

Lecture 2: A Simple General Equilibrium Model

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Lecture overview

- This lecture will introduce a first, very simple macro model
“Robinson Crusoe” economy. Robinson lives alone on an island, and produces + consumes coconuts.
- It will have the two hallmarks of macro models that we’ll see throughout:
Macroeconomics as the interaction of optimizing (micro) agents in many markets.
 1. Micro-foundations: preferences, technology, constraints, optimizing behavior
 2. General equilibrium: prices in all markets (labor, goods, assets, ...) adjust to balance supply and demand in all markets (“equilibrium”)
- Objectives of this lecture
 - Methodological: learn how to set up and solve general equilibrium macro models
 - Conceptual: optimal resource allocation (“planning solution”) vs. competitive equilibrium, welfare theorems, “invisible hand”

Outline

1. Environment

2. Optimal resource allocation

3. Competitive equilibrium

Definition and characterization

Welfare theorems

Robinson Crusoe island economy

- Setting: one good (coconuts) produced with one input (hours worked)

1. Preferences [utility function]

$$U(c, h) = u(c) - v(h)$$

- c : consumption of coconuts, U is increasing in c
- h : hours worked, U is decreasing in h

2. Technology

$$y = f(n)$$

- y : output, i.e., production of coconuts
- n : hours employed, f is increasing in n

3. Resource constraints

$$c = y, \quad n = h$$

- What do we think will happen? How much is produced, worked, eaten?

Organizing production

We will compare two different ways of organizing production in this economy:

1. Optimal resource allocation

- This is the literal Robinson Crusoe interpretation. Robinson decides how much to work h to maximize utility $U(c, h)$, with consumption given as $c = f(h)$.
- There are no markets here, no prices. Robinson just chooses what to do. Extended to bigger economies, this is a “planning” solution: the allocation gets chosen.

2. Decentralized competitive equilibrium

- Robinson sees prices for coconuts and wages. He decides how much to work and how many coconuts to buy. Crusoe Inc. sees prices for coconuts and wages, and decides how many workers to hire and how many coconuts to sell. [clearly odd, but bear with me ...]
- Prices adjust to clear the output (coconut) and labor markets. This is a market economy.

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Solving the model in one slide

- Robinson Crusoe faces a simple problem: figure out how much to work and thus eat. I.e., he solves

$$\max_{c,n} u(c) - v(n)$$

subject to the resource constraint

$$c = f(n)$$

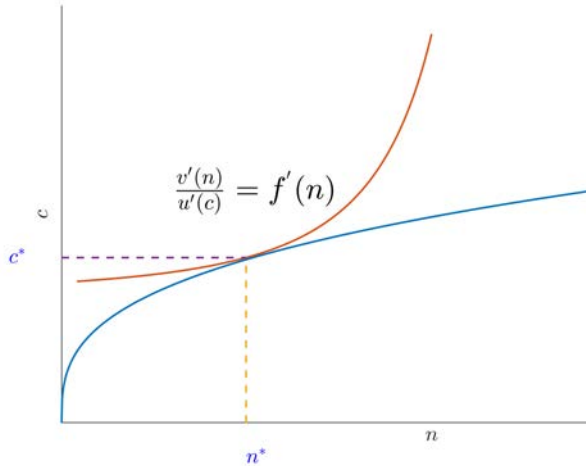
We have used that $c = y$ and $n = h$. That distinction will resurface in the competitive economy.

