenever x is small enough. Using this fact, we can write

$$\log \beta = -\log(1 + \rho) \approx -\rho \quad \text{and} \quad \log[1 - \delta + f'(k)] \approx f'(k) - \delta$$

We thus obtain the following approximation to the Euler condition:

$$\Delta \log c_t \equiv \log c_{t+1} - \log c_t \approx \frac{1}{\theta} [f'(k_{t+1}) - \delta - \rho]$$

- This approximation turns out to be exact when time is continuous. Indeed, the continuous-time version of the Euler condition is given by

$$\frac{d \log c}{dt} = \frac{1}{\theta} [f'(k_t) - \delta - \rho]$$

Heuristic solution

- We can equivalently write this as

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta} [f'(k_t) - \delta - \rho]$$

Consumption growth is proportional to the difference between the MPK, net of depreciation, and the subjective discount rate.

- Next, consider the resource constraint. This can be rewritten as

$$k_{t+1} - k_t = f(k_t) - \delta k_t - c_t,$$

and its continuous-time version is

$$\dot{k}_t = f(k_t) - \delta k_t - c_t.$$

Exactly as we saw before, in the Solow growth model.

- Summary: to get to continuous time just (i) write differences as rates of change & (ii) use the approximation $\log(1 + x) \approx x$

Continuous-time equilibrium dynamics

We thus reach the following characterization of the optimal (and so also the competitive equilibrium) dynamics:

Proposition

Consider the continuous-time version of the model. The equilibrium path $\{c_t, k_t\}_{t \in [0, \infty)}$ is the unique solution to the following ODE system:

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= \frac{1}{\theta} [f'(k_t) - \delta - \rho] \\ \dot{k}_t &= f(k_t) - \delta k_t - c_t,\end{aligned}$$

for all t , with initial condition $k(0) = k_0 > 0$ as well as the terminal condition $\lim_{t \rightarrow \infty} k_t = k^$, where k^* is the steady-state capital stock.*

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The phase diagram

Comparative statics & dynamics

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Phase diagram

- Using the continuous-time equations, we can visualize how the economy will evolve over time using a tool called the phase diagram. Key are two loci:

- The $\dot{k} = 0$ locus is given by (c, k) such that

$$\dot{k} = f(k) - \delta k - c = 0 \quad \Leftrightarrow \quad c = f(k) - \delta k$$

- The $\dot{c} = 0$ locus is given by (c, k) such that

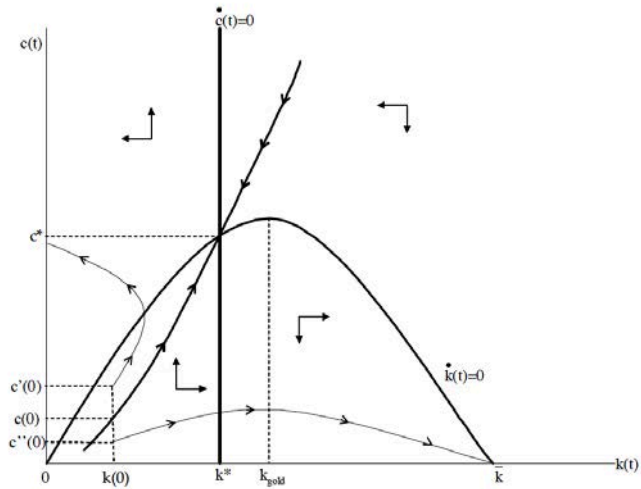
$$\dot{c} = c \frac{1}{\theta} [f'(k) - \delta - \rho] = 0 \quad \Leftrightarrow \quad k = k^* \text{ or } c = 0$$

- The steady state is simply the intersection of the two loci:

$$\dot{c} = \dot{k} = 0 \quad \Leftrightarrow \quad \{(c, k) = (c^*, k^*) \text{ or } (c, k) = (0, 0)\}$$

where $k^* \equiv (f')^{-1}(\rho + \delta)$ and $c^* \equiv f(k^*) - \delta k^*$.

Phase diagram



Phase diagram

The two loci partition the (c, k) space in four regions. We now examine the direction of change in c and k in each of these four regions.

- c dimension (sign of $\dot{c}$)
 1. If $0 < k < k^*$, then $\dot{c} > 0$. This is because this region identifies the points where the marginal product of capital exceeds the discount rate.
 2. Conversely, in the region to the right of the $\dot{c}$ locus, we have $\dot{c} < 0$.

We conclude that the direction of change in the c dimensions is as indicated by the *vertical* arrows.

