

Lecture 16: Permanent-Income Consumption

Christian Wolf

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Consumption – overview

- Next two lectures will zero in on **household consumption**. Main reasons:
 1. Will allow us to think about “demand shocks” in a more principled way than just as “household impatience”
 2. Prepare analysis of policy: effect of changes in interest rates (monetary policy) and transfer income (stimulus checks) through household spending
- **Structure:**
 - Today: **permanent-income** consumption model
 - Consumers in our competitive decentralization of the neoclassical growth model were what's called “permanent-income.” Will dig deeper into this.
 - Focus: how consumption responds to interest rate & income changes
 - Next time: borrowing constraints, **hand-to-mouth** consumption

Outline

1. A Simple Two-Period Model

2. Income Changes

Theoretical Predictions

Empirical Evidence

3. Interest Rate Changes

Theoretical Predictions

Empirical Evidence

Environment

- Household lives two periods, $t \in \{0, 1\}$ with discount rate ρ
 - Utility $u(\bullet)$ for consumption at dates $t = 0$ and $t = 1$
 - Constant interest rate, $r_0 = r_1 = r$. Household receives income each period, y_0, y_1 .
 - The household might also start with some assets a_0 (debt if negative). But to simplify notation, we will suppose that $a_0 = 0$.
Think of a_0 as part of y_0 .
- Household decision problem: consume & save
 - More precisely: household chooses consumption, c_0, c_1 , and assets a_1 brought into $t = 1$
 - Note: there is no benefit to saving at $t = 1$, so we set $a_2 = 0$. Implicitly there is a constraint that rules out *borrowing* in period $t = 1$ (“no-Ponzi constraint”).

The household problem

- To summarize, the **household's problem** is

$$\begin{aligned} \max_{c_0, c_1 \geq 0, a_1} \quad & u(c_0) + \frac{1}{1 + \rho} u(c_1) \\ \text{s.t.} \quad & c_0 + a_1 = y_0 \\ & c_1 = (1 + r)a_1 + y_1 \end{aligned} \tag{1}$$

- Note that this is a special version of the consumption-savings problem in the RBC model: two periods, time-invariant return r , exogenous income
- Why are we doing this? zeroing on the consumption-savings problem (without thinking about the supply side of the economy) transparently reveals addt'l economic insights
- Key assumption: a_1 is unconstrained, can be positive or negative
If $a_1 > 0$, then the household is saving, and if $a_1 < 0$, then the household is borrowing.
- Can simplify this problem using the household's *lifetime budget constraint*
Note: this will be possible precisely because a_1 is unconstrained.

Lifetime budget constraint

- Note that there is a budget constraint for each period:

$$\begin{aligned}c_0 + a_1 &= y_0 \\c_1 &= (1 + r)a_1 + y_1\end{aligned}$$

- With a_1 unconstrained, we can eliminate a_1 to get a single **lifetime budget constraint**:

$$\underbrace{c_0 + \frac{1}{1+r} c_1}_{\text{pv of consumption}} = \underbrace{y_0 + \frac{1}{1+r} y_1}_{\text{pv of income}}$$

- The right-hand side is household lifetime wealth, i.e., the present value of the household's lifetime income
 - The left-hand side is the present value of household lifetime consumption
- Note that the two per-period budget constraints imply the lifetime budget constraint. The constraints are *equivalent* only when a_1 is unconstrained.

Simplified household problem

- Using the lifetime budget constraint, we can re-write the household problem as follows:

$$\begin{aligned} \max_{c_0, c_1 \geq 0} \quad & u(c_0) + \frac{1}{1+r} u(c_1) \\ \text{s.t.} \quad & c_0 + \frac{1}{1+r} c_1 = y_0 + \frac{1}{1+r} y_1 \end{aligned} \tag{2}$$

- This is a standard consumer problem, with rel. price of date 1 consumption of $1/(1+r)$
- You already know how to solve this: consumer will equate marginal rate of substitution between c_0 and c_1 to relative price, use budget constraint to determine c_0, c_1 , then recover savings that implements this, given via

$$a_1 = y_0 - c_0$$

Next slides will have the details ...

