

Lecture 15: The Real Business Cycle Model

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Real business cycles

- Now: **full-blown RBC model**. Changes relative to last class:
 - Infinite-horizon, rather than just two periods
 - Consider competitive eq'm and thus also get prices. Will however still be equivalent to planning solution.
Yet again: fundamental theorems of welfare economics = invisible hand.
- Roadmap:
 1. Model set-up
 2. Labor-only model
 3. Adding capital: analytical solutions & simulations
 4. Assessment: successes, limitations, and way forward

Outline

1. Model Set-Up

- Competitive Equilibrium
- Planning Problem

2. The Labor-Only RBC Model

3. Full Model Solution

- A Special Case
- Numerical Analysis

4. Assessment

- Successes
- Limitations

Overview

- Same environment as before:

Again featuring demand shocks θ_t and supply shocks A_t . They now follow some general stochastic (e.g., ARMA) processes (vs. deterministic in previous lecture note). Details not important for us.

- **Preferences**

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \theta_{t+s} U(c_{t+s}, \ell_{t+s}) \right]$$

Expectations reflect uncertainty about future consumption, hours worked, and demand. Uncertain because of stochasticity in θ_t and A_t .

- **Technology**

$$y_t = A_t F(k_t, \ell_t)$$

- **Resource constraints**

$$c_t + k_{t+1} = y_t + (1 - \delta)k_t, \quad k_0 > 0 \text{ given}$$

- But now market interactions: price-taking firms & households

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Households

- Households consume, supply labor, and save in capital and risk-free bonds. They take prices as given.

- Preferences**

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^s \theta_t U(c_t, \ell_t) \right]$$

- Constraints**

- Budget:

$$c_t + k_{t+1} + b_{t+1} = (1 - \delta + r_t)k_t + (1 + R_t)b_t + w_t \ell_t + \pi_t, \quad \forall t \geq 0$$

- Feasibility: $\ell_t \geq 0$, $c_t \geq 0$, $k_{t+1} \geq 0$, $b_{t+1} \geq \underline{b}$
- Initial conditions: $b_0 = 0$ given, $k_0 > 0$ given

- The choice variables are the sequences $\{c_t, \ell_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$.

Note: these are now stochastic = random variables. We omit technicalities.

Households

- This gives the date- t Lagrangean

$$\mathcal{L} = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \{ \theta_{t+s} U(c_{t+s}, \ell_{t+s}) + \lambda_{t+s} [(1 - \delta + r_{t+s})k_{t+s} + (1 + R_{t+s})b_{t+s} \right. \\ \left. + w_{t+s}\ell_{t+s} + \pi_{t+s} - c_{t+s} - k_{t+s+1} - b_{t+s+1}] \} \right]$$

- The date- t optimality conditions are

$$\partial c_t : \theta_t U_c(c_t, \ell_t) = \lambda_t \quad (1)$$

$$\partial \ell_t : -\theta_t U_\ell(c_t, \ell_t) = \lambda_t w_t \quad (2)$$

$$\partial k_{t+1} : \lambda_t = \beta \mathbb{E}_t [\lambda_{t+1} (1 - \delta + r_{t+1})] \quad (3)$$

$$\partial b_{t+1} : \lambda_t = \beta (1 + R_{t+1}) \mathbb{E}_t [\lambda_{t+1}] \quad (4)$$

I am using that the bond return is risk-free, so R_{t+1} goes out of the expectation.

Households

- Combining (1) and (2):

$$-U_\ell(c_t, \ell_t) = U_c(c_t, \ell_t)w_t \quad (5)$$

- Equate marginal disutility of work to marginal return (= wage), evaluated at marginal utility of consumption. Echoes the two-period analysis.

- Combining (1) and (3):

$$\theta_t U_c(c_t, \ell_t) = \beta \mathbb{E}_t [\theta_{t+1} U_c(c_{t+1}, \ell_{t+1})(1 - \delta + r_{t+1})] \quad (6)$$

- Equate marginal utility of consumption today and expected marginal utility tomorrow, adjusted for expected returns. Echoes long-run growth analysis.

- Combining (1) and (4):

$$\theta_t U_c(c_t, \ell_t) = \beta(1 + R_{t+1}) \mathbb{E}_t [\theta_{t+1} U_c(c_{t+1}, \ell_{t+1})] \quad (7)$$

- “Arbitrage” condition for returns on bonds and capital. Just now in expected returns.