- k dimension (sign of $\dot{k}$)
 1. If $c < f(k) - \delta k$, then $\dot{k} > 0$. This is because this region identifies the points where consumption is less than GDP net of capital replacement cost.
 2. Conversely, in the region about the $\dot{k}$ locus, we have $\dot{k} < 0$.

We conclude that the direction of in the k dimension is as indicated by the *horizontal* arrows.

Moving along the phase diagram

How will the economy move along the phase diagram?

- Pick an arbitrary initial point for $x = (c, k)$, call it $x_A = (c_A, k_A)$, and ask what's going to be the value of x , call it x_B , after Δt amount of time has passed.
- For Δt small enough, we have that

$$x_B = x_A + \frac{dx}{dt} \Delta t$$

That is, starting from any given point in the phase diagram, you can find the point at the next instant by using the ODE. Continuing like this, you can draw the entire path.

- The above was possible to do for arbitrary initial point $x_0 = (c_0, k_0)$. But what x_0 is the relevant one? We can think of k_0 are fixed by history. But what about c_0 ?
- The initial consumption, c_0 , is not fixed. It is optimally chosen.

Moving along the phase diagram

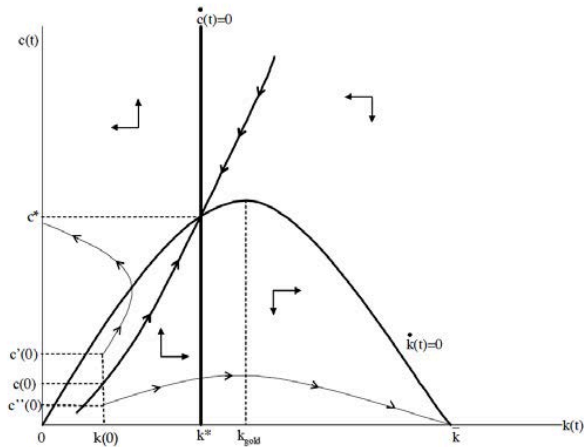
- Since there is a unique eq'm, there is a function $c = c(k)$ such that $c(0) = c(k(0))$ for any initial $k(0)$. This $c(0)$ is such that we move to the steady state as $t \rightarrow \infty$.
- Mathematically: the system is saddle path-stable, not stable.
 - A stable system is one where, for *any* starting point, we converge to steady state.
 - A saddle path-stable system is instead one in which we only go to a resting point on a lower-dimensional manifold (the saddle path). Here that's $c = c(k)$.
- The saddle path is the solid, upward-sloping line. Why does it look like that?
 - Suppose you started in the top left or lower right quadrants. Move away from steady state.
 - So has to go from lower left to upper right. But why a line, not an area? Because for any $k(0)$, a $c(0)$ that takes us to steady state is part of an eq'm – and there's only one.

Moving along the phase diagram

What happens away from the saddle path?

1. Any c_0 that puts the economy *above* the saddle path leads to zero capital in finite time, thus violating the resource constraint.
2. Any c_0 that puts the economy *below* the saddle path leads to so much capital and so little consumption that the boundary condition at ∞ [recall Lecture 8] is violated.
 - Can show: the economy will cross the golden rule and is thus dynamically inefficient.
 - Once the economy reaches k_{gold} , where $f'(k_{gold}) - \delta = 0$, continuing on the path is dominated by an alternative path – stop investing and consume $c = f(k_{gold}) - \delta k_{gold}$.
 - In other words, the economy is wasting too much resources in investment and it would better increase consumption.

Back to the phase diagram



Make sure you now understand all lines in this figure.

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Comparative statics and dynamic responses

- Remember our initial objective: describe how the economy will be affected by changes in the environment E.g., more gov't purchases, taxes, foreign aid, ...
- So far today: figure out how our *baseline* NGM unfolds over time
 1. We know how to derive the optimality conditions in discrete time:

$$\begin{aligned}k_{t+1} - k_t &= f(k_t) - \delta k_t - c_t \\ \frac{c_{t+1}}{c_t} &= [\beta(f'(k_{t+1}) + (1 - \delta))]^{\frac{1}{\theta}}\end{aligned}$$

2. We have seen how to map this system into continuous time:

$$\begin{aligned}\dot{k}_t &= f(k_t) - \delta k_t - c_t \\ \frac{\dot{c}_t}{c_t} &= \frac{1}{\theta} [f'(k_t) - \delta - \rho]\end{aligned}$$

We can then study this continuous-time system using the phase diagram.