- Key: as long as a_1 is unconstrained (allowed to be positive or negative), problems (1) and (2) are equivalent

Euler equation

- Equating marginal rates of substitution and the relative price:

Make sure you could derive this from the Lagrangean of the consumer problem.

$$\frac{1}{1 + \rho} \frac{u'(c_1)}{u'(c_0)} = \frac{1}{1 + r}$$

- But this is just a version of the standard Euler equation

$$u'(c_0) = \frac{1 + r}{1 + \rho} u'(c_1)$$

With *constant elasticity* preferences, $u'(c) = c^{-\gamma}$ for $\gamma > 0$, this becomes,

$$\frac{c_1}{c_0} = \left(\frac{1 + r}{1 + \rho} \right)^{1/\gamma} \Leftrightarrow \log \frac{c_1}{c_0} \approx \frac{r - \rho}{\gamma}$$

- Consumption grows if returns are greater than impatience, encouraging the household to save, and vice-versa
- Note that this is close to the Euler equation that we also found in the growth model
Difference: there returns were ultimately linked to the marginal product of capital, here exogenous.

Solution

- To complete the solution, it remains to find c_0, c_1 satisfying (i) the Euler equation

$$\frac{c_1}{c_0} = \left(\frac{1+r}{1+\rho} \right)^{1/\gamma} \quad (3)$$

and (ii) the lifetime budget constraint

$$c_0 + \frac{1}{1+r} c_1 = y_0 + \frac{1}{1+r} y_1 \quad (4)$$

- Note that there are two equations in two unknowns, c_0, c_1 . First equation gives consumption growth, second gives consumption level.
- Will now solve this, giving us c_0, c_1 as a function of y_0, y_1 , and r .
- Next part of the slides: special cases to illustrate economic mechanisms
 1. How does consumption respond to changes in income, y_0 and y_1 ?
 2. How does consumption respond to changes in interest rates, r ?

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Consumption and income changes

- Consider the special case

$$r = \rho = 0$$

The insights are much more general. But this special case is instructive.

- In this case, the Euler equation simplifies to

$$\frac{c_1}{c_0} = 1^{1/\gamma} \quad \Rightarrow \quad c_0 = c_1$$

and the lifetime budget constraint simplifies to

$$c_0 + c_1 = y_0 + y_1$$

- It follows that the solution is

$$c_0 = c_1 = \underbrace{\frac{y_0 + y_1}{2}}_{\text{permanent income}} \equiv y^p$$

In words: consumption each period equals av'g lifetime income = “**permanent income**”

Key force: consumption smoothing

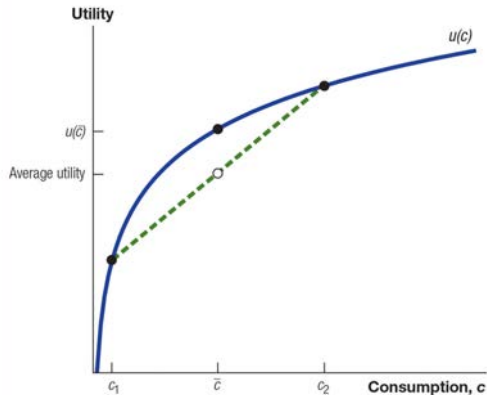
- What's going on? Because of the **concavity** of the utility function $u(c)$, the household prefers a relatively smooth consumption path
- In our special case, where the interest rate compensates for discounting, $r = \rho$, the household chooses a perfectly flat consumption path
- Consequently, the household's current consumption, c_0 , is not tightly linked to fluctuations in her current income, y_0
- Instead, consumption is equal to her **permanent income**

$$c_0 = \frac{y_0 + y_1}{2} \equiv y^p$$

Key force: consumption smoothing

FIGURE 16.2

The Desire to Smooth Consumption



Because $u(c)$ is concave, the utility of the average, $u(\bar{c})$, is greater than average utility.

Implications for saving & borrowing

- We can then back out the household's asset position

$$a_1 = y_0 - c_0 = \frac{1}{2}(y_0 - y_1)$$

- Suppose $y_0 > y^P$, equivalently, $y_0 > y_1$, income is *shrinking*. Then:
 - Consume $c_0 < y_0$
 - Save $a_0 > 0$ to smooth consumption
- Now suppose $y_0 < y^P$, equivalently, $y_0 < y_1$, income is *growing*. Then:
 - Consume $c_0 > y_0$
 - Borrow $a_0 < 0$ to smooth consumption

Intuition for saving & borrowing

- If the household has higher income now than in the future, she *saves* to bring her consumption in the future up to par with consumption in the present.
- Conversely, if the household has lower income now than in the future, she *borrow*s to bring her consumption in the present up to par with her consumption in the future.
- In either case, the household is smoothing consumption, and saving/borrowing is merely the instrument that facilitates this smoothing.

