

Lecture 14: Towards Real Business Cycles

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Real business cycles

- Last time: how to think about “demand” & “supply” shocks at the aggregate level, in general equilibrium
- Next two lectures: adding demand and supply shocks to the **neoclassical growth model**. Do we see something that looks like business cycles?
 - Note the methodological philosophy: model of optimizing households and firms, built for long run. Does it also work for short run?
 - This is known as the “real business cycle” model. Modeling philosophy has remained dominant, but the specific model itself has too many limitations.
- **Roadmap**
 1. Today: 2-period model, planning solution
 2. Next time: infinite-horizon model, competitive decentralization

Outline

1. Model Set-Up

2. Solving the Planner's Problem

3. Demand & Supply Shocks

Labor-only model

Full model

Overview

- **Real business cycle model** = **neoclassical growth model** plus two changes
 1. Augmented to feature **shocks**. We will look at basic supply and demand shocks.
 2. Endogenous **labor supply** (rather than just exogenously fixed at 1). Allows us to generate employment fluctuations.
 - For the time being, no distinction between extensive and intensive margins of employment (# workers vs. hours per worker)
- **Objective:** study response of economy to short-run demand & supply shocks. Can we generate business cycles?
 - vs. before: focus on long-run capital accumulation.
 - Solution concept: competitive eq'm + no frictions = planning solutions.

Thus, if we see fluctuations, they will be “efficient.” Important methodological insight.
- Today: simple two-period version

Demand and supply shocks

- **Demand:** demand shock = shock to *intertemporal preferences*
 - Think of drop in consumer spending during a recession as a sudden urge to consume less today relative to tomorrow.
 - Proxy for changes in consumer expectations or consumer credit. Will later formalize this.
- **Supply:** supply shock = shock to *total factor productivity* (TFP)
 - Drop in business production during a recession as a sudden reduction in the economy's ability to produce.
 - Examples: Covid-19 shock, credit crunch causing firms to go out of business, development of new general-purpose technology (AI)

Environment

- **Preferences**

$$\theta_1 U(c_1, \ell_1) + \theta_2 U(c_2, \ell_2)$$

- Notation: c_t = consumption, ℓ_t = hours worked, θ_t = preference shock for period t , U = utility function, strictly concave and increasing in c , decreasing in ℓ
- Same as in basic growth model, but now with time preference shocks and labor disutility

- **Technology**

$$y_t = A_t F(k_t, \ell_t)$$

- Notation: k_t = capital, ℓ_t = hours worked, A_t = technology shock, y_t = output, F = production function, strictly increasing and concave
- Same as in growth model, but now with TFP shocks

- **Resource constraints** [w/ full depreciation]

$$c_1 + k_2 = y_1$$

$$c_2 + k_3 = y_2$$

Equilibrium concepts

- Down the road we will again consider the **competitive equilibrium**
 1. Optimizing behavior:
 - Households supply labor and save in capital (as well as bonds), taking prices as given
 - Firms hire labor and rent capital, also taking prices as given
 2. Markets clear: supply = demand for labor, capital, output
- Eq'm will again be efficient (all cond's required for welfare theorems are satisfied), so today we just look at the planner's problem to get quantities
- Next time: infinite-horizon planning problem + competitive eq'm

The competitive eq'm will again additionally give prices, just like in our long-run analysis.

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Planner's problem: formulation

- The planner chooses a feasible allocation to **maximize welfare**:

$$\max_{\{c_1, \ell_1, c_2, \ell_2, k_2\}} \theta_1 U(c_1, \ell_1) + \theta_2 U(c_2, \ell_2) \quad (1)$$

$$s.t. \quad c_t + k_{t+1} = y_t \quad t \in \{1, 2\} \quad (2)$$

$$y_t = AF_t(k_t, \ell_t) \quad t \in \{1, 2\} \quad (3)$$

$$k_1 = \bar{k} > 0 \text{ (exogenously given)}, k_3 = 0 \quad (4)$$

- Interpretation: (1) is welfare, (2) and (3) are the resource constraints and production technology that determine feasibility, and finally (4) is initial & boundary condition
- Same as the planner's problem in the neoclassical growth model, except: we only have two periods, we have endogenous labor supply, and we have added shocks

We can thus use exactly the same techniques as in our long-run analysis for business-cycle questions.