- This suggests a line of attack to figure out the effects of changes in the environment ...

Comparative statics and dynamic responses

1. Figure out how a given change in the **environment/policy** affects (i) the resource constraint and (ii) the savings decision (Euler equation)
 - You already know how to do this in discrete time. Will show many examples now.
 - Then get the continuous-time analogue of this new system.
2. **Steady state:** use comparative statics on the steady state
 - Can do this using either the discrete-time or continuous-time equations. Same result.
 - Note: here the phase diagram is not needed. Already feasible two lectures ago.
3. **Dynamics:** how does the economy evolve from old to new steady state?
 - Draw old and new resource constraint, Euler equation and saddle paths.
 - Use the diagram to describe the evolution of the economy: $k(0)$ is given, then follow saddle path of the new economy. Compare with old economy.

Comparative statics & dynamics

Application I: change in preferences

Change in preferences

Experiment: suppose that the discount factor β falls permanently (or equivalently ρ increases). Think of this as a reduction in the willingness to save.

Interpretation: something that improves incentives to save, e.g. better property rights.

- Let's first figure out how the two equations are affected:
 - The resource constraint of the economy is not affected by this change. As a result, the $\dot{k} = 0$ locus does not change.
 - The Euler condition is affected: for any given MPK, optimal consumption growth is lower the higher the discount rate. It follows that the $\dot{c} = 0$ locus shifts to the left.
- Now let's turn to the steady state. We have

$$\frac{\partial k^*}{\partial \rho} = \frac{1}{f''(k^*)} < 0, \quad \frac{\partial c^*}{\partial \rho} = [f'(k^*) - \delta]k'(\rho) < 0$$

Aside: how does the conclusion for c^* relate to dynamic inefficiency?

Change in preferences

- Now let's use the phase diagram for dynamics.
- Main question: what happens on impact? Starting from the old steady state, where ρ was low, how does the economy respond over time to a permanent increase in ρ ?
- Let's draw the phase diagram together on the board. Summary:
 - $\dot{c} = 0$ locus has shifted left. New saddle path must lie above the old one. [Why? Hint: look at the phase diagram arrows. Must go from lower left to upper right quadrant.]
 - Start at old k^* . Then consumption must initially jump up, before then going down to lower new steady state c^{**} . [Intuition?]

Comparative statics & dynamics

Application II: additive endowment

Additive endowment

Experiment: suppose that the economy receives an exogenous endowment of the consumption good, equal to $e > 0$, in each period. [e.g., unconditional foreign aid]

- How are the two key equations affected?
 - The budget constraint of the planner becomes

$$k_{t+1} - k_t = f(k_t) + e - \delta k_t - c_t, \quad \text{or} \quad \dot{k} = f(k) + e - \delta k - c$$

- Since the endowment is assumed to be lump sum, the incentives the households face when deciding how much to consume and save are not affected, so we still have

$$\frac{c_{t+1}}{c_t} = \{\beta[1 + f'(k_{t+1}) - \delta]\}^{\frac{1}{\theta}} \quad \text{or} \quad \frac{\dot{c}_t}{c_t} = \frac{1}{\theta}[f'(k_t) - \delta - \rho],$$

Additive endowment

- Let's begin again with the steady state. Since the Euler equation is unaffected we get

$$\begin{aligned}\frac{\partial k^*}{\partial e} &= 0 \\ \frac{\partial c^*}{\partial e} &= 1\end{aligned}$$

- Make sure you see the intuition.
 - Incentives for savings are unaffected, so k^* doesn't change.
 - Foreign aid just 1-1 increases consumption.
 - Recall our Solow model discussion: so this kind of foreign aid doesn't work as a “big push” policy. Saving rate goes down 1-1 with every dollar of additional aid.

Additive endowment

- Let's now do dynamics. Consider a permanent decrease in the aid package, say from $e = e_{high}$ to $e = e_{low}$.
- Analysis is really easy: if initially at k^* , then you will stay there. Consumption just immediately drops 1-1. No transition.