Extension to many periods

- The results extend straightforwardly beyond the two-period case. To see this, suppose the household lives for T periods $\{0, 1, \dots, T-1\}$.

- Continue to assume $r = \rho = 0$. Lifetime budget constraint becomes

$$c_0 + c_1 + \dots + c_{T-1} = y_0 + y_1 + \dots + y_{T-1}$$

- Household's utility function is:

$$u(c_0) + u(c_1) + \dots + u(c_{T-1})$$

- Maximizing utility subject to the budget constraint gives:

Make sure you see this!

$$c_0 = \dots = c_{T-1} = \underbrace{\frac{y_0 + y_1 + \dots + y_{T-1}}{T}}_{T \text{ period permanent income}} \equiv y^p$$

- Thus, for large T , the relationship between y_0 and y^p is even weaker

So we have the same result as before but even more dramatic.

Taking stock

- Key takeaway: because of consumption smoothing, c_0 is equal to *permanent income* y^p as opposed to *current income* y_0
 - Current income y_0 affects c_0 *only* via its effect on y^p
 - This is the **permanent income hypothesis** of Friedman (1957)
- We will now dig deeper into two implications of the model
 1. **Life-cycle**: how is consumption predicted to evolve over a household's life-cycle, from young age to prime working years to retirement?
 2. **Transfers**: how is consumption predicted to respond to lump-sum income gains, like stimulus checks?

Life-cycle implications

- Income makes predictable changes over the life cycle, it is likely to increase early in life and likely to decrease later in life:
 - Young people are likely to have $y_t < y^p$
 - Middle-aged people are likely to have $y_t > y^p$

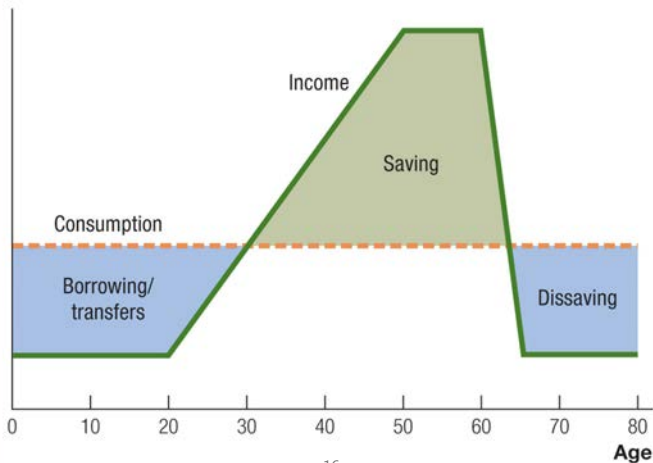
Permanent-income hypothesis says: consumption should not reflect these predictable income movements.

- Instead, people should use borrowing/savings to smooth consumption:
 - Young should borrow against their future
 - Middle-aged should pay back earlier borrowing and save for retirement

Life-cycle implications

FIGURE 11.5

The Life-Cycle Model of Consumption



Transfer implications

- Suppose you receive a (small) lump-sum transfer today, e.g., a stimulus check. How should your consumption respond?
- Permanent-income prediction: very little. Why?

- Recall that

$$c_0 = \underbrace{\frac{y_0 + y_1 + \cdots + y_{T-1}}{T}}_{T \text{ period permanent income}} \equiv y^p$$

- Thus: \$100 lump-sum gain will have little effect on consumption today, because it is smoothed over an entire lifetime
- Suggests that lump-sum transfers will not stimulate consumption much
In fact matters are even worse: if people expect higher taxes in the future, then consumption will not respond at all. Will discuss in the fiscal policy notes. Key concept will be “Ricardian Equivalence.”