- This gives the optimality condition

$$\frac{v'(n)}{u'(c)} = f'(n) \tag{1}$$

- (1) together with $c = f(n)$ pins down the optimal (n^*, c^*) [two equations, two unknowns]
- Intuition: Robinson's valuation of time vs. coconuts [LHS] equals his ability to turn time into coconuts [RHS]

Graphical representation



An increase in productivity

- Now suppose that Robinson gets more productive:

$$y = A \times f(n)$$

and A increases from A_0 to $A_1 > A_0$

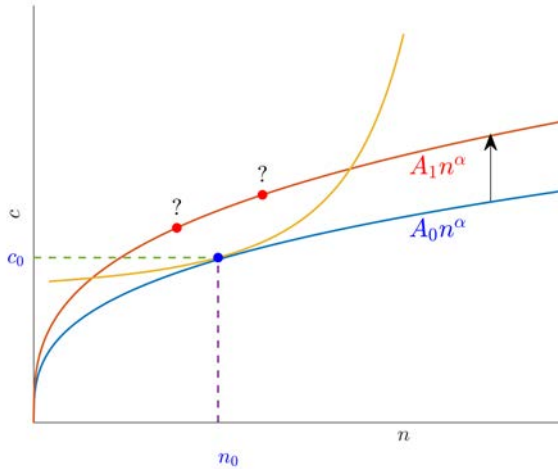
For example, Robinson figures out how to use ladders.

- Will he eat more? Will he work more?

$$\frac{v'(n^*)}{u'(A \times f(n^*))} = A \times f'(n^*) \quad (2)$$

- Could use implicit function th'm on (2) to figure out $\frac{\partial n^*}{\partial A}$. Will do so in later classes.
- Economic intuition:
 - Substitution effect: working gives more coconuts, so leisure is more expensive, so work more
 - Income effect: get more coconuts for given work, so richer, consume more leisure

Graphical representation



A worked-out example

- Let's consider a specific parametric example:

- The production function is

$$f(n) = An^\alpha, \quad 0 < \alpha < 1, A > 0$$

- The utility function is

$$u(c) = \log(c), \quad v(n) = \theta \frac{n^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}, \quad \theta, \epsilon > 0$$

- Then the optimality condition is

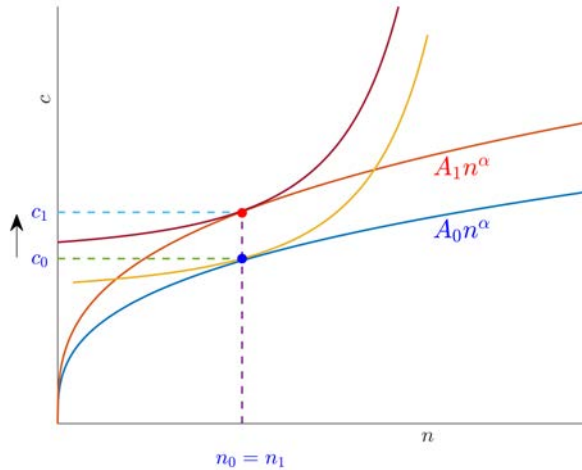
$$\frac{v'(n)}{u'(c)} = f'(n) \quad \Rightarrow \quad \frac{\theta n^{\frac{1}{\epsilon}}}{\frac{1}{c}} = \alpha An^{\alpha-1}$$

Combining this with the production function we get

$$n^* = \left(\frac{\alpha}{\theta}\right)^{\frac{\epsilon}{1+\epsilon}}, \quad c^* = A \left(\frac{\alpha}{\theta}\right)^{\frac{\alpha\epsilon}{1+\epsilon}}$$

Note: n^* does not depend on A . Income and substitution effects exactly cancel ...

Graphical representation



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Competitive equilibrium

- We now consider an alternative set-up: **competitive equilibrium**
- Market structure
 - Robinson Crusoe buys coconuts at price p and supplies labor at wage w . Chooses how much to buy and how much to work. Takes the prices as given—market prices.
 - Crusoe Inc. sells coconuts at price p and hires workers at wage w . Chooses how much to produce/sell and workers to hire. Pays profits Π back to the owner, Robinson.
 - Prices adjust so that, for both coconuts and labor, demand = supply.
- Discussion & interpretation
 - Taken literally, for the Robinson Crusoe economy, this is of course very odd.
 - More palatable interpretation: there are many identical Robinsons, and many identical Crusoe Inc.'s. They are all small, and all take prices as given.