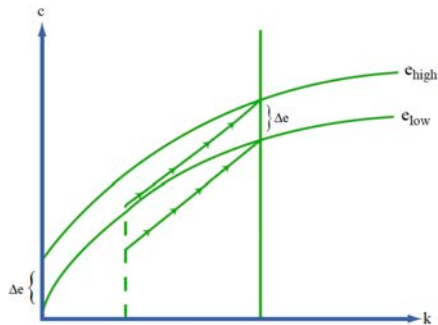


Figure 3.2

Comparative statics & dynamics

Application III: distortionary taxation

Distortionary taxation

- There are many good reasons why societies may redistribute through policies.
 1. One simple reason is fairness considerations and aversion to inequality.
 2. The other big consideration is that incentives of firms and households may *not* be such that their joint optimality at the same time also guarantees planner optimality.
 - Last time we studied an economy in which incentives are indeed aligned perfectly. But externalities, monopoly power, etc all break this. May want to use taxes to correct this. E.g. for externalities: “Pigouvian” tax.
- However, taxation and redistribution also involve a distortion of private incentives to save, invest, and accumulate wealth. Let us abstract from the beneficial effects and let's understand these distortionary effects.

Distortionary taxation

- The government taxes labor and capital income at a flat tax rate $\tau \in (0, 1)$. The government then redistributes the proceeds from this tax uniformly (lump-sum) across households. Let T_t be the transfer made in period t .
- First question: how does this affect our two key differential equations?
- Can't use planner solution anymore. Now we need to figure out how households respond to the tax. Won't be Pareto-optimal anymore.

Distortionary taxation

- The household budget is

$$c_t + k_{t+1} = (1 - \tau)(w_t + r_t k_t) + (1 - \delta)k_t + T_t,$$

- Let's first think about the household consumption-savings decision. Saving returns are now lower. The FOC becomes

$$u_c(c_t) = \beta[1 + (1 - \tau)r_{t+1} - \delta]u_c(c_{t+1}).$$

Exercise: write the Lagrangian of the household, as we did in the previous lecture note, and derive the condition, to make sure you understand how it is obtained.

- Now, combining the above with $r_t = f'(k_t)$, and assuming again the power CES specification for u we get

$$\frac{c_{t+1}}{c_t} = \{\beta[(1 - \tau)f'(k_{t+1}) + 1 - \delta]\}^{\frac{1}{\theta}}$$

Distortionary taxation

- What about the budget/resource constraint?
- Combining the budget constraint of the household with the zero-profit condition of the firms gives

$$c_t + k_{t+1} = (1 - \tau)f(k_t) + (1 - \delta)k_t + T_t$$

The government budget on the other hand is

$$T_t = \tau(w_t + r_t k_t) = \tau f(k_t)$$

- Combining we get the resource constraint of the economy:

$$c_t + k_{t+1} = f(k_t) - (1 - \delta)k_t$$

Observe that the tax scheme does not at all appear in the resource constraint of the economy—it is only redistributive and does not absorb resources.

Distortionary taxation

- We conclude that the discrete-time pair of difference equations becomes

$$\begin{aligned}\frac{c_{t+1}}{c_t} &= \{\beta[1 + (1 - \tau)f'(k_{t+1}) - \delta]\}^{\frac{1}{\theta}} \\ c_t + k_{t+1} &= f(k_t) - (1 - \delta)k_t\end{aligned}$$

- The analogous continuous-time system is

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= \frac{1}{\theta}[(1 - \tau)f'(k_t) - \delta - \rho], \\ \dot{k}_t &= f(k_t) - \delta k_t - c_t.\end{aligned}$$

Distortionary taxation

- Let's begin again with the steady state. We have

$$\begin{aligned}\frac{\partial k^*}{\partial \tau} &= \frac{f'(k^*)}{(1-\tau)f''(k^*)} < 0 \\ \frac{\partial c^*}{\partial \tau} &= [f'(k^*) - \delta] \frac{\partial k^*}{\partial \tau} < 0\end{aligned}$$

- Thus, higher taxes lead to lower investment, and so to a lower capital stock and from there necessarily to less consumption.

Dynamics: unanticipated permanent tax cut

- Now turn to dynamics, and consider an unanticipated permanent tax cut.
- The $\dot{k} = 0$ locus does not change, but the $\dot{c} = 0$ locus shifts right. Saddle path thus shifts right. Exactly the same as in the preference change case.

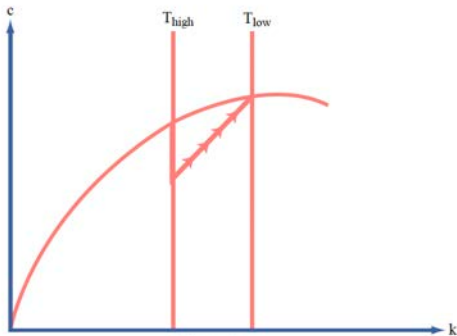


Figure 3.3

Comparative Statics & Dynamics

Application IV: institutions

- In the preceding analysis, we interpreted τ literally as the rate of taxation.
- More generally, we can think of τ as a proxy for various forces that make private agents (consumers, entrepreneurs, firms) not receive the full return on their investments.
 - Confiscation by an authoritarian government
 - Theft, weak property protection
 - Weak/corrupt courts and regulators
 - Unfair competitive environment
 - Uncertainty/instability
 - ...