- The firm problem is essentially unchanged compared to our earlier long-run growth model analysis:

Why? The problem is still static, uncertainty does not matter.

$$\pi_t \equiv \max_{k_t^f, \ell_t^f} [A_t F(k_t^f, \ell_t^f) - r_t k_t^f - w_t \ell_t^f]$$

- Optimal behavior is thus again characterized by the relations

$$\begin{aligned}w_t &= A_t F_\ell(k_t^f, \ell_t^f) \\r_t &= A_t F_k(k_t^f, \ell_t^f)\end{aligned}$$

and as usual profits are zero

Equilibrium

Definition

Given stochastic processes for aggregate demand $\{\theta_t\}_{t=0}^{\infty}$ and supply $\{A_t\}_{t=0}^{\infty}$, an equilibrium is a collection of stochastic paths of prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, household var's $\{c_t, \ell_t, k_t, b_t\}_{t=0}^{\infty}$, and firm var's $\{y_t, k_t^f, \ell_t^f, \pi_t\}_{t=0}^{\infty}$, such that the following are true:

- (i) Given demand, prices, and firm profits, household choices are optimal. Similarly, given supply and prices, firm choices are optimal.
- (ii) All markets clear. That is:

$$\ell_t = \ell_t^f, \quad k_t = k_t^f, \quad b_t = 0, \quad \forall t$$

- Same triplet as before. Only difference – now everything is stochastic.
- Randomness in shocks means randomness in macroeconomic outcomes. **Q**: do the resulting fluctuations on average look like a typical business cycle?

Equilibrium characterization

- To characterize the mapping from shocks to macro outcomes, we proceed as in the growth model to remove prices. This gives (now stochastic!) difference equations:
 - Combining the consumer budget constraint, market-clearing, and firm optimality, we obtain the resource constraint [Make sure you see this!]

$$c_t + k_{t+1} = A_t F(k_t, \ell_t) + (1 - \delta)k_t \quad (8)$$

- Combining the capital Euler equation, firm optimality, and market-clearing, we obtain the aggregate Euler equation

$$\theta_t U_c(c_t, \ell_t) = \beta \mathbb{E}_t [\theta_{t+1} U_c(c_{t+1}, \ell_{t+1}) (1 - \delta + A_{t+1} F_k(k_{t+1}, \ell_{t+1}))] \quad (9)$$

- Combining optimal labor supply and labor demand, we obtain

$$-U_\ell(c_t, \ell_t) = U_c(c_t, \ell_t) A_t F_\ell(k_t, \ell_t) \quad (10)$$

- Combining the system of *expectational* difference equations (8), (9), and (10) with initial conditions and suitable boundary cond's, obtain an eq'm
- Next: first show that planning problem also gives (8) - (10), then solve the system

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Planning problem

- Now consider again a benevolent planner. The problem is as follows:

$$\max_{\{c_t, k_{t+1}, \ell_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \theta_t U(c_t, \ell_t) \right]$$

subject to the resource constraint

$$c_t + k_{t+1} = A_t F(k_t, \ell_t) + (1 - \delta)k_t$$

as well as $k_{t+1}, c_t \geq 0$ and the initial condition giving k_0

- This is identical to the planning problem in the neoclassical growth model, except for (i) stochasticity of θ_t and A_t , and (ii) the labor supply choice
- vs. competitive eq'm: as usual the planner is only bound by the resource constraint, not by the constraints of the market

Planning problem

- The planning problem gives the date- t Lagrangian

$$\mathcal{L} = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \{ \theta_{t+s} U(c_{t+s}, \ell_{t+s}) + \lambda_{t+s} [A_{t+s} F(k_{t+s}, \ell_{t+s}) + (1 - \delta)k_{t+s} - c_{t+s} - k_{t+s+1}] \} \right]$$

- The optimality conditions are

$$\partial c_t : \theta_t U_c(c_t, \ell_t) = \lambda_t \quad (11)$$

$$\partial \ell_t : -\theta_t U_\ell(c_t, \ell_t) = \lambda_t A_t F_\ell(k_t, \ell_t) \quad (12)$$

$$\partial k_{t+1} : \lambda_t = \beta \mathbb{E}_t [\lambda_{t+1} (1 - \delta + A_{t+1} F_k(k_{t+1}, \ell_{t+1}))] \quad (13)$$