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Decision problems

- **Consumer problem**

- The problem of Robinson is to maximize his utility, taking as given prices, wages and the profits that he receives:

$$\max_{c,h} u(c) - v(h) \quad \text{subject to} \quad pc = wh + \Pi$$

- This gives coconut demand and labor supply functions $c(p, w, \Pi)$ and $h(p, w, \Pi)$

- **Firm problem**

- The problem of Crusoe Inc. is to maximize profits, taking as given prices and wages:

$$\Pi = \max_n pf(n) - wn$$

- This gives coconut supply and labor demand functions $y(p, w)$ and $n(p, w)$

Competitive equilibrium definition

Definition

A competitive equilibrium in the Robinson Crusoe economy is a set of quantities $\{c, y, n, h\}$ and prices $\{p, w\}$ such that:

1. **Utility maximization:** taking as given w , p , and Π , Robinson chooses (c, h) to solve

$$\max_{c, h} u(c) - v(h) \quad \text{subject to} \quad pc = wh + \Pi$$

2. **Profit maximization:** taking as given w and p , Crusoe Inc. chooses n and y to solve

$$\Pi = \max_n pf(n) - wn, \quad y = f(n)$$

3. **Market-clearing:** demand = supply for both coconuts and labor

$$\text{coconut market : } c = y, \quad \text{labor market : } n = h$$

Some remarks

- Price normalization
 - Note that we can, without loss of generality, normalize one of the prices, say $p = 1$. This makes coconuts the “numeraire” of the economy.
 - Simple logic is that only relative prices matter, coconuts vs. hours worked. Can double all prices without anything changing. [verify this!]
- Walras' law
 - In our eq'm characterization it will suffice to check that one market clears (either coconuts or labor), the other will then clear automatically, using budget constraints
 - To see this, suppose that $c = y$, and note that

$$pc = wh + \Pi = wh + py - wn$$

so $h = n$ follows, where I have used the consumer budget and the definition of Π .

Characterization

- I will from now on impose $p = 1$. Utility maximization then gives

$$\frac{v'(h)}{u'(c)} = w \quad (3)$$

and profit maximization gives

$$f'(n) = w \quad (4)$$

- Market-clearing gives

$$y = f(n) \quad (5)$$

$$c = y \quad (6)$$

$$n = h \quad (7)$$

- This is five equations in five unknowns, looking promising ...

Characterization

- Equating (3) and (4), and using (6), we get

$$\frac{v'(n)}{u'(c)} = f'(n)$$

Using (5) and (6),

$$c = f(n)$$

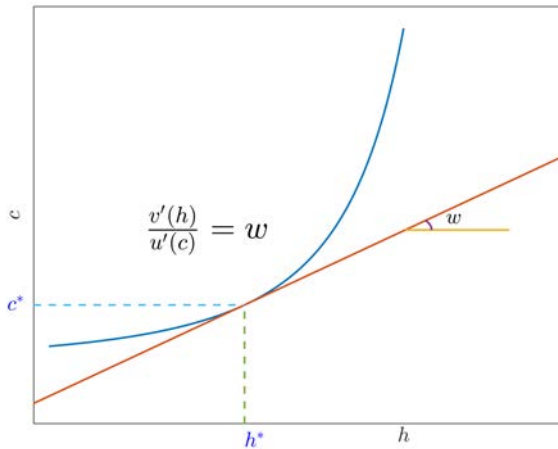
Equilibrium allocation is pair (c, n) that satisfies these two relations.