Planner's problem: solution

- The Lagrangian for this problem can be written

$$\begin{aligned}\mathcal{L} = & \theta_1 U(c_1, \ell_1) + \theta_2 U(c_2, \ell_2) \\ & + \lambda_1 \left[\underbrace{A_1 F(k_1, \ell_1)}_{y_1} - c_1 - k_2 \right] + \lambda_2 \left[\underbrace{A_2 F(k_2, \ell_2)}_{y_2} - c_2 \right]\end{aligned}$$

- Key FOCs:

$$\text{for } t \in \{1, 2\}, \quad \frac{\partial \mathcal{L}}{\partial c_t} = 0 \quad \Rightarrow \quad \theta_t U_c(c_t, \ell_t) = \lambda_t \quad (5)$$

$$\text{for } t \in \{1, 2\}, \quad \frac{\partial \mathcal{L}}{\partial \ell_t} = 0 \quad \Rightarrow \quad -\theta_t U_\ell(c_t, \ell_t) = \lambda_t \underbrace{A_t F_\ell(k_t, \ell_t)}_{\frac{\partial y_t}{\partial \ell_t} = MPL_t} \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial k_2} = 0 \quad \Rightarrow \quad \lambda_1 = \lambda_2 \underbrace{A_2 F_k(k_2, \ell_2)}_{\frac{\partial y_2}{\partial k_2} = MPK_2} \quad (7)$$

Planner's problem: solution

- Using (5) to substitute λ_t out of (6) and (7) gives:

Proposition

An allocation $(c_1, c_2, \ell_1, \ell_2, k_2)$ is feasible if it satisfies (2) at $t = 1, 2$. It is a solution to the planner's problem—and hence also an equilibrium outcome—if and only if it satisfies:

$$-\frac{U_\ell(c_t, \ell_t)}{U_c(c_t, \ell_t)} = A_t F_\ell(k_t, \ell_t) \quad t \in \{1, 2\} \quad (8)$$

$$\frac{\theta_1 U_c(c_1, \ell_1)}{\theta_2 U_c(c_2, \ell_2)} = A_2 F_k(k_2, \ell_2) \quad (9)$$

- It then remains to solve this system – i.e., to construct the mapping from shocks $\{\theta, A\}$ to outcomes $\{c, \ell, y, k\}$
- We can then ask whether the shocks generate something that looks like business cycles. But first: interpret the optimality cond's.

Efficiency: $MRS = MRT$

- What are the economic interpretations of conditions (8) and (9)?
 - (8) equates the marginal rate of substitution (MRS) between consumption and leisure in t with the corresponding marginal rate of transformation (MRT), which is simply the marginal product of labor (MPL) at t
 - (9) equates the MRS between consumption “today” and consumption “tomorrow” to the corresponding MRT, which is simply the marginal product of capital (MPK) tomorrow
- Conditions (8) and (9) reflect the usual efficiency condition, which requires the equality of MRS and MRT for any pair of goods
 - Here the pairs are consumption and labor today, and consumption today and tomorrow.
 - Note how this combines the margins of both the Robinson Crusoe economy [cons. vs. labor] and the growth model [cons. today vs. tomorrow]

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The effects of demand and supply shocks

- So our objective is now to understand the propagation of **demand** & **supply** shocks in this **RBC economy**
- Our approach:
 - Set $A_2 = 1$ and $\theta_2 = 1$ (“long run”), focus on how outcomes in period 1 (“short run”) vary with A_1 (“supply shock”) and θ_1 (“demand shock”).
 - Aside on interpretation: a high θ_1 means a preference for consuming in the first period, or an increase in the demand for goods today vs. tomorrow. A high A_1 means a higher supply of goods today.
- We will proceed in two steps:
 1. Study effects of the shocks in a benchmark version of the model *without investment*
This will essentially be the Robinson Crusoe economy we studied early on!
 2. Discuss what happens with investment

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Removing investment

- Suppose no investment $k_2 = 0$ and that the production function uses only labor, $F(\ell)$. The resource constraint is then

$$c_1 = y_1 = A_1 F(\ell_1)$$

- Then write the optimality condition for labor

$$-\frac{U_\ell(c_1, \ell_1)}{U_c(c_1, \ell_1)} = A_1 F'(\ell_1)$$

These two equations pin down c_1 and ℓ_1 and hence y_1 in terms of the supply shock A_1 , but the demand shock θ_1 has dropped out.