- Solving out λ_t , we see that we have arrived at the same conditions as in the competitive eq'm – yet again, invisible hand
 - Thus allocations in the competitive eq'm and planning problem of the RBC model coincide

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Model

- Now consider a special case of the above model, without any capital. Then we just have the following pair of equations:

1. Resource constraint:

$$c_t = y_t = A_t F(\ell_t)$$

2. Optimal labor supply:

$$-U_\ell(c_t, \ell_t) = U_c(c_t, \ell_t) A_t F_\ell(\ell_t)$$

- Note that this is the same as before, in our 2-period model!
 - Capital is the only thing making our infinite-horizon model dynamic – it's the intertemporal savings decision = consumption today vs. consumption tomorrow
 - Without capital it's just a sequence of static economies with a labor supply decision – labor-consumption trade-off. So we just repeat our Robinson Crusoe economy for each t .
- Next: characterize allocations & prices

Allocations

- To arrive at a closed-form solution we return to our special assumptions:

- **Preferences:**

$$U(c, \ell) = \frac{c^{1-\gamma} - 1}{1-\gamma} - \frac{\ell^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}, \quad \gamma, \varphi > 0$$

- **Technology:**

$$y_t = A_t \ell_t$$

- Then we again get

$$c_t^\gamma \ell_t^{\frac{1}{\varphi}} = A_t, \quad c_t = y_t = A_t \ell_t$$

and so

$$\ell_t = A_t^{\frac{1-\gamma}{\frac{1}{\varphi}+\gamma}}, \quad c_t = y_t = A_t^{\frac{\frac{1}{\varphi}+1}{\frac{1}{\varphi}+\gamma}}$$

- We have solved for the entire time path $\{c_t, y_t, \ell_t\}_{t=0}^\infty$ as a function of the time path of A_t and θ_t (which didn't matter)!

Prices

- So far nothing new. Now let's look at the prices that support this allocation. Wages:
 - From the firm problem we get $w_t = A_t F_\ell(\ell_t)$. A_t increases wages, while higher ℓ_t decreases wages if there are decreasing returns to scale.
 - With our as'n's we just have $w_t = A_t$ and so positive supply shocks increase wages
- Now consider the intertemporal price – interest rates:
 - Households can save and borrow in bonds. Interest rates adjust to make sure that they don't want to save or borrow anything in eq'm:

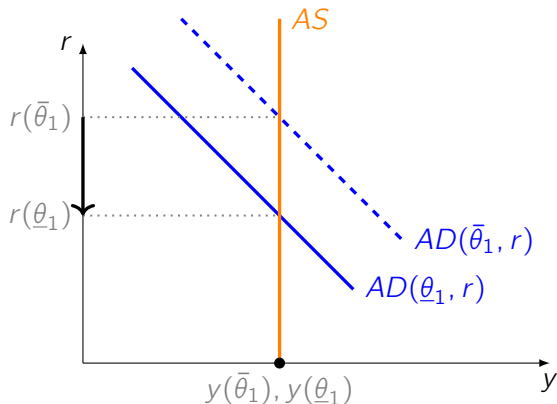
$$1 + R_{t+1} = \frac{1}{\beta} \cdot \frac{\theta_t U_c(c_t, \ell_t)}{\mathbb{E}_t [\theta_{t+1} U_c(c_{t+1}, \ell_{t+1})]}$$

- Thus rates are moved by demand shocks – R moves to offset θ ! E.g., with our preferences:

$$1 + R_{t+1} = \frac{1}{\beta} \cdot \frac{\theta_t c_t^{-\gamma}}{\mathbb{E}_t [\theta_{t+1} c_{t+1}^{-\gamma}]}$$

Note: if only demand shocks then $c_t = c_{t+1}$. Thus $1 + R_{t+1} = \frac{1}{\beta} \frac{\theta_t}{\mathbb{E}_t [\theta_{t+1}]}$.

A visual representation



- Demand decreases from $\bar{\theta}_1$ to $\underline{\theta}_1$. Eq'm rates just drop to offset this and stabilize demand, leaving output unaffected.
- Fully *supply-determined*: vertical aggregate supply curve. Demand only moves prices.

Summary of labor-only economy

- We have affirmed our conclusions from the two-period mini-model:
 - Supply shocks lead to co-movement. They generate something that looks like business cycles [minus investment of course]
 - Demand shocks are irrelevant.

This was as easy as before. Without investment the model is static – sequence of static problems.