- But these are exactly the same equations as in the planning problem before, so

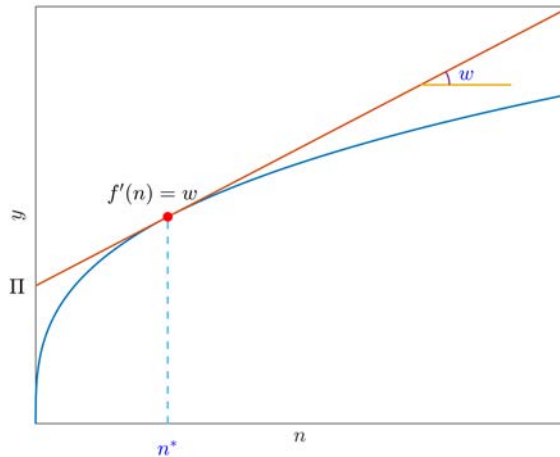
CE allocation = optimal allocation

- What is going on?

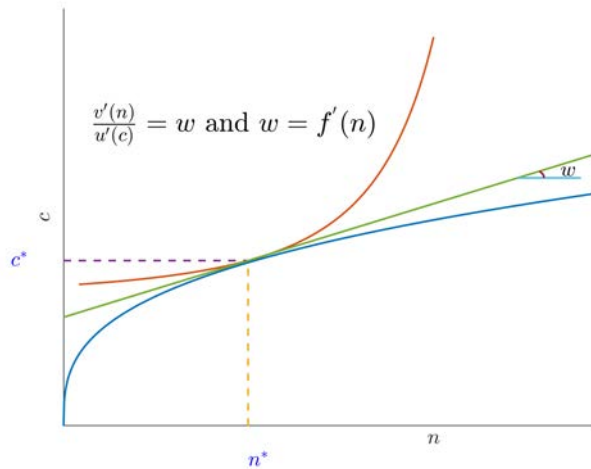
Utility maximization



Profit maximization



Competitive equilibrium



Summary and discussion

- Studied eq'm in which households and firms trade in competitive markets and maximize, taking as given prices. Somehow the result equals the optimal allocation.
- So even though everyone just acts in their self-interest, the resulting allocation is jointly optimal. What is going on?
 - Next up: this is an example of the so-called welfare theorems
 - “Invisible hand”: optimality of market outcomes
- Natural questions: is this true more generally? when does it fail?

Outlook: in this class you'll see some macro models where the welfare theorems apply, and some where they fail. The practically relevant case is the latter, but it's useful to understand the benchmark.

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Pareto efficiency/optimality

- To state the welfare theorems, we introduce a concept of optimality: **Pareto efficiency**
 - An allocation is said to be Pareto efficient/optimal if it is impossible to find another feasible allocation that improves everyone's welfare
Feasible allocation = allocation that respects resource constraints.
 - In our Robinson Crusoe economy there is just one individual (Robinson), and so Pareto efficiency (PE) = maximize his utility

$$u(c) - v(h)$$

subject to feasibility

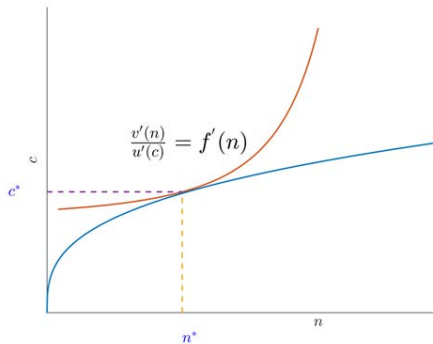
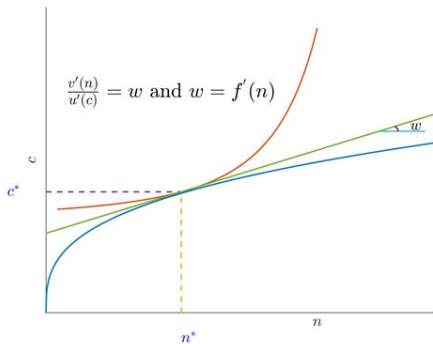
$$y = f(n), \quad c = y, \quad n = h$$

- More generally, in economies with many individuals, PE means that there is no feasible allocation that makes any individual better off without making someone else worse off
- In the Robinson Crusoe economy, **CE allocation** = **PE allocation**. True in general?