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Evidence on life-cycle implications

- Carroll-Summers (1989): study consumption vs. income over the life cycle of households
- Find: **mixed-bag evidence**
 - Consumption is indeed smoother than income:
 - Households seem to be inclined to borrow at young age, when permanent income is typically higher than transitory income
 - They seem to save and plan for retirement at middle age, when permanent income is typically lower than transitory income
 - However, consumption still tracks income too much

Evidence on life-cycle implications

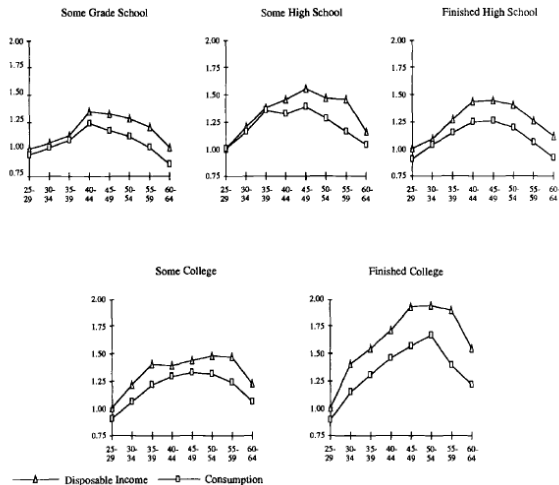


Fig. 10.8a Income and consumption profiles by educational group, 1972-73
CES

Source: Calculations by authors using CES tapes.

Evidence on life-cycle implications

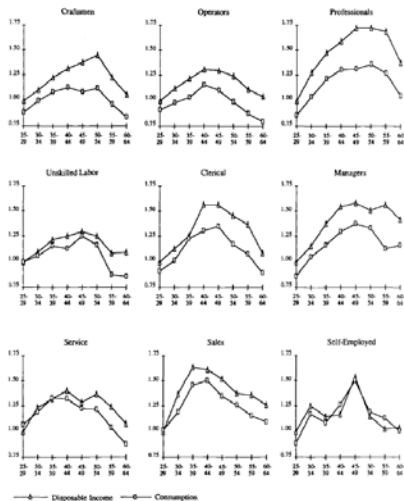


Fig. 10.8b Income and consumption profiles by occupational group, 1972-73
CES

Source: Calculations by authors using CES tapes.

Evidence on transfer implications

- In 2008, the US economy was on the brink of recession. The **Economic Stimulus Act** was passed to stimulate the economy.
 - About \$100 billion sent to 130 million tax filers as rebates. Each individual filer received between \$300-\$600.

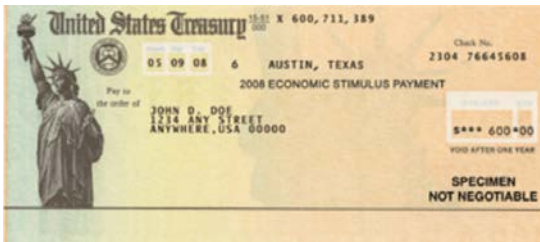


Image is in the public domain.

- Similar [if even bigger] rebates in response to Covid-19
- How did **consumers** respond to these stimulus checks?

Empirical evidence

- Parker, Souleles, Johnson, and McClelland (2013) provide an answer using micro data on individual household consumption
- Key idea: exact timing of the receipt of the rebate was *exogenous*
Depends on the last two digits of the SSN — effectively random.

- *Treatment group*: consumers that received windfall in early May.
- *Control group*: consumers that would receive windfall later (e.g., July).

Example of a “natural experiment:” random assignment of who gets the transfer when.

- Note: both groups knew early on that they were going to receive the rebate, so both of them could re-optimize at the same time.
- PIH: consumption response should be small & identical for the two groups
Make sure you see both points: the “small” and the “identical.”

Empirical evidence

Dependent Variable:	Dollar change in			
	Food	Strictly Non- durables	Non- durable spending	Total spending
Estimation method:	OLS	OLS	OLS	OLS
<i>ESP</i>	0.016 (0.027)	0.079 (0.046)	0.121 (0.055)	0.516 (0.179)

- Coefficient 0.12 means, for each dollar a household received, they spent about 12 cents on non-durables in that same month.
- They spent 51 cents in total including durables (e.g., cars).