- We did however learn something additional about prices:
 - Productivity shocks lead to procyclical wages. Will also tend to lead to procyclical interest rates, as high A_t raises current consumption relative to future consumption.
[Make sure you see this in the algebra.]
 - How is the irrelevance of demand shocks supported? lower demand (for example) is offset by low interest rates, encouraging households to spend more, so nothing happens

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Adding back investment

- We now return to the full economy, featuring investment
- Challenge: analytical solutions in general not available. Our two ways forward:
 1. Analytical solution for a very particular special case
 2. Numerical simulations
- Objective: show that our main conclusions extend
 - **Supply shocks** can generate something that looks like business cycles ...
 - ... while **demand shocks** can't

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A restricted environment

- We will now specialize the RBC model environment in several ways:
 1. **Preferences:** no labor supply disutility (so $\ell_t = \bar{\ell} = 1$ at all times), log utility for consumption (so $\gamma = 1$)
 2. **Technology:** Cobb-Douglas production function ($y_t = A_t k_t^\alpha$), full depreciation ($\delta = 1$)
 3. **Shocks:** only allow for supply shocks, no demand shocks
- The model difference equations then become [verify!]

$$\begin{aligned}\frac{1}{c_t} &= \alpha\beta\mathbb{E}_t\left[\frac{1}{c_{t+1}}A_{t+1}k_{t+1}^{\alpha-1}\right] \\ c_t + k_{t+1} &= A_t k_t^\alpha\end{aligned}$$

Does this remind you of anything we've seen before? Lecture Note 8?

Model solution

- We will guess a constant savings rate:

$$c_t = (1 - s)A_t k_t^\alpha$$

This is why, for this closed-form special case, we can't have demand shocks – with those the savings rate would not be constant.

- Also note that this implies

$$k_{t+1} = sA_t k_t^\alpha$$

- Plug the guess into the Euler equation and cancel terms: from

$$\frac{1}{(1 - s)A_t k_t^\alpha} = \alpha\beta\mathbb{E}_t \left[\frac{1}{(1 - s)A_{t+1} k_{t+1}^\alpha} A_{t+1} k_{t+1}^{\alpha-1} \right]$$

we get

$$s = \alpha\beta$$

Model solution

- We thus overall have:

$$\begin{aligned}k_{t+1} &= \alpha\beta A_t k_t^\alpha, & c_t &= (1 - \alpha\beta)A_t k_t^\alpha \\r_t &= \alpha A_t k_t^{\alpha-1}, & w_t &= (1 - \alpha)A_t k_t^\alpha \\y_t &= A_t k_t^\alpha, & i_t &= k_{t+1} - (1 - \delta)k_t = \alpha\beta A_t k_t^\alpha\end{aligned}$$

Note: I use the notation $i_t = k_{t+1} - (1 - \delta)k_t = k_{t+1}$ to indicate investment.

- We thus indeed get the typical business-cycle patterns:
 - Consumption, investment, and output all co-move, following TFP A_t
 - Prices are also procyclical: in good times (high A_t) wages and interest rates go up
- Note that the model features *amplification*: higher A_t increases output both directly and by inducing more capital accumulation, exactly as we discussed in the growth model

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Numerical analysis

- We now return to the full model and analyze it *numerically*. 3 steps:
 1. We commit to functional form assumptions for preferences, the production function, and the stochastic shock processes:

- For preferences we consider our usual specification:

$$U(c, \ell) = \frac{c^{1-\gamma} - 1}{1-\gamma} - \frac{\ell^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}, \quad \gamma, \varphi > 0$$

- For production we consider Cobb-Douglas: $y_t = A_t k_t^\alpha \ell_t^{1-\alpha}$
 - For the shock processes we assume what's known as an autoregressive (AR) process.
Intuition: shocks move demand/supply for a couple of years.