First and second welfare theorems

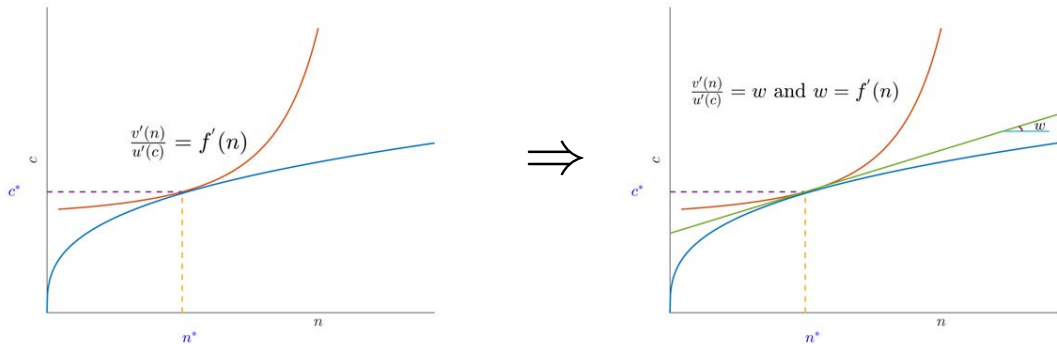
- Rough statements:
 - First welfare th'm: under some strong assumptions CE allocations are PE, i.e., $CE \Rightarrow PE$
 - Second welfare th'm: under some even stronger assumptions any PE allocation can be obtained as a CE, i.e., $PE \Rightarrow CE$
 - Together they establish conditions such that $CE \Leftrightarrow PE$
- Next: some intuition from the Robinson Crusoe economy, then dig a bit deeper into first welfare theorem [= “invisible hand”]
- If you're interested in formal treatments I can provide references.

Graphical intuition: $CE \Rightarrow PE$



Idea: if indifference curves and production possibilities are tangent to price line, then they are also tangent to each other and hence the allocation is optimal

Graphical intuition: $PE \Rightarrow CE$



Idea: if indifference curves and production possibilities are tangent to each other, then can find a price line that separates them in a tangent fashion

First welfare theorem: “invisible hand”

- The first welfare theorem reflects the famous idea of the “invisible hand”
 - In our Robinson Crusoe economy the social optimum was easy to compute. Very hard in large economies with many individuals, het. tastes, rich production technologies, ...
 - First welfare th'm says: just rely on private markets, prices will send the right signals so that “everyone acting optimally $\Rightarrow$ overall optimality”, no need to do any planning
- A sketch of the assumptions underlying it:
 1. Perfect competition [individuals and firms are price takers]
 2. Individuals and firms are rational
 3. No externalities
 4. Perfect information
- How should we think about this result?

First welfare theorem: “invisible hand”

- Should we take away that markets always deliver optimal outcomes and there is no room for gov't intervention?
- No—key idea instead is to understand better *why* gov't intervention may be desirable
 - Violations of the assumptions underlying the theorem are called “market failures” or “frictions”. Everywhere in the real world, and also in the models we will cover.
 - Helps you discipline your thinking: what market failures are relevant for the problem at hand? how can we address them?

What's Next

What's next

- We will now begin with the first main part of the course—long-run growth
- Key question: can saving lead to long-run growth? how much should an economy save?
Today's mini model had a consumption-leisure trade-off. We'll now turn to a different trade-off: optimal savings = consumption today vs. tomorrow. But the methodological tools will be similar.
 - Warm-up: Solow growth model. Fix savings decision, then see how economy evolves over time. Can there be long-run growth?
 - Then turn to the main analysis: Ramsey growth problem. Endogenize savings decision, figure out how much an economy should save.
 - Finally apply similar logic to consumption vs. R&D, also look at implications for inequality, e.g., related to AI.

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