Institutions

- From this perspective, we can also think of the savings/growth effects of a higher τ as a proxy for the savings/growth effects of weaker institutions.
- Key prediction: better institutions map to a higher steady-state y^*
 - Here: the effect goes through only capital accumulation. In richer models and in the real world. the effect is also likely to go through A .
 - We will revisit this later, and discuss evidence supportive of this prediction.
 - Relates to “big push”: to increase savings unconditional aid doesn't work, but e.g. improving legal frameworks does

Comparative statics & dynamics

Application V: productivity

Productivity shocks: a prelude to RBC

- We now consider the effect of a shock to total factor productivity.

Aside: the stochastic version of this is the Real Business Cycle (RBC) model. Will return to this in the business-cycle half of the class.

- Let output be given by

$$y_t = A_t f(k_t)$$

where A_t denotes TFP.

- Solving either the planner's problem or the decentralized economy, we get the eq'm dynamics as [do it!]

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta} [A_t f'(k_t) - \delta - \rho], \quad \dot{k}_t = A_t f(k_t) - \delta k_t - c_t.$$

Note that TFP A_t affects both the production possibilities frontier of the economy (the resource constraint) and the incentives to accumulate capital (the Euler condition).

Productivity shocks: a prelude to RBC

- As usual we begin with the steady state.
- First of all we find

$$\frac{\partial k^*}{\partial A} = -\frac{f'(k^*)}{Af''(k^*)} > 0$$

Returns are higher, so households save more. The steady-state level of capital is higher.

- Next we have

$$\frac{\partial c^*}{\partial A} = f(k^*) + [Af'(k^*) - \delta] \frac{\partial k^*}{\partial A} > 0$$

More production for any given k + greater k^* both push c^* up.

Aside: why is the term in brackets necessarily positive?

Unanticipated permanent productivity shock

- Now let's study dynamics, for a permanent increase in A .
- The $\dot{k} = 0$ locus shifts up and the $\dot{c} = 0$ locus shifts right, permanently.
- What can we say about dynamics? Harder now!
 - c_0 may either increase or fall, depending on whether wealth or substitution effects dominate. Note how our strategy of putting saddle paths into the right quadrants doesn't yield a unique answer in this application.
 - Along the transition, both c and k are increasing towards the new higher steady state.
 - Key insight: consumption will change more for any change in capital if $\frac{1}{\theta}$ is big $\rightarrow$ steeper saddle path

Unanticipated permanent productivity shock

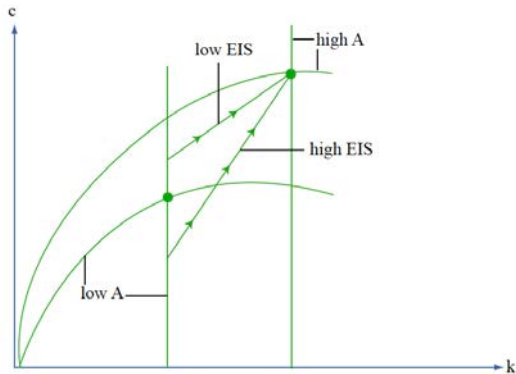


Figure 3.5 (A)

Intuition: high EIS $\rightarrow$ very willing to consume less today and enjoy the high returns $\rightarrow$ fast convergence to the new steady state

Comparative statics & dynamics

Application VI: Government spending

Government spending

- For our final twist we introduce a government that collects taxes in order to finance some exogenous level of government spending.
- Will consider two kinds of revenue collection:
 1. Lump-sum taxation
 2. Distortionary taxation
- Question: how are steady state & dynamics affected by (i) the presence of gov't spending and (ii) the mode of its financing?

Government spending: lump-sum taxation

- Suppose the government finances its expenditure with lump-sum taxes. The household budget is

$$c_t + k_{t+1} = w_t + r_t k_t + (1 - \delta)k_t - T_t,$$

implying that the Euler condition remains

$$\frac{u_c(c_t)}{u_c(c_{t+1})} = \beta[1 + r_{t+1} - \delta] = \beta[1 + f'(k_{t+1}) - \delta]$$

That is, taxes do not affect the savings choice.