- Why does the demand shock θ_1 have **no effect** on consumption or employment?
 - Intuition: without investment to link periods, the economy is static and a shock to the relative demand for date $t = 1$ has no effect
 - Later, in competitive eq'm: interest rates fall to induce more consumption today, offsetting the demand shock. I.e., prices neutralize the shock.

What about supply shocks?

- What about the effect of A_1 ? Will provide intuitive discussion here, and a worked-out parametric example on the next slide
- Higher A_1 = labor is more productive. What does that do?
 - **Substitution effect**: return to labor is higher, so work more. **Income effect**: richer overall, so try to both consume more and work less (= more leisure).
 - We will suppose that income effects are not strong, so ℓ_1 increases with A_1 . Then consumption and output $c_1 = y_1 = A_1 F(\ell_1)$ also increase with A_1 .
This will also be the case in our later dynamic models since business cycle is *transitory* up or down, so income effects will be moderate.
- Summary: supply shocks can cause booms as well as recessions, whereas demand shocks are irrelevant.

A worked-out example

- A commonly used preference specification is

$$U(c, \ell) = \frac{c^{1-\gamma} - 1}{1-\gamma} - \frac{\ell^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}, \quad \gamma, \varphi > 0$$

Furthermore for simplicity suppose a linear production function $F(\ell) = \ell$.

- Then the conditions pinning down consumption and employment reduce to

$$c_1^\gamma \ell_1^{\frac{1}{\varphi}} = A_1, \quad c_1 = y_1 = A_1 \ell_1$$

Solving for employment then consumption and output gives

$$\ell_1 = A_1^{\frac{1-\gamma}{\frac{1}{\varphi} + \gamma}}, \quad c_1 = y_1 = A_1^{\frac{\frac{1}{\varphi} + 1}{\frac{1}{\varphi} + \gamma}}$$

Consumption and output are unambiguously increasing in the supply shock A_1 .

- The sign of employment response depends on whether the substitution effect dominates ($\gamma < 1$) or the income effect dominates ($\gamma > 1$).

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Adding investment back in

- We now return to the full model
- Approach: skip the math and focus on the intuition
 - Closed-form solution not very tractable
 - Will study numerical solutions in infinite-horizon economy next time
- Preview: what changes relative to no-investment case?
 - Supply shock still generates something that looks like business cycles. c , ℓ , y , and now k all co-move, as they should
 - Things are even worse for demand shocks: a *positive* demand shock now causes a *recession*!

Demand shocks

- Suppose there is a **positive demand shock**, i.e., θ_1 increases. What happens?
- Households value consumption “today” more than consumption “tomorrow”. How can the planner satisfy this desire?
 - By reducing investment — i.e., transferring goods from tomorrow to today. In short, current consumption c_1 rises but investment k_2 falls.
 - Moreover, households also value leisure today more than leisure tomorrow. This is achieved by reducing employment today. Since employment ℓ_1 falls, output $y_1 = A_1 F(\bar{k}, \ell_1)$ falls.
- Bottom line: increase in θ_1 increases c_1 but decreases ℓ_1 , y_1 , and k_2 .

Supply shocks

- Suppose there is a **positive supply shock**, A_1 increases. What happens?
- The economy is more productive “today” than “tomorrow”. Thus, even fixing factor inputs, today’s production y_1 increases.
 - Increase in production allows for more consumption c_1 .
 - Consumption should be smoothed, so the increase in c_1 is less than 1-for-1 with the increase in y_1 . $k_2 = y_1 - c_1$ invested, so y_2 is also higher, allowing c_2 to be higher.
 - Because the increase in consumption is moderated, there is a weaker income effect on labor supply, so employment ℓ_1 increases, further increasing y_1 .
- Bottom line: increase in A_1 increases c_1 *and* ℓ_1 , y_1 , and k_2 .

Taking stock & outlook

- Two main takeaways from our analysis:
 1. Supply shocks in the RBC model generate something that looks like business cycles – we get broad-based co-movement of all real quantities
 2. Demand shocks don't work: no recession after drop in consumer spending!
So our micro-101 intuition – as well as Keynesian accounts of business cycles – were misleading.
- Where we're going next:
 - RBC model with infinite horizon and competitive eq'm decentralization
Show our conclusions extend + allow us to understand where the demand results come from.
 - Dig deeper into micro consumption and investment problems
Objective: develop more satisfactory account of demand shocks than “impatience.”
 - Figure out how to make demand matter (in the “right” way) in GE
From there we can finally do modern policy analysis: interest rate policy, stimulus checks, ...

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