Empirical evidence

- Parker et al. thus find **significant effects** on consumption:
 - 12-30% of the rebate spent on non-durable consumption
 - 50-90% of the rebate spent in total—including durables
- These results suggest that the **marginal propensity to consume** (MPC) out of current disposable income (or liquid wealth) is quite high
- Not consistent with the PIH, which predicts that small transitory income shocks should have almost no effect on consumption

The way forward

- What is going on?
 - **Life-cycle**: you have high income potential, i.e., high y^p . What does the PIH suggest you should be doing? Are you doing that?
 - **Transfers**: how did you respond to stimulus checks?
- Two natural explanations:
 1. **Borrowing constraints**: would like to borrow, but can't. So consumption will track current income.
 2. **Bounded rationality**: maybe don't optimize for small income changes?
- Next time: study implications of borrowing constraints

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Interest rates & consumption

- So far we have focused on how consumption c depends on **income y** . We now turn attention to how consumption c depends on **interest rates r** .
- Why? down the road this will be key for understanding the effects of *monetary policy*
 - Monetary policy sets a specific short-term *nominal* interest rate—the Federal Funds Rate
 - But this indirectly affects other nominal rates, such as treasuries and mortgage rates
 - Given *nominal rigidities*, which we will discuss at length in classes to come, changes in nominal rates pass through to changes in real rates
- Today, we abstract from these issues, as well as any general equilibrium considerations.
- We instead treat r as exogenous and simply study the comparative statics of consumption c with respect to r .

Interest rates & consumption

- We will decompose the total effect of r on c into two parts:
 1. **Substitution effect**
 2. **Income/wealth effects**
- Bottom line will be that an increase in r reduces consumption today via 1., while the effect of 2. is ambiguous
- Overall we will conclude:
 - An increase in r tends to reduce c (holding y constant)
 - “Aggregate demand tends to fall when interest rates rise”

This connects with our discussion in Lecture Note 15: negative demand shocks in the RBC model are offset by falling interest rates.

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Revisiting the consumption-savings problem

- Return to the consumer problem, now making an explicit the distinction between the return on initial assets, r_0 , and the return between the today & tomorrow, r ($\equiv r_1$):

$$\begin{aligned} \max_{c_0, c_1 \geq 0} \quad & u(c_0) + \frac{1}{1 + \rho} u(c_1) \\ \text{s.t.} \quad & c_0 + a_1 = a_0(1 + r_0) + y_0 \\ & c_1 = a_1(1 + r) + y_1 \end{aligned}$$

- Equivalently, in terms of the lifetime budget constraint, we can write this as

$$\begin{aligned} \max_{c_0, c_1 \geq 0} \quad & u(c_0) + \frac{1}{1 + \rho} u(c_1) \\ \text{s.t.} \quad & c_0 + \frac{1}{1 + r} c_1 = a_0(1 + r_0) + y_0 + \frac{1}{1 + r} y_1 \end{aligned}$$

- Note: r_0 affects current financial wealth while r affects the relative price of future consumption (i.e., return to saving) and the present value of future income.

For now we treat the two as independent. Revisit later.

Solution

- Recall that the Euler equation is given by

$$u'(c_0) = \left(\frac{1+r}{1+\rho} \right) u'(c_1)$$

With *constant elasticity* preferences, $u'(c) = c^{-\gamma}$ for $\gamma > 0$, this becomes

$$\frac{c_1}{c_0} = \left(\frac{1+r}{1+\rho} \right)^{1/\gamma} \Leftrightarrow \log \frac{c_1}{c_0} \approx \frac{r-\rho}{\gamma}$$

- This gives the *growth* of consumption. Pin down the *level* via the lifetime budget:

$$c_0 + \frac{1}{1+r} c_1 = a_0(1+r_0) + y_0 + \frac{1}{1+r} y_1$$

Recall that once we solve for c_0 and c_1 , saving is implicitly determined by the flow budget constraint, $a_1 = a_0(1+r_0) + y_0 - c_0$.

- Q:** What happens to consumption c_0 (and saving a_1) as r increases?

Substitution effect

- The household still tends to smooth consumption. However, she also responds to r (relative to ρ).
- Specifically, from the Euler equation, a higher r implies *higher consumption growth*. I.e., household increases c_1 relative to c_0 .
- This is a *substitution effect*, a movement along an indifference curve, as the relative price $1/(1+r)$ of c_1 to c_0 changes. The strength of this effect is governed by γ .
- Does this mean that current consumption, c_0 , is necessarily decreasing in r ?

Income and wealth effects

- Not necessarily, because there are also **income/wealth effects**, depending on whether the household is a borrower or a lender
- To see this clearly, let $a_0 = 0$ and $y_1 = 0$, so that all income is in the present. The lifetime budget constraint is now given by:

$$c_0 + \underbrace{\frac{c_1}{1+r}}_{a_1 > 0} = y_0$$

The household is saving, $a_1 > 0$, to finance future consumption.