- The government budget is $T_t = g_t$, where g_t denotes gov't spending.
- The resource constraint of the economy becomes

$$c_t + g_t + k_{t+1} = f(k_t) + (1 - \delta)k_t$$

Note that g_t absorbs resources from the economy.

Government spending: lump-sum taxation

- We conclude

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= \frac{1}{\theta}[f'(k_t) - \delta - \rho], \\ \dot{k}_t &= f(k_t) - \delta k_t - c_t - g_t\end{aligned}$$

- In the steady state, k^* is independent of g and c^* moves 1-1 with $-g$.
- Dynamics: suppose the economy is initially at steady state. Then a sudden change in g 1-1 feeds through to consumption—it immediately jumps to the new steady-state level.
- Clearly, the effect of government spending financed with lump-sum taxes is isomorphic to a negative endowment shock.

Government spending: distortionary taxation

- Suppose the government finances its expenditure with distortionary income taxation. The household budget is

$$c_t + k_{t+1} = (1 - \tau)(w_t + r_t k_t) + (1 - \delta)k_t,$$

implying

$$\frac{u_c(c_t)}{u_c(c_{t+1})} = \beta[1 + (1 - \tau)r_{t+1} - \delta] = \beta[1 + (1 - \tau)f'(k_{t+1}) - \delta].$$

That is, taxes now distort the savings choice.

- The government budget is

$$g_t = \tau f(k_t)$$

and the resource constraint of the economy is

$$c_t + g_t + k_{t+1} = f(k_t) + (1 - \delta)k_t.$$

Government spending: distortionary taxation

- We conclude

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= \frac{1}{\theta}[(1 - \tau)f'(k_t) - \delta - \rho], \\ \dot{k}_t &= (1 - \tau)f(k_t) - \delta k_t - c_t.\end{aligned}$$

- In the steady state, k^* is a decreasing function of g (equivalently, τ) and c^* decreases more than one-to-one with g .
- Dynamics: suppose the economy is at steady state. Then a sudden increase in g and so τ leads to a rightward shift of the $\dot{k} = 0$ locus and a leftward shift of the $\dot{c} = 0$ locus.
 - Clearly, the effect of government spending financed with distortionary taxes is isomorphic to a negative endowment shock *plus* a distortionary tax shock.
 - Exercise: fill in the details of the analysis.

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Taking stock

This concludes the first major part of the course. What have we learned?

- Methodology of macroeconomics
 - Modeling: optimization + general equilibrium
 - First macro model: Robinson Crusoe economy
- We then studied how economies *accumulate capital* over time via:
 1. **Solow growth model**
Mechanical accumulation of capital via a constant savings rate.
 2. **Neoclassical (or Ramsey) growth model**
Savings rate as the outcome of a planning problem (for the optimum) or of market interactions of households and firms.

We used these models to think about two very important questions.

Taking stock

1. Is **capital accumulation** alone enough to ensure **long-term growth**?

- The (perhaps surprising) answer was “no”. Under our assumptions, capital accumulation alone cannot be the engine of growth.
- We were able to already see this in the simple Solow growth model. The richer Ramsey problem doesn't change this basic insight. Can't overcome decreasing returns to scale.

2. How much of its income **should** a nation **save**? What determines how much is saved?

- Go beyond Solow. Optimal saving = Ramsey problem. Compared the steady state to the “Golden Rule” and characterized transitional dynamics.
- Main application: how do gov't policies affect savings rates?
Go beyond the planning problem, study competitive eq'm. First characterize the steady state and then use the phase diagram for dynamics.

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2. Characterizing equilibrium dynamics

The phase diagram

Comparative statics & dynamics

3. Taking Stock

4. Midterm discussion

Midterm discussion

- Midterm next week. Closed book, 85mins. Structure similar to past years:
 1. Relatively short yes/no/uncertain questions with brief discussion
Taken from entire class, up to the lecture before the exam.
 2. Two longer questions [model-based: Robinson Crusoe, Solow, or Ramsey]
- Main skills
 - Robinson Crusoe: consumption-leisure trade-off, planning problem vs. competitive eq'm
 - Solow: technology/resource constraint, dynamics of the capital stock (incl. steady state), Golden rule & dynamic inefficiency, interpretation of Solow growth model regressions
 - Ramsey: how to derive the Euler equation, planning problem vs. competitive eq'm, phase diagram for dynamics, effects of (policy) changes on resource constraint & Euler equation

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14.05 Intermediate Macroeconomics
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