

Lecture 18: Investment

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14.05, Fall 2025

Investment – overview

- Last few lectures: a model of **consumer demand**

- Today: dig deeper into **investment demand**

About 15-20% of GDP. Smaller than consumption, but more volatile, and so potentially key for understanding business cycles.

1. Models

- Recap the investment problem that is embedded in the baseline neoclassical growth model/RBC model

Analogous to what we did last time, when we recapped and further analyzed the consumption problem embedded in the RBC model.

- Two simple extensions: adjustment costs & liquidity-constrained investment

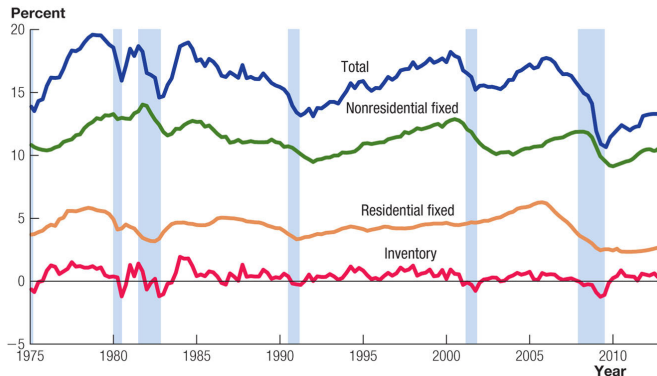
2. Review **empirical evidence** on firm investment behavior

- Putting the pieces together, we will arrive at an improved overall model of **aggregate demand** – consumption + investment

Investment in the data

FIGURE 17.4

Components of Investment (as a Share of GDP)



Residential: New houses, renovations, *Nonresidential:* Equipment, machinery, plant, structures. For intuition in what follows, think nonresidential investment. Residential investment behaves similarly.

Outline

1. A Simple Model of Investment

The Neoclassical Benchmark

Adding Adjustment Costs

Financially Constrained Firms

2. Empirical Evidence

3. Summary: Aggregate Demand

Overview

- We will begin as always with theoretical benchmarks
- First: revisit the **investment problem** embedded in the **neoclassical growth model**
 - This goes back to Lecture Note 9: recall the decentralization where firms own the capital stock and decide how much to invest
 - We will revisit that problem and draw attention to some of its starker implications
Just like last time we drew attention to the income and interest rate response implications of the permanent-income consumption model.
- We will then generalize the baseline in two ways:
 1. Add **adjustment costs**. This will slow down investment.
 2. Make firms **financially constrained** [i.e., no negative dividends]. This will tie investment to current cash flows.

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The investment problem

- Recall the decentralization of the neoclassical growth model with firms owning the capital stock, as covered in Lecture Note 9.
- Ignoring for simplicity the static labor hiring decision, the problem becomes:
You can think of $f_t(\bullet)$ as already embedding the optimal labor choice. Wages would enter this function via the time subscript t .

$$\max_{\{k_{t+1}, \pi_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{s=0}^t \frac{1}{1+r_s} \right) \pi_t \right]$$

subject to the profit equation

$$\pi_t = f_t(k_t) + (1 - \delta)k_t - k_{t+1} \quad \forall t \geq 0$$

as well as $k_{t+1} \geq 0 \quad \forall t \geq 0$, k_0 given

- For future reference: define investment i_t as

$$i_t = k_{t+1} - (1 - \delta)k_t$$

Optimal investment

- Optimal investment is characterized by the optimality condition

$$\underbrace{1 + r_{t+1}}_{\text{marginal cost}} = \underbrace{\mathbb{E}_t [f'_{t+1}(k_{t+1}) + (1 - \delta)]}_{\text{marginal benefit}}$$

For example, with $f_t(k_t) = A_t k_t^\alpha$, this gives

$$k_{t+1} = \left[\frac{\alpha \mathbb{E}_t [A_{t+1}]}{r_{t+1} + \delta} \right]^{\frac{1}{1-\alpha}} \quad (1)$$

- Some observations:
 - Capital demand and so investment is increasing in expectations about future productivity A_{t+1} , and decreasing in the cost of capital r_{t+1}
 - Capital demand does not depend on current conditions (e.g., k_t), nor on expectations about the future beyond $t + 1$. All that matters are current returns vs. current cost of capital.
 - Can show from (1): investment will be *very* responsive to changes in r_{t+1}
1% change in r_{t+1} can move investment by hundreds of %. Argument on board.

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The investment problem

- Now consider adding investment adjustment costs. Specifically, the problem becomes:

$$\max_{\{k_{t+1}, i_t, \pi_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\prod_{s=0}^t \frac{1}{1+r_s} \right) \pi_t \right]$$

subject to

$$\begin{aligned}\pi_t &= f_t(k_t) - i_t - \phi(i_t) \quad \forall t \geq 0, \\ k_{t+1} &= (1 - \delta)k_t + i_t\end{aligned}$$

as well as $k_{t+1} \geq 0 \quad \forall t \geq 0$, k_0 given

- Here $\phi(\bullet)$ captures *investment adjustment costs*
 - $\phi(0) = 0$, and $\phi(i) > 0$ if $i \neq 0$. Example:

$$\phi(i) = \frac{\phi}{2} i^2$$

- Interpretation: costs of installing equipment, planning, ...