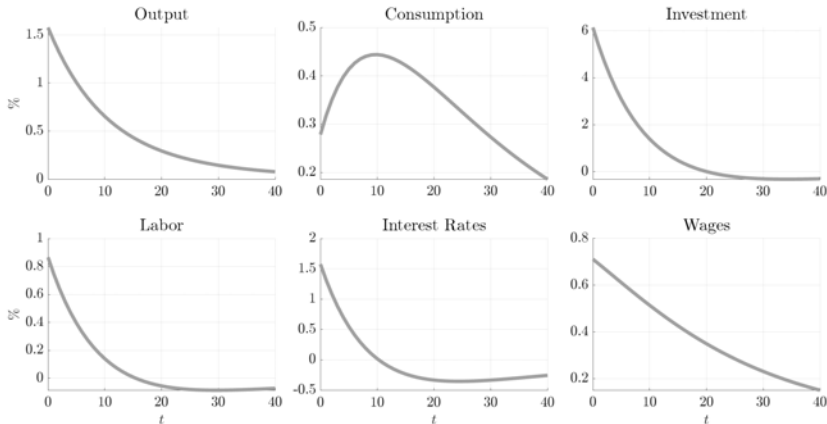
2. We arrive at values for all model parameters. E.g., $\alpha = 1/3$. Comes from micro- and macroeconomic evidence.
3. We *solve* the model. I.e., we find stochastic processes for macro var's so that our difference equations hold. Do this on the computer.

- Next slides: numerical results. No need for you to know how to do this.

Supply shock

At $t = 0$ productivity A increases, then gradually returns to baseline. **Q:** What happens?

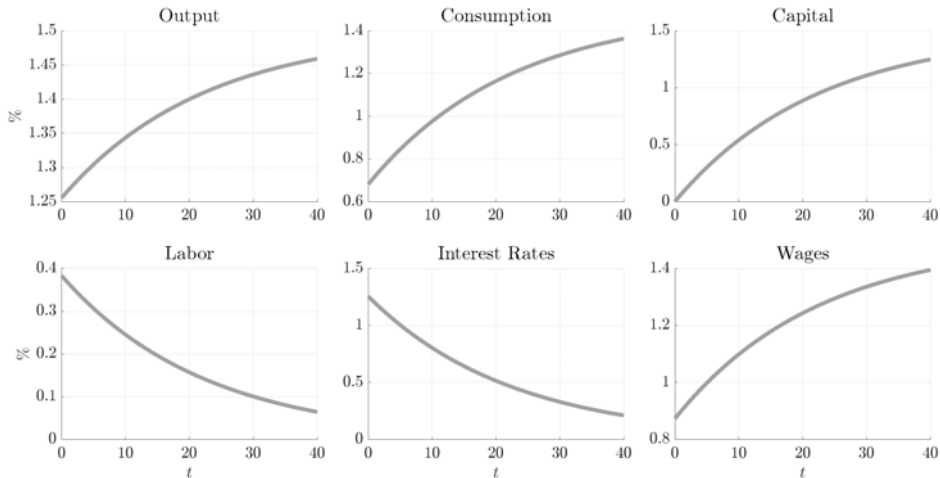
Known as an *impulse response function*. Compute how economy responds to impulse of higher A .



Supply shock: a detour

- Did this remind you of anything? How about if A increases *forever*?
- This is exactly what we studied in the **growth model**! Remember what we saw there:
 - y jumps up immediately, then keeps on increasing. k increases to the new steady state.
 - The impact consumption response was ambiguous, depends on strength of substitution vs. income effects.
- The experiment here was slightly different. A increased **persistently**, not **permanently**. Also we here allowed for a labor supply response. But matters less.
- Next slide: numerical results for *permanent* increase in A
Takeaway: those results echo our earlier phase diagram analysis. Everything does indeed hang together!

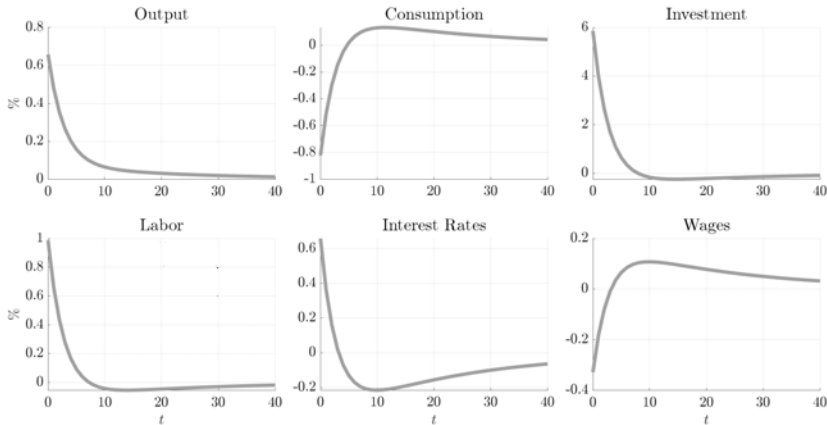
Supply shock: a detour



You will study this case further in the problem set.