- As r increases, the household becomes richer in the sense that she earns more on her savings. So she increases c_0 (as well as c_1).
- Equivalently, the present value of the households' planned consumption path declines (by more than her lifetime wealth).

Income and wealth effects

- Of course the converse could also happen—the present value of the households' planned consumption path could decline by less than her lifetime wealth.
- To see this, let $a_0 = 0$ and $y_0 = 0$, so that all income is in the future. The lifetime budget constraint is now given by:

$$\underbrace{c_0}_{a_1 < 0} + \frac{c_1}{1+r} = \frac{y_1}{1+r}$$

Here the household uses debt, $a_1 = -c_0 < 0$, to finance current consumption.

- Higher r makes her poorer as she needs to pay more interest on her debt. As the household is poorer, she tends to decrease c_0 and c_1 .
 - Taken together, these income/wealth effects depend on income vs. planned spending profiles. Is the household a net borrower or lender?

Initial returns

- Finally consider initial returns, r_0 . The key thing to note is that $a_0(1 + r_0)$ enters the consumption-savings problem just like y_0
- We thus know what happens for higher r_0 : cash-on-hand has increased, so consumption will increase. Standard **wealth effect**.
- Note however that there is no substitution effect:
 - What enters the Euler equation is r , not r_0
 - Changing r_0 is giving a lump-sum income gain/loss to the household. Does not affect decisions on the margin.
- Remaining question: how does r_0 respond to r ?

Returns and asset prices

- To link r_0 and r , we need to briefly talk about asset prices.
- Note that $a_0(1 + r_0)$ is the value of your initial wealth inclusive of returns. What is that value, and how will it respond to changes in r ?

- Basic asset pricing insight: price = discounted present value of future dividends.
- This gives

$$\text{wealth} = d_0 + \frac{d_1}{1 + r}$$

where d_0 captures current dividends and d_1 captures future dividends.

- We conclude: a low r increases the value of wealth today, which would also tend to push up consumption, via the wealth effect.

Summary

- We've seen multiple channels on how interest rates affect consumption:
 1. **Substitution effect**: high r decreases c_0 relative to c_1
 2. **Income/wealth effects**: high r increases c_0 and c_1 for net lenders, and decreases consumption for net borrowers
 3. **Initial wealth/asset effects**: high asset prices [through low r] increase c_0 and c_1
- Net effect is complicated. Which effect dominates?
 - **Theory I**: in simple general equilibrium models we have $c = y$ and no savings, so there is only a substitution effect. Unambiguously negative.
If interested, see the discussion in Auclert (2019).
 - **Theory II**: will look at a nice special case to keep in mind, *log utility*
 - Afterwards: empirics

Consumption-savings with log utility

- Suppose the household has log utility, $u(c) = \log c$ (equivalently, $\gamma = 1$).
- Then the household problem is

$$\begin{aligned} \max_{c_0, c_1 \geq} \quad & \log c_0 + \frac{1}{1+\rho} \log c_1 \\ \text{s.t.} \quad & c_0 + \frac{1}{1+r} c_1 = a_0(1+r_0) + y_0 + \frac{1}{1+r} y_1 \end{aligned}$$

- Easy to show that current consumption is given by
Verify this by combining Euler equation and budget constraint.

$$c_0 = \frac{1+\rho}{2+\rho} \underbrace{\left(a_0(1+r_0) + y_0 + \frac{y_1}{1+r} \right)}_{\text{lifetime wealth}}.$$

- We see that c_0 here is unambiguously decreasing in r .

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Empirical evidence

- Empirical evidence suggests that decreases in interest rates boost short-run consumption
Some recent examples: Di Maggio et al. (2017), Agarwal et al. (2017), Chodorow-Reich et al. (2021).
- Main mechanisms:
 1. **Wealth effects** related to initial returns—lower interest rates boost home prices & stock market valuations, increasing consumption
 2. **Substitution effects** + **wealth/income effects** for borrowing high-MPC households
+ outside of our model: mortgage refinancing.
- In our later theoretical analysis of monetary policy we will take as given that short-term rate cuts boost consumer spending
Note: in our stylized models this will always be the case because of the substitution effect.

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