Optimal investment

- The investment optimality condition is now

$$\underbrace{(1 + r_{t+1})}_{\text{cost of capital}} \cdot \underbrace{(1 + \phi'(i_t))}_{\text{adjustment cost}} = \underbrace{\mathbb{E}_t [f'_{t+1}(k_{t+1}) + (1 - \delta)(1 + \phi'(i_{t+1}))]}_{\text{marginal benefit}}$$

- For example, if $\phi(i) = \frac{\phi}{2}i^2$ and $i_{t+1} = 0$ [or $\phi = 0$ from tomorrow on], then

$$i_t = \frac{\frac{\mathbb{E}_t [f'_{t+1}(k_{t+1}) + (1 - \delta)]}{1 + r_{t+1}} - 1}{\phi}$$

If ϕ is small then close gap between marginal returns and marginal costs very aggressively. Otherwise more sluggish.

- General observations:
 - Adjustment costs do not change the determinants of investment. But make things more sluggish – equalize marginal costs and returns more slowly.
 - Thus i_t doesn't just depend on date- t costs and returns. Past and future also matter.

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Investment for constrained firms

- Note that, in the investment problem we solved so far, it could be that firm profits in some individual periods t are very negative, i.e., $\pi_t \ll 0$
 - For example, suppose that k_t is very small, but the k_{t+1} that solves

$$1 + r_{t+1} = \mathbb{E}_t [f'_{t+1}(k_{t+1}) + (1 - \delta)]$$

is very large. Then $\pi_t = f_t(k_t) + (1 - \delta)k_t - k_{t+1} \approx -k_{t+1} \ll 0$. Is this feasible?

- This would mean either negative dividends or borrowing by the firm. Maybe financial frictions prevent that. In that case would set

$$i_t = f_t(k_t)$$

i.e., all firm cash-on-hand is immediately re-invested. Thus investment co-moves with *current* productivity/firm cash-on-hand

- Note the analogy to the consumption problem:
 1. Standard **neoclassical firms** are like permanent-income consumers
 2. **Constrained firms** are like hand-to-mouth households

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Empirical evidence & model implications

1. **Cost of capital** and **expected returns** are important to explain investment

- Empirical tests based on a statistic called “Tobin’s Q”: measure of how far return vs. cost of capital ratio is away from 1, i.e., $\frac{\mathbb{E}_t[f'_{t+1}(k_{t+1}) + (1-\delta)]}{1+r_{t+1}}$
- Finding at the aggregate level: investment does broadly co-move with this return vs. cost of capital measure [see next slide]

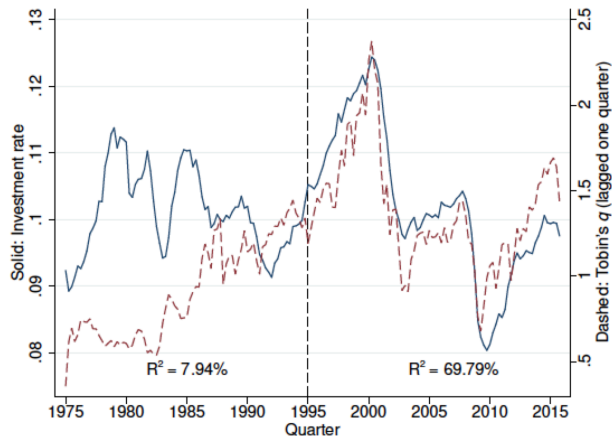
2. But the response of investment to changes in returns/cost of capital is rather **muted**

- Simplest neoclassical model suggests very large elasticities. vs. empirical evidence: 1% change in cost of capital → investment responds by around 5%
For example see Zwick & Mahon (2018) or Gormsen & Huber (2024).
- This calls for relatively large investment adjustment costs ϕ

3. For some firms—e.g., small ones—**current cash flows** also matter a lot

- Inconsistent with simple neoclassical model. Instead calls for firm-level financial frictions.

Tobin's Q and investment



At the aggregate level, investment strongly co-moves with Tobin's Q.

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Our model of investment

- Discussion today suggests an **investment demand function** of the form

$$i_t = \text{constant} + \underbrace{\beta_q \cdot q}_{\text{unconstrained firms}} + \underbrace{\beta_{\text{cash}} \cdot \text{current cash} + \beta_{\text{credit}} \cdot \text{borr. limit}}_{\text{constrained firms}}$$

- **Unconstrained firms** respond to (expected) returns and the current cost of capital. q is decreasing in r and increasing in future A .
 - **Constrained firms** respond to current cash on hand, which is increasing in current A . They also invest less if financial constraints tighten.
- This in particular suggests a deeper model of “**investment demand shocks**”
 - Suppose that borrowing constraints tighten. This will decrease investment demand.
 - This is analogous to our discussion on the consumption side: financial frictions as a theory of demand contractions.

Summary: aggregate demand

- Combining our understanding of consumption and investment, we conclude that both components of **aggregate demand** – both c and i – tend to ...
 - ... decrease with interest rates r
via substitution & wealth effects for c and via q for i
 - ... increase with expectations of future productivity or future output
via permanent income for c and via q for i
 - ... increase with current output
via current income for constrained consumers and via current cash flows for constrained firms
 - ... increase with the borrowing limit, or decrease with a credit crunch
These would be examples of shocks to consumer or firm demand.
- But of course we did already see that, in the neoclassical growth model, demand shocks do not generate **typical business cycles**.
- So up next: adjust the supply side to make demand matter—get **non-vertical supply**

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14.05 Intermediate Macroeconomics
Fall 2025

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