Demand shock

At $t = 0$ demand θ drops, then gradually returns to baseline. **Q:** What happens?



Again echoes what we discussed for the two-period model last class.

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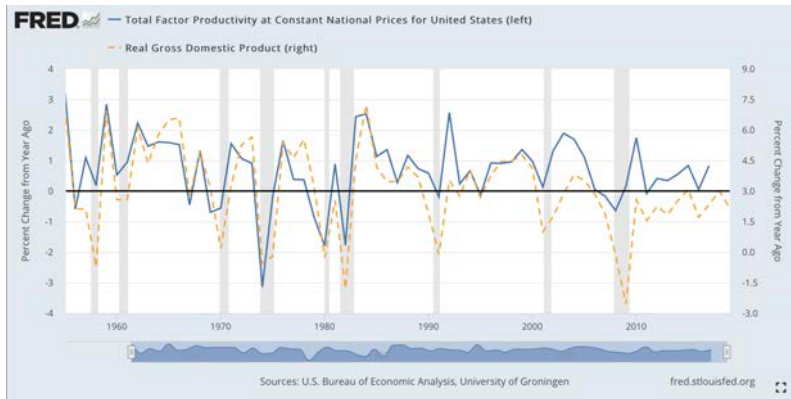
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The RBC paradigm

- Main result of “real business cycle” literature: supply shocks in neoclassical growth model can even *quantitatively* account for business cycles
 - Pioneering work of Kydland-Prescott (1982), Nobel Prize in 2004
 - Provocative finding: model with no role for demand—in which business cycles are efficient responses to changing supply A_t —kind of looks like the data
- We have all the tools to understand how this result was established:
 1. Write down the model, commit to specific functional forms, get model parameters from micro- and macroeconomic evidence
 - For shocks: fit a process that matches empirically observed fluctuations in TFP = Solow residual [next slide: measured cyclical TFP data]
 2. Solve the model, i.e., figure out the effects of supply shocks on macroeconomic outcomes.
 3. Simulate data from the model, and provide a statistical comparison of those data with real world data. E.g., compare the variance of output.

TFP in the data



- We see: TFP in the data is volatile and procyclical
- Feeding that process into the model, we get fluctuations that *quantitatively* look like U.S. business cycles [more or less]. This is the core result of Kydland-Prescott (1982).

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1. Quantitative success?

- Bar to assess “quantitative fit” is relatively low: just broadly match variances and covariances of some key U.S. macro aggregates
- Even that bar is not quite cleared, e.g., hours worked are only half as volatile in the model compared to the data

2. Are **TFP shocks** really “**supply shocks**”?

- Variation in aggregate TFP can result from variation in (i) capacity utilization [workers work overtime, machines get used more] and (ii) efficiency of resource allocation across firms
- But (i) and (ii) may be driven by “demand” shocks, like a credit crunch for firms [like 2008]
- Controlling for these effects, TFP becomes a lot less procyclical ...

Limitations

3. What about **inflation**?

- Want to explain inflation, but RBC model is silent on it – price level is irrelevant
- All that is determined are relative prices. No role for money. Could double all prices and nothing would happen.

We never said anything about money. We normalized the price of output today to 1, and then had relative prices over time (interest rates) and for labor (wages). Remember Robinson Crusoe.

- Of course this means the model could trivially *match* inflation. But that's just because it could match *any* path of inflation.

4. Empirical evidence on the **importance of demand**

- There is compelling empirical evidence that monetary policy can influence real interest rates r and output y – but not in the RBC model
- There are popular accounts that some past recessions were due to deficient demand, e.g. the Great Recession of 2008.

Where do we go from here?

1. Where do demand shocks **come from**?

- We will take a step back and look in more detail at how households make consumption decisions, and firms make investment decisions
- This will allow us to move beyond “demand shocks” as fluctuations in “patience”, like our θ_t today. Can talk about things like credit.

2. How can we get demand shocks **to matter**?

- RBC model is an example of a (largely) supply-determined economy. Classical dichotomy holds, price level irrelevant, monetary policy irrelevant.
- We will allow for demand in general and monetary policy in particular to matter by introducing “nominal rigidities” – make firm prices “sticky”

3. What are the **implications for policy**?

- What are the effects of monetary and fiscal policies in such economies?
- What *should* monetary and fiscal policy